

## **Econ 594SA: Organization**

- Class Page: [www.econ.ucsb.edu/~bohn/Summer/SummerMacro.html](http://www.econ.ucsb.edu/~bohn/Summer/SummerMacro.html)
  - Information is updated throughout the quarter.
  - Check for announcements. Class page announcements are assumed known.
- Open door policy for graduate students.
  - Official office hours: MW 2:30-3:30. NH3016.
- E-mail: [bohn@econ.ucsb.edu](mailto:bohn@econ.ucsb.edu). Put “Econ 594” in the subject line.
- Grading for Part I: two problem sets, plus “midterm” exam.

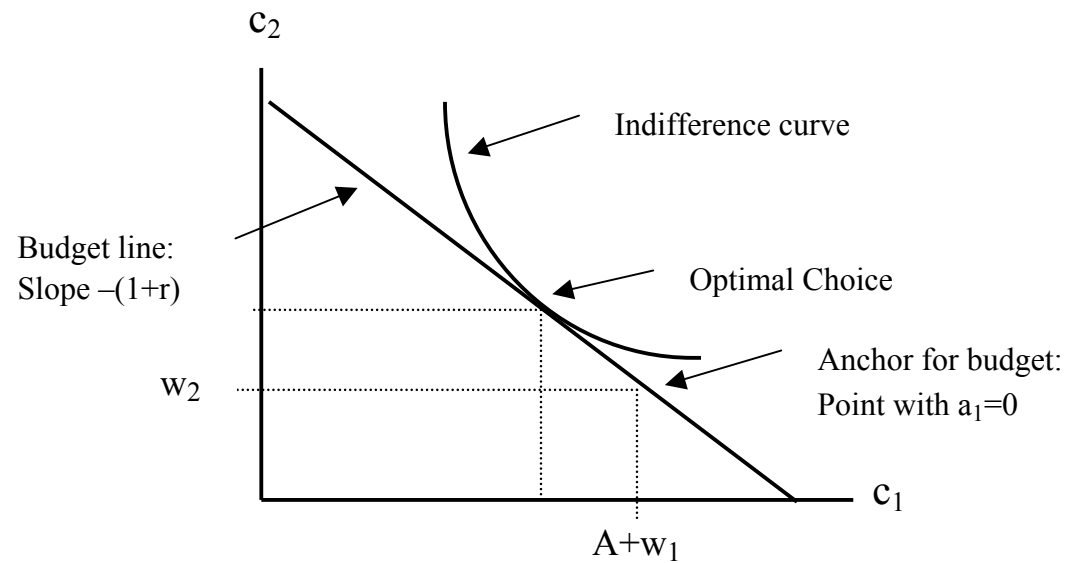
## Macroeconomics

- Objectives of macroeconomics:
  - To analyze the economy as a whole, to explain economic **growth** and economic **fluctuations**, and to assess economic **policy**.
- Outline of 594SA – Part I
  - **Introduction**: Intertemporal choice problems.
    - Tools: Constrained optimization. Graphical analysis. (Known already?)
  - **The Solow Model**: The Mechanics of Economic Growth.
    - Tools: Differential equations. Comparative statics. Linear approximations.
  - **Optimal Growth**: Consumption and savings/investment choices over time.
    - Tools: Optimal control. Systems of differential equations. Phase diagrams.
    - Applications to Fiscal Policy.
    - If time: Application to money.

## Intertemporal Choice: Consumption

- Individual decision problem:
  - Given a series of wage incomes  $w_t$ . Given real interest rate  $r$  (assume constant).
  - Individuals choose consumption  $c_t$  and asset holdings at
  - Subject to budget equations:  $a_t = (1 + r)a_{t-1} + w_t - c_t$
  
- Building intuition: Two-period setting with graphical analysis. Then generalize.
  - Consumption now ( $c_1$ ) vs. consumption later ( $c_2$ ).
  - Assume given initial wealth  $A = (1 + r)a_0$ .
  
- Budget equations imply an **intertemporal budget constraint (IBC)**:
  - use  $a_1 = (1 + r)a_0 + w_1 - c_1 = A + w_1 - c_1$  and
  - $a_2 = (1 + r)a_1 + w_2 - c_2$
  - impose the **terminal condition**  $a_2 = 0$ :
    - $\Rightarrow a_1 = -\frac{1}{1+r}[w_2 - c_2] \Rightarrow 0 = A + w_1 - c_1 + \frac{1}{1+r}[w_2 - c_2]$
    - $\Rightarrow$  IBC:  $c_1 + \frac{1}{1+r}c_2 = w_1 + \frac{1}{1+r}w_2 + A.$
  - Means: **Present value of consumption = Present value of income plus initial wealth.**

## Two Periods: Graphical Analysis



- Budget line has slope  $-(1+r)$ . Increase in  $r \Rightarrow$  steeper slope.

Feasible set: Area under the budget line.

- Endowment point is  $(A+w_1, w_2)$ . Higher  $A, w_1, w_2 \Rightarrow$  budget line shifts “out”.

## Two Periods: Math

- Optimization problem: maximize  $U = u(c_1) + \beta u(c_2)$

- subject to IBC: 
$$c_1 + \frac{1}{1+r} c_2 = A + w_1 + \frac{1}{1+r} w_2$$

- Approach #1: substitute constraint into objective. Problem is:

$$\text{Max} \quad U = u\left(A + w_1 + \frac{1}{1+r} w_2 - \frac{1}{1+r} c_2\right) + \beta u(c_2).$$

$$\Rightarrow \text{FOC for } c_2: -\frac{1}{1+r} u'(c_1) + \beta u'(c_2) = 0 \quad \Rightarrow \frac{1}{1+r} u'(c_1) = \beta u'(c_2)$$

- Approach #2: use Lagrangian; define shadow value =  $\lambda$ . Problem is:

$$\text{Max} \quad L = u(c_1) + \beta u(c_2) + \lambda \cdot \left(A + w_1 + \frac{1}{1+r} w_2 - c_1 - \frac{1}{1+r} c_2\right)$$

$$\Rightarrow \text{FOC for } c_1 \text{ \& } c_2: u'(c_1) = \lambda \text{ and } \beta u'(c_2) = \lambda \cdot \frac{1}{1+r} \quad \Rightarrow \frac{1}{1+r} u'(c_1) = \beta u'(c_2)$$

- Same conditions. If utility is strictly concave, the solution  $(c_1, c_2)$  is unique.

## Interpretation: Consumption Smoothing

- **Intuition:** Suppose time preference is approximately equal to the interest rate:

$$\beta \approx \frac{1}{1+r} \Rightarrow u'(c_1) \approx u'(c_2) \Rightarrow c_1 \approx c_2$$

- Insight: *Consumption is a “smooth” series.* True even if the income series varies.

- **Benchmark case:** suppose  $c_1 \approx c_2$ . Then  $c_1 + \frac{1}{1+r} c_2 = c_1 \cdot (1 + \frac{1}{1+r}) = A + w_1 + \frac{1}{1+r} w_2$

$$\Rightarrow c_1 = c_2 = \frac{1}{1+(1+r)^{-1}} \cdot (A + w_1 + \frac{1}{1+r} w_2)$$

- Find: *Consumption is approximately a fraction of total lifetime resources.*

- Note: If disposable income were a constant  $y$ , then  $\frac{w_1 + \frac{1}{1+r} w_2}{1+(1+r)^{-1}} = \frac{y + \frac{1}{1+r} y}{1+(1+r)^{-1}} = y$ .

- **Permanent Income** = Annuity equivalent of the income actual stream (Friedman 1957)

$$\text{Here } y^P = \frac{w_1 + \frac{1}{1+r} w_2}{1+(1+r)^{-1}}. \text{ Then } c_1 = y^P + \frac{1}{1+(1+r)^{-1}} \cdot A.$$

- Find: *Consumption  $\approx$  Permanent Income plus a fraction of initial wealth.*

## Interpretation: Savings Incentives

- **Savings incentives:** High interest rates provide incentives to consume less & save more:
  - If  $1+r > 1/\beta$ , then  $u'(c_1) > u'(c_2) \Rightarrow c_1 < c_2$ . (Caveat: High  $r$  also reduces  $y^P$ )
  - High interest rates “tilt” the consumption path upwards. Consumption grows over time.
  - Growing consumption must start at a lower level to satisfy the IBC.
    - $\Rightarrow$  Initial consumption is (slightly) less than permanent income intuition would suggest.
  
- **Example (Power utility):**  $u(c) = \frac{1}{1-\theta} c^{1-\theta}$  with  $\theta > 0, \theta \neq 1$ 
  - Then  $\frac{1}{1+r} u'(c_1) = \beta u'(c_2) \Leftrightarrow \frac{1}{1+r} c_1^{-\theta} = \beta c_2^{-\theta} \Leftrightarrow c_2 = [(1+r)\beta]^{1/\theta} c_1$
  - Combine with  $A + w_1 + \frac{1}{1+r} w_2 = c_1 + \frac{1}{1+r} c_2 = c_1 + \frac{1}{1+r} [(1+r)\beta]^{1/\theta} c_1$
  - Obtain  $c_1 = \frac{1}{1 + \frac{1}{1+r} [(1+r)\beta]^{1/\theta}} (A + w_1 + \frac{1}{1+r} w_2)$

## Many Periods: Permanent Income Model

- Generalize to arbitrary number of periods  $n$ . Terminal condition  $a_n = 0$ .

- Intertemporal budget constraint: 
$$\sum_{t=1}^n \frac{1}{(1+r)^{t-1}} c_t = A + \sum_{t=1}^n \frac{1}{(1+r)^{t-1}} w_t$$

- If consumption is about constant (meaning:  $\beta \approx 1/(1+r)$ ), the budget constraint implies

$$c_1 = (A + \sum_{t=1}^n \frac{1}{(1+r)^{t-1}} w_t) / (\sum_{t=1}^n \frac{1}{(1+r)^{t-1}})$$

Interpretation:  $\sum_{t=1}^n \frac{1}{(1+r)^{t-1}} = \frac{1+r}{r} [1 - (1+r)^{-n}] =$  present value of a fixed annuity.

For large  $n$  and small  $r$ ,  $1/\sum_{t=1}^n \frac{1}{(1+r)^{t-1}} \approx \frac{r}{1+r} \approx r$

*=> Consumption is approximately a fraction  $r$  of lifetime resources.*

- Implications of permanent income theory:

1. **Distinguish changes in current income** (holding future income constant) from changes in permanent income (current and future income): Permanent changes have a much greater impact than temporary changes.

2. **Future income matters** => Expectations about future income matter.

[Aside: uncertainty would complicate matters – ignored here.]

3. **Consumption growth** depends on interest rates relative to the rate of time preference

## Intertemporal Choice with Production

- Output is produced with capital and labor:  $Y_t = F(K_t, L_t)$ 
  - Properties: Increasing; concave; constant returns to scale. (More later.)
  - Firm profits = Output – wage cost – cost of capital. (All in real terms.)
  - Real wage = Marginal product of labor:  $w_t = F_L(K, L)$
  - Interest rate = Marginal product of capital – depreciation rate.
  
- Warm-up: A static model. Given K. No capital investment.
  - Time constraint: available hours = h. Time for work (l) plus leisure (h-l)
  - Households maximize utility from consumption and leisure:  $U = u(c, h-l)$   
 subject to a budget constraint:  $c = w \cdot l + \pi$  [given other funds  $\pi$ ]  
 FOC:  $u_1(c, h-l) \cdot w = u_2(c, h-l)$   
 => Consumption-leisure graph with indifference curves and budget line (slope  $-w$ ).
  - Assume firms are owned by households:  $\Pi = F(K, L) - w \cdot L$ . Population = N.
  - Market equilibrium with identical households:  $l = \frac{1}{N} L$ ,  $\pi = \frac{1}{N} \Pi$   
 =>  $c = w \cdot \frac{L}{N} + \frac{1}{N} \Pi = \frac{1}{N} [w \cdot L + F(K, L) - w \cdot L] = \frac{1}{N} F(K, L)$   
 => Aggregate tradeoff between per-capita consumption and leisure is concave.

- What if a “**social planner**” (government) made all the decisions?
  - Social planner maximizes household utilities subject to production constraint.  
 Maximize  $U = u(c, h - l)$  s.t.  $c = \frac{1}{N} F(K, N \cdot l)$ .  
 FOC:  $u_1(c, h - l) \cdot \frac{1}{N} F_L \cdot N = u_1(c, h - l) \cdot F_L = u_2(c, h - l)$ . Same as household FOC.
  - Note: Planning solutions are **Pareto Optimal**. (Here simple: all households identical.)  
 => Social planner is a useful “device” to compute Pareto-optimal allocations.
  - **First fundamental welfare theorem: Competitive equilibrium is Pareto optimal.**  
 (Assuming price-taking/competitive behavior, increasing utility.)
  - **Second fundamental welfare theorem: All Pareto optimal solutions can be implemented as market equilibrium.** (In general: with lump sum transfers).  
 (Requires a concave production function and a concave utility function. Here no transfers needed because households are identical.)
- Interpret the social planner as a **representative household** who also operates a firm.
  1. Solving a representative agent problem is a convenient way to obtain market allocations.
  2. Given the optimal allocation, market clearing “prices” follow from the FOC.  
 [Here: optimal  $(c, l)$  implies unique  $w = u_2(c, h - l) / u_1(c, h - l)$ .]

## General Case: Production Economy with discrete time periods

(Here a sketch only)

- Resource constraints:  $Y_t = F(K_t, L_t) = I_t + C_t$  and  $K_{t+1} = I_t + (1 - \delta) \cdot K_t$ .
  - Household utility:  $U = u(c_1, h - l_1) + \beta \cdot u(c_2, h - l_2) + \dots$
  - Alternative interpretations:
    1. Market allocation with firms and households; markets for labor, goods, capital.
    2. Social planner maximizes  $U$  subject to production constraints.
    3. Representative household maximizes  $U$  s.t. per-capita resource constraints.
  - Observations:
    - Marginal increase in  $l_t$  allows a marginal increase in  $c_t$  by  $F_L$ .  
 $\Rightarrow \frac{\partial u}{\partial c_t} F_L(K_t, L_t) = \frac{\partial u}{\partial l_t}$  must apply – as in the static production model; also:  $F_L = w_t$ .
    - Marginal increase in  $K/N$  reduces  $c_t$  by same amount & increases  $c_{t+1}$  by  $F_K + (1 - \delta)$ .  
 $\Rightarrow$  Optimality condition is  $\frac{\partial u}{\partial c_t} = \beta \frac{\partial u}{\partial c_{t+1}} [F_K(K_t, L_t) + 1 - \delta]$ ; also:  $F_K + 1 - \delta = 1 + r$ .

*Interpretation: Marginal product of capital = interest + depreciation = cost of capital*

[Note: Concave production  $\Rightarrow$  aggregate tradeoff between  $c_t$  and  $c_{t+1}$  is concave.]
  - Key challenge: Dynamics of capital and output:  $K_t \rightarrow Y_t \rightarrow I_t \rightarrow K_{t+1} \rightarrow Y_{t+1} \dots$
- How does such the economy evolve over time? How does economic growth occur?*

## Learning Objectives

- Conceptual:
  1. Macroeconomics is based on microeconomic principles – optimal choices subject to constraints, market equilibrium, welfare theorems.
  2. Microeconomic intuition: income and substitution effects.
  3. Macroeconomic intuition from simplified models:
    - Intertemporal consumption choices and permanent income.
    - Static production models and consumption-leisure tradeoff.
  
- Technical skills:
  1. Optimization – with and without constraints.
  2. Solving and interpreting intertemporal and static choice problems.

*Problem sets for practice.*