## Notes on Obstfeld-Rogoff Ch. 1

- Open Economy = domestic economy trading with ROW
- Macro level: focus on intertemporal issues (not: multiple good, added later)

OR 1.1-1.2: Small economy = Easiest setting to convey basic ideas

- Two periods $t=1$ (now) and $t=2$ (future)
- Representative agents in each country; given incomes.
- Small economy: takes international prices are given; incl. interest rate r .
$1 /(1+r)=$ relative price of period- 2 consumption
- Individual problem (person i):

$$
\begin{align*}
& U_{1}^{i}=u\left(c_{1}^{i}\right)+\beta u\left(c_{2}^{i}\right), \quad 0<\beta<1 .  \tag{1}\\
& c_{1}^{i}+\frac{c_{2}^{i}}{1+r}=y_{1}^{i}+\frac{y_{2}^{i}}{1+r} . \tag{2}
\end{align*}
$$

- Problem:

$$
\begin{align*}
& \max _{c_{1}^{i}} u\left(c_{1}^{i}\right)+\beta u\left[(1+r)\left(y_{1}^{i}-c_{1}^{i}\right)+y_{2}^{i}\right] . \\
& u^{\prime}\left(c_{1}^{i}\right)=(1+r) \beta u^{\prime}\left(c_{2}^{i}\right),  \tag{3}\\
& \frac{\beta u^{\prime}\left(c_{2}^{i}\right)}{u^{\prime}\left(c_{1}^{i}\right)}=\frac{1}{1+r} . \tag{4}
\end{align*}
$$

- Indifference curve diagram: MRS $=$ relative price.
- Special case of $\beta=1 /(1+r)=>c_{1}=c_{2}$.
- Macroeconomics: Solution to country problem with identical individuals
= Solution to individual problem.
- Notation: Capital letters for country
(in per capita units, or normalize population $=1$ )
- Definition of Current Account $=$ income - consumption $=$ net lending.

$$
\begin{equation*}
C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}, \tag{6}
\end{equation*}
$$

with $B_{t}=$ foreign assets

- Decompose: Trade balance + Net factor incomes from abroad.
- Application to the two period model:

$$
\begin{aligned}
& \qquad C A_{2}=Y_{2}+r B_{2}-C_{2}=Y_{2}+r\left(Y_{1}-C_{1}\right)-C_{2} \\
& =-\left(Y_{1}-C_{1}\right)=-B_{2}=-C A_{1}, \\
& \text { because } \mathrm{B}_{1}=0, \mathrm{~B}_{2}=\mathrm{Y}_{1}-\mathrm{C}_{1}, \mathrm{~B}_{3}=0 .
\end{aligned}
$$

- Distinction: GDP vs. GNP (Data: See Table 1, p.7)
- Here: GDP $=Y_{2}$ vs. GNP $=Y_{2}+r B_{2}$
- Comparison to Autarchy (Key graph: Fig.1.1, p.8)
- Define the autarchy rate $\mathrm{r}^{\mathrm{A}}=$ equilibrium rate in closed economy $\left(\mathrm{Y}_{\mathrm{t}}=\mathrm{C}_{\mathrm{t}}\right)$

$$
\begin{equation*}
\frac{\beta u^{\prime}\left(Y_{2}\right)}{u^{\prime}\left(Y_{1}\right)}=\frac{1}{1+r^{\mathrm{A}}} \tag{7}
\end{equation*}
$$

- Special case of $\beta=1 /(1+r)$ with $r=$ world interest rate.

$$
\frac{u^{\prime}\left(Y_{2}\right)}{u^{\prime}\left(Y_{1}\right)}=\frac{1+r}{1+r^{A}}
$$

- If $\mathrm{r}^{\mathrm{A}}>\mathrm{r}$, then current resources are scarce $=>$ borrow; if $\mathrm{r}^{\mathrm{A}}<\mathrm{r}$, lend.
- Variations in endowments: $Y_{1}$ up or $Y_{2}$ down $=>r^{A}$ down, borrow less Find $r^{A}=r$, iff $Y_{1}=Y_{2}$. Only output fluctuations motivate $C A<>0$.


## - Principle of comparative advantage:

- "import" goods that have a relatively high domestic price (here $\mathrm{C}_{1}$ if $\mathrm{r}^{\mathrm{A}}>\mathrm{r}$ )
- welfare gain if $\mathrm{r}^{\mathrm{A}}<>\mathrm{r}$, regardless of sign.


Figure 1.1
Consumption over time and the current account

- Extension to government consumption G:
- Assume balanced budget, lump-sum taxes, Ricardian neutrality.
- G exogenous or separable in utility

$$
\begin{align*}
& C_{1}+\frac{C_{2}}{1+r}=Y_{1}-G_{1}+\frac{Y_{2}-G_{2}}{1+r}  \tag{8}\\
& C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}-G_{t}
\end{align*}
$$

- Effects of variations in G like reductions in Y.
- Caveat: effects differ if $u(C, G)$ is non-separable
- Extension to production model
$\mathrm{Y}=\mathrm{F}(\mathrm{K})$, holding labor input constant. Ignore depreciation.

$$
\begin{equation*}
K_{t+1}=K_{t}+I_{t} \tag{11}
\end{equation*}
$$

- Budget equation:

$$
\begin{align*}
& B_{t+1}+K_{t+1}-\left(B_{t}+K_{t}\right)=Y_{t}+r_{t} B_{t}-C_{t}-G_{t} \\
& C A_{t}=B_{t+1}-B_{t}=Y_{t}+r_{t} B_{t}-C_{t}-G_{t}-I_{t} \tag{12}
\end{align*}
$$

- Define national savings:

$$
\begin{align*}
& S_{t} \equiv Y_{t}+r_{t} B_{t}-C_{t}-G_{t}  \tag{13}\\
& C A_{t}=S_{t}-I_{t} \tag{14}
\end{align*}
$$

- Two period model (See Figure 1.3, p.20)

$$
\begin{align*}
& B_{2}=Y_{1}-C_{1}-G_{1}-I_{1} \\
& -B_{2}=Y_{2}+r B_{2}-C_{2}-G_{2}-I_{2} \\
& C_{1}+I_{1}+\frac{C_{2}+I_{2}}{1+r}=Y_{1}-G_{1}+\frac{Y_{2}-G_{2}}{1+r} .  \tag{15}\\
& I_{2}=K_{3}-K_{2}=0-K_{2}=-K_{2} . \\
& \max _{C_{1}, I_{1}} u\left(C_{1}\right)+\beta u\left\{(1+r)\left[F\left(K_{1}\right)-C_{1}-G_{1}-I_{1}\right]\right. \\
& \left.\quad+F\left(I_{1}+K_{1}\right)-G_{2}+I_{1}+K_{1}\right\} . \tag{16}
\end{align*}
$$

- Optimality condition:

$$
F^{\prime}\left(K_{2}\right)=r
$$

$=>$ separation of consumption and investment choices!


Figure 1.3
Investment and the current account

- Comparison to Autarchy:

$$
\begin{aligned}
& C_{2}=F\left[K_{1}+F\left(K_{1}\right)-C_{1}\right]+K_{1}+F\left(K_{1}\right)-C_{1} \\
& \frac{\mathrm{~d} C_{2}}{\mathrm{~d} C_{1}}=-\left[1+F^{\prime}\left(K_{2}\right)\right]
\end{aligned}
$$

- Autarchy point: MRS = marginal product of capital.
- Characterization of optimal CA: Borrow iff $\mathrm{r}^{\mathrm{A}}>\mathrm{r}$ !
- New motive to borrow: whenever $F^{\prime}(\mathrm{K})$ is high.


## OR 1.3: World economy with two region

- both "large" - meaning domestic changes affect world prices
- Assume savers in both regions take r as given $=>$ Competitive behavior
- Endowment economy without government. (Foreign variables $=*$ )
- Goods market equilibrium: $\quad \mathrm{CA}+\mathrm{CA}^{*}=0$.

$$
Y_{t}+Y_{t}^{*}=C_{t}+C_{t}^{*} . \quad S_{t}+S_{t}^{*}=0 .
$$

- Example in Figure 1.5: $\mathrm{S}=\mathrm{S}(\mathrm{r}), \mathrm{S}^{*}=\mathrm{S}^{*}(\mathrm{r})=>$ Equilibrium r .


## Example with $\mathrm{r}^{\mathrm{A}}>\mathrm{r}^{\mathrm{A}^{*}}$ : Home $\mathrm{S}=\mathrm{CA}>0$.



Figure 1.5
Global exchange equilibrium

- Behavior of savings functions depends on the elasticity of intertemporal substitution ( $\sigma$ ).

$$
\mathrm{d} \log \left(\frac{C_{2}}{C_{1}}\right)=\sigma \mathrm{d} \log (1+r) . \quad \sigma(C)=-\frac{u^{\prime}(C)}{C u^{\prime \prime}(C)}
$$

- CES preferences:

$$
u(C)=\frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad \sigma>0
$$

- Impact of changes in $r$ on consumption: Income + substitution effect

$$
\frac{\mathrm{d} C_{1}}{\mathrm{~d} r}=\frac{\left(Y_{1}-C_{1}\right)-\sigma C_{2} /(1+r)}{1+r+\left(C_{2} / C_{1}\right)}
$$

- OR discuss wealth effect = Impact of $r$ on PV of income
- commonly included in income effect
- Extension to production model

$$
Y=A F(K), \quad Y^{*}=A^{*} F^{*}\left(K^{*}\right)
$$

- Market equilibrium:

$$
\begin{aligned}
& Y_{1}+Y_{1}^{*}=C_{1}+C_{1}^{*}+I_{1}+I_{1}^{*} \\
& S_{1}+S_{1}^{*}=I_{1}+I_{1}^{*} \\
& C A_{1}+C A_{1}^{*}=0
\end{aligned}
$$

- Figure 1.7: Savings - investment diagrams in two countries.

$$
\frac{\mathrm{d} C_{1}}{\mathrm{~d} r}=\frac{\left(Y_{1}-C_{1}-I_{1}\right)-\sigma C_{2} /(1+r)}{1+r+\left(C_{2} / C_{1}\right)}
$$

- Impact of productivity changes:

$$
\left.\frac{\mathrm{d} I_{1}}{\mathrm{~d} A_{2}}\right|_{r \text { constant }}=-\frac{F^{\prime}\left(K_{2}\right)}{A_{2} F^{\prime \prime}\left(K_{2}\right)}>0
$$

Example with CA surplus in home country


Figure 1.7
Global intertemporal equilibrium with investment

- Application 1: Lower discount factor in Home: SS shifts left.
- Application 2: Higher current output in Home: SS shifts right.
- Application 3: Higher future productivity in Home: SS->left; II->right.


Figure 1.8
A rise in future Home productivity

- Application 4: Higher discount factor in Foreign: $S * S *$ shifts right.
- Broader question \#1: What may explain the U.S. current account deficit?
- Bernanke's hypothesis: "The Global Savings Glut"

Current account balance as a percent of GDP, 1960-2005


- Context: Growing international trade. Growing financial integration


- Potentially relevant disturbances to the current account:
- Slow economic growth in Japan \& Europe: Low consumption, high savings.

Low foreign demand for U.S. goods.

- Relatively good investment opportunities in the U.S.?
(Problem: Substantial share went into housing)
- Higher oil prices: More saving by oil exporters.
- Increased saving by developing countries:

A puzzle: LDCs with low capital should have high MPK!
Risk aversion ("precautionary saving")? Political risk?

- Observation: Interest rates were unusually low in early 2000s
- Bernanke's conclusion: "A World Saving Glut"
- Shift right in foreign supply of savings $=>$ low world interest rate.


## Real Interest Rates



- Implication of CA deficit: Declining net asset position.

Has the US net asset position declined at an exponential rate?
Find: Surprising stability - until 2008.


## Data Analysis

$$
\underline{2006}=\text { typical year } \quad(\text { vintage data })
$$

$\underline{2008}=$ exception or break?

| Dec. 2006 | US Assets | US Liabilities | Net Position |
| :--- | ---: | ---: | ---: |
| Private | 12,284 | 12,346 | -62 |
| FDI | 2,856 | 2,099 | 756 |
| Portfolio Equity | 4,252 | 2,539 | 1,713 |
| Portfolio Other | 5,177 | 7,708 | $-2,530$ |
| Official | 292 | 2,770 | $-2,478$ |
| Total | 12,576 | 15,116 | $-2,540$ |


| Dec. 2008 | US Assets | US Liabilities | Net Position |
| :--- | ---: | ---: | ---: |
| Private | 12,505 | 13,021 | -516 |
| FDI | 3,699 | 2,647 | 1,052 |
| Portfolio Equity | 2,851 | 1,838 | 1,014 |
| Portfolio Other | 5,955 | 8,537 | $-2,581$ |
| Official | 918 | 3,871 | $-2,954$ |
| Total | 13,423 | 16,892 | $-3,469$ |


| Net Position Dec. 2005 |  |  | $\mathbf{- 2 , 2 3 8}$ |
| :--- | ---: | ---: | ---: |
| US assets | 10,444 |  |  |
| US liabilities | $-12,683$ |  | -812 |
| Current account balance | $\mathbf{- 8 5 5}$ |  |  |
| Everything but asset incomes | 647 | $6.2 \%$ |  |
| Income on US assets | -604 | $4.8 \%$ |  |
| Income paid on US liabiities |  |  | 532 |
| Changes in Valuation, net: | 1,106 | $10.6 \%$ |  |
| On US assets | -574 | $4.5 \%$ |  |
| On US liabilities |  |  | $\mathbf{- 2 1}$ |
| Statistical Discrepancy\&Capital Balance |  |  | $\mathbf{- 2 , 5 4 0}$ |


| Net Position Dec. 2007 |  |  | $\mathbf{- 2 , 1 4 0}$ |
| :--- | ---: | ---: | ---: |
| US assets | 15,791 |  |  |
| US liabilities | $-17,931$ |  | $\mathbf{- 7 0 6}$ |
| Current account balance | -832 |  |  |
| Everything but asset incomes | 762 | $4.8 \%$ |  |
| Income on US assets | -636 | $3.5 \%$ |  |
| Income paid on US liabiities | $-2,397$ | $-15.2 \%$ | -824 |
| Changes in Valuation, net: | 1,573 | $-8.8 \%$ |  |
| On US assets |  |  | 201 |
| On US liabilities |  |  | $\mathbf{- 3 , 4 6 9}$ |
| Statistical Discrepancy\&Capital Balance |  | $-10.4 \%$ |  |
| Net Position Dec.2008 | $-5.2 \%$ |  |  |
| Memo: Total return on US assets |  |  |  |
| $\quad$ Total return on US liabilities |  |  |  |

- Applied question \#2: To what extent is capital investment financed abroad?
- The Feldstein-Horioka puzzle:

The Relaton betwen Domath Saving Rabier and
Dotedtic Frevitudy Rates

| Sample periad | Gross saving and ünestment |  |  | Net saving and invetment |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cotatint | $3 / T$ | E | Camestrt | SY | $\underline{S N}^{2}$ |
| 1960-74 | $\begin{aligned} & 0 \cdot 095 \\ & (02018) \end{aligned}$ | $\frac{099_{7}}{(0.324)}$ | 4911 | $\begin{aligned} & a \cdot n y \\ & n+3 \end{aligned}$ | $\begin{gathered} 9.938 \\ 4095 \end{gathered}$ | 687 |
| 4960-64 | $\begin{gathered} 6029 \\ (4015) \end{gathered}$ | 6geg <br> (006to | 094 | $\begin{aligned} & 00 t 7 \\ & \text { [00tit } \end{aligned}$ | $\begin{aligned} & 0.14 \\ & 50674 \end{aligned}$ | 691 |
| 495-59 | $4+098$ | $\begin{aligned} & 4672 \\ & \{0 \mathrm{cos}\} \end{aligned}$ | 063 | $\begin{aligned} & 0+02 a \\ & (0+020) \end{aligned}$ | $\begin{gathered} 0+9 \mathrm{~B} \\ \cos 133 \end{gathered}$ | 45 |
| 1989-74 |  | - 017 <br> (aroget | -18 | $\begin{gathered} \text { and } \\ \text { (oven } \end{gathered}$ | $\begin{gathered} 0+3^{2} \\ (0-c a y) \end{gathered}$ | 083 |

- OR 1.4: Optimal taxation in a "large" economy

Home period 2 consumption, $\mathrm{C}_{2}$


Figure 1.11
The optimal tax on foreign borrowing

- Supply of foreign savings:

$$
S_{1}^{*}(r)=Y_{1}^{*}-C_{1}^{*}(r)=\frac{\beta^{*}}{1+\beta^{*}} Y_{1}^{*} \frac{1}{\left(1+\beta^{*}\right)(1+r)} Y_{2}^{*}
$$

- Offer curve:

$$
1+r=\frac{Y_{2}^{*}}{\left(1+\beta^{*}\right)\left(Y_{1}-C_{1}\right)+\beta^{*} Y_{1}^{*}}
$$

- Welfare problem is to maximize:

$$
C_{2}=Y_{2}+\frac{Y_{2}^{*}}{\left(1+\beta^{*}\right)\left(Y_{1}-C_{1}\right)+\beta^{*} Y_{1}^{*}}\left(Y_{1}-C_{1}\right)
$$

- Optimal strategy of borrower: Reduce borrowing relative to competitive amount => Borrow at reduced interest rates. Welfare gain. Loss abroad. Implementation: Tax.
- OR 1.5: Factor price equalization via labor mobility
- Savings decision in period 1; labor allocation in period 2

$$
\begin{aligned}
& C_{1}=Y_{1}-K_{2} \\
& C_{2}=L_{2} f\left(K_{2} / L_{2}\right)-w\left(L_{2}-L^{\mathrm{H}}\right)+K_{2}
\end{aligned}
$$

- Constant returns to scale: international wage $w$ determines $K / L=k$.
- FOC:

$$
u^{\prime}\left(C_{1}\right)=\beta\left[1+f^{\prime}\left(k_{2}\right)\right] u^{\prime}\left(C_{2}\right)
$$

- Autarchy line:

$$
C_{2}=F\left(Y_{1}-C_{1}, L^{\mathrm{H}}\right)+Y_{1}-C_{1} .
$$

- With mobility:

$$
C_{2}=[1+r(w)]\left(Y_{1}-C_{1}\right)+w L^{\mathrm{H}}
$$

Period 2 consumption, $\mathrm{C}_{2}$


Period 1 consumption, $\mathrm{C}_{1}$

Figure 1.12
Trade in labor services

Table 1: The Impact of Capital and Labour Mobility on Taxes - A Numerical Example

| Mobility of capital: Mobility of labour: | Fixed <br> Fixed |  | Mobile Fixed |  | Mobile Mobile |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Externality: | $1 \%$ | $2 \%$ | $1 \%$ | $2 \%$ | $1 \%$ | $2 \%$ |
|  | Elasticities with respect to (l- $\tau$ ) |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Capital tax | 0.38 | 0.38 | 1.64 | 1.64 | 44.30 | 22.90 |
| Labour tax | 0.12 | 0.12 | 0.50 | 0.50 | 100.00 | 50.00 |
| Labour force |  |  |  |  |  |  |
| Capital tax | 0.05 | 0.05 | 0.21 | 0.21 | 42.90 | 21.30 |
| Labour tax | 0.45 | 0.45 | 0.50 | 0.50 | 100.00 | 50.00 |
| Capital-labour ratio |  |  |  |  |  |  |
| Capital tax | 0.33 | 0.33 | 1.43 | 1.43 | 1.43 | 1.43 |
| Labour tax | -0.33 | $-0.33$ | 0.00 | 0.00 | 0.00 | 0.00 |
| Output |  |  |  |  |  |  |
| Capital tax | 0.15 | 0.15 | 0.64 | 0.64 | 42.70 | 21.60 |
| Labour tax | 0.35 | 0.34 | 0.49 | 0.49 | 98.60 | 49.30 |

Note: Values $>1$ are highlighted in bold.
Source: author's calculations

- Example (Bohn 2006): Small economy with congestion effect
- TFP depends on absolute population with elasticity $\varepsilon$
- Compute responses to tax changes

