Appendix to:
Ownership Risk, Investment, and the Use of Natural Resources

by Henning Bohn and Robert T. Deacon

Department of Economics
University of California at Santa Barbara
Santa Barbara, CA 93106

Revised, November 1998

This appendix shows that the optimal decisions rules for capital investment and natural resources exploitation have the properties claimed in the text. Section A1 covers optimal capital investment, Section A2 examines oil production and exploration, and Section A3 considers forestry.

A1. The Optimal Capital Investment Policy

Existence and uniqueness of the optimal policy can be shown along the lines of Stokey-Lucas (1989, ch.9), conditional on $\xi_t=0$. The value function collapses to zero for $\xi_t=1$, which implies that the optimal decision conditional on $\xi_t=1$ is indeterminate, but irrelevant. Without loss of generality, one may set $I_t=0$ for $\xi_t=1$. The partial derivatives of the optimal policy function K^* and the optimal investment per worker are obtained by taking the total differential of equation (7).

Notation

The following additional notation and preliminary transformations are convenient. Let $i_t = (I/N)_t$, $k_t = (K/N)_t$, and $y_t = (Y/N)_t$ be the investment, capital, and output per worker. Constant returns to scale and our investment cost function imply that

$$V(K_t,N_t,H_t,x_t,\pi_t,\xi_t) = N_t \cdot v(k_t,H_t,x_t,\pi_t,\xi_t),$$

so $V(\cdot)$ is proportional to population times a “per-capita” value function that depends on capital and population only through the ratio $K/N=k$.

The assumption that x_t and π_t are Markov processes can be formalized by writing $x_{t+1} = f^x(x_t,\varepsilon^x_{t+1})$ and $\pi_{t+1} = f^\pi(\pi_t,\varepsilon^\pi_{t+1})$, where ε^x_{t+1} and ε^π_{t+1} are white noise processes. The integral $\int V_K\, dG$ in eq. (7) can then be written as

$$\int V_K dG = (1-\pi_t)\cdot\int v_k(k_{t+1},H_{t+1},f^x(x_t,\varepsilon^x_{t+1}),f^\pi(\pi_t,\varepsilon^\pi_{t+1}),0)\cdot dG^\varepsilon(\varepsilon^x_{t+1},\varepsilon^\pi_{t+1})$$

where the r.h.s. integral is over the marginal distributions of the innovations to x and π. We have used the fact that V=0 if $\xi_{t+1}=1$, which occurs with probability π_t.

The Total Differential of the First Order Condition

The total differential of eq. (7) is then

$$\left[2\cdot c'(i_t/k_t)/k_t+i_t/k_t^2\cdot c''(i_t/k_t)\right]\cdot(di_t-(i_t/k_t)\cdot dk_t)$$

$$= -[\frac{1}{1+r}\cdot\int v_k dG^{\varepsilon}]d\pi_t + [\frac{1-\pi_t}{1+r}\cdot\int v_{kk}dG^{\varepsilon}]\cdot dk_{t+1} + [\frac{1-\pi_t}{1+r}\cdot\int v_{kH}dG^{\varepsilon}]\cdot dH_{t+1}$$
$$+ [\frac{1-\pi_t}{1+r}\cdot\int v_{kx}\cdot f^x{}_x dG^{\varepsilon}]\cdot dx_t + [\frac{1-\pi_t}{1+r}\cdot\int v_{k\pi}\cdot f^{\pi}{}_{\pi}dG^{\varepsilon}]\cdot d\pi_t, \quad \text{(A1.1)}$$

where subscripts denote partial derivatives. The differential and the expressions therein can be written as follows. Define

$$\Omega_{cc} \equiv \left[2\cdot c'(i_t/k_t)/k_t + i_t/k_t{}^2\cdot c''(i_t/k_t)\right] > 0,$$
$$\Omega_c \equiv \int v_k dG^{\varepsilon} = c(i_t/k_t) + i_t/k_t\cdot c'(i_t/k_t) > 0,$$

and note that both expressions are positive. To replace dk_{t+1} and dH_{t+1} by t-dated variables, we can exploit the dynamics of physical capital and human capital, which are

$$dk_{t+1} = 1/(1+n)\cdot di_t + (1-\delta)/(1+n)\cdot dk_t$$

and $$dH_{t+1} = \left[\beta\cdot H_t{}^{\beta-1}\cdot h(x_t) + 1-\delta_h\right]\cdot dH_t + [H_t{}^{\beta}\cdot h'(x_t)]\cdot dx_t.$$

Equation (A1.1) can therefore be rewritten as

$$\left[\frac{\Omega_{cc}}{k_t} - \frac{1-\pi_t}{1+r}\cdot\int v_{kk}dG^{\varepsilon}/(1+n)\right]\cdot di_t$$
$$= \left[\Omega_{cc}\cdot(i_t/k_t) + \frac{1-\pi_t}{1+r}\cdot\int v_{kk}dG^{\varepsilon}\cdot(1-\delta)/(1+n)\right]\cdot dk_t$$
$$+ \frac{1-\pi_t}{1+r}\cdot\int v_{kH}dG^{\varepsilon}\cdot\left(\beta\cdot H_t{}^{\beta-1}\cdot h(x_t) + 1-\delta_h\right) dH_t$$
$$+ \frac{1-\pi_t}{1+r}\cdot\left[\int v_{kx}\cdot f^x{}_x dG^{\varepsilon} + \int v_{kH}dG^{\varepsilon}\cdot H_t{}^{\beta}\cdot h'(x_t)\right]\cdot dx_t$$
$$+ \left[-\frac{\Omega_c}{1+r} + \frac{1-\pi_t}{1+r}\cdot\int v_{k\pi}\cdot f^{\pi}{}_{\pi}dG^{\varepsilon}\right]\cdot d\pi_t$$

Define $$\Omega_i = \frac{\Omega_{cc}}{k_t} - \frac{1-\pi_t}{1+r}\cdot\int v_{kk}dG^{\varepsilon}/(1+n),$$

and note that $\Omega_i>0$ is the second order condition for value maximization. Provided $v_{kk}\leq 0$ (to be verified below), the second order condition is satisfied, and $\Omega_i>0$. The derivatives of the optimal policy function $i^*(\cdot)$ can then be read off the above differential as

$$\frac{\partial i^*{}_t}{\partial k_t} = \frac{\Omega_{cc}}{\Omega_i}\cdot\frac{i_t}{k_t} + \frac{1-\delta}{\Omega_i}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{kk}dG^{\varepsilon}/(1+n) \quad \text{(A1.2a)}$$
$$\frac{\partial i^*{}_t}{\partial H_t} = \frac{1}{\Omega_i}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{kH}dG^{\varepsilon}\cdot\left(\beta\cdot H_t{}^{\beta-1}\cdot h(x_t) + 1-\delta_h\right) \quad \text{(A1.2b)}$$
$$\frac{\partial i^*{}_t}{\partial x_t} = \frac{1}{\Omega_i}\cdot\frac{1-\pi_t}{1+r}\cdot\left[\int v_{kx}\cdot f^x{}_x dG^{\varepsilon} + \int v_{kH}dG^{\varepsilon}\cdot H_t{}^{\beta}\cdot h'(x_t)\right] \quad \text{(A1.2c)}$$
$$\frac{\partial i^*{}_t}{\partial \pi_t} = -\frac{1}{\Omega_i}\cdot\frac{\Omega_c}{1+r} + \frac{1}{\Omega_i}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{k\pi}\cdot f^{\pi}{}_{\pi}dG^{\varepsilon}. \quad \text{(A1.2d)}$$

The signs of these derivatives depend on the second derivatives of the value function $v(\cdot)$. The envelope theorem implies that

$$v_k(\cdot) = V_k(\cdot) = \frac{\partial PR_{t+1}}{\partial K_{t+1}}$$

$$= (1-\alpha)\cdot H_{t+1}^{\alpha}\cdot k_{t+1}^{-\alpha} + (1-\delta)\cdot c(\frac{i_{t+1}}{k_{t+1}}) + \frac{i_{t+1}}{k_{t+1}}\cdot[\frac{i_{t+1}}{k_{t+1}}+1-\delta]\cdot c'(\frac{i_{t+1}}{k_{t+1}})$$

can be written as a function of i_{t+1}, k_{t+1}, and H_{t+1}. The second partial derivatives of $v(\cdot)$ can therefore be computed as

$$v_{kk} = -\alpha(1-\alpha)\cdot H_{t+1}^{\alpha}\cdot k_{t+1}^{-\alpha-1} + \left[-\frac{i_{t+1}}{k_{t+1}} + \frac{\partial i^*_{t+1}}{\partial k_{t+1}}\right]\cdot \Omega_k \qquad \text{(A1.3a)}$$

where $$\Omega_k = 2\cdot(\frac{i_{t+1}}{k_{t+1}} + 1-\delta)\cdot c'(\frac{i_{t+1}}{k_{t+1}})$$

$$+ \frac{i_{t+1}}{k_{t+1}}\cdot(\frac{i_{t+1}}{k_{t+1}} + 1-\delta)\cdot c''(\frac{i_{t+1}}{k_{t+1}})\Big]/k_{t+1} > 0;$$

$$v_{kH} = \alpha(1-\alpha)\cdot H_{t+1}^{\alpha-1}\cdot k_{t+1}^{-\alpha} + \frac{\partial i^*_{t+1}}{\partial H_{t+1}}\cdot \Omega_k; \qquad \text{(A1.3b)}$$

$$v_{kx} = \frac{\partial i^*_{t+1}}{\partial x_{t+1}}\cdot \Omega_k; \text{ and} \qquad \text{(A1.3c)}$$

$$v_{k\pi} = \frac{\partial i^*_{t+1}}{\partial \pi_{t+1}}\cdot \Omega_k \qquad \text{(A1.3d)}$$

Inserted into (A1.2a-d), these equations imply that the derivatives of the policy function at time t depend on the derivatives of the policy function at time t+1.

To evaluate the derivatives, we use a limit argument. Consider the finite horizon analog of the above infinite horizon problem, i.e., assume the economy ends at some known terminal date T periods ahead. We will determine the derivatives of the optimal policy in the finite horizon problem through a backward recursion, starting at the terminal date, and then exploit the fact that the finite horizon policy converges to the infinite horizon policy as $T\to\infty$.

The Infinite Horizon Problem as Limit of a Finite Horizon Problem

Since the finite horizon problem has time-dependent policy and value functions, let superscripts denote the number of remaining periods (e.g., V^n denote the value function with n periods to go.) In the final period, there is no investment. Hence, $i_T^*=0$ and its derivatives are identically zero. In period t=T-1, we therefore have

$$v_{kk}^1 = -\alpha(1-\alpha)\cdot H_{t+1}^{\alpha}\cdot k_{t+1}^{-\alpha-1} < 0 \qquad \text{(A1.4a)}$$

$$v_{kH}^1 = \alpha(1-\alpha)\cdot H_{t+1}^{\alpha-1}\cdot k_{t+1}^{-\alpha} > 0 \qquad \text{(A1.4b)}$$

$$v_{kx}^1 = 0;\ v_{k\pi}^1 = 0, \qquad \text{(A1.4c,d)}$$

and

$$\Omega_i^1 = \frac{\Omega_{cc}}{k_t} - (1-\pi_t)\cdot\int v_{kk}^1 dG^{\varepsilon}/(1+n) > 0. \qquad \text{(A1.4e)}$$

Since $v_{kk}^1<0$ and $\Omega_{cc}>0$, we have $0<\Omega_{cc}/\Omega_i^1<1$. This is useful to evaluate the derivatives of the policy function,

$$\frac{\partial i^*_t}{\partial k_t} = \frac{\Omega_{cc}}{\Omega_i^1}\cdot\frac{i_t}{k_t} + \frac{1-\delta}{\Omega_i^1}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{kk}^1 dG^{\varepsilon}/(1+n) < \frac{i_t}{k_t} \qquad \text{(A1.5a)}$$

$$\frac{\partial i^*_t}{\partial H_t} = \frac{1}{\Omega_i^1}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{kH}^1 dG^{\varepsilon}\cdot\left(\beta\cdot H_t^{\beta-1}\cdot h(x_t) + 1-\delta_h\right) > 0 \qquad \text{(A1.5b)}$$

$$\frac{\partial i^*_t}{\partial x_t} = \frac{1}{\Omega_i^1}\cdot\frac{1-\pi_t}{1+r}\cdot\int v_{kH}^1 dG^{\varepsilon}\cdot H_t^{\beta}\cdot h'(x_t) > 0 \qquad \text{(A1.5c)}$$

$$\frac{\partial i^*_t}{\partial \pi_t} = -\frac{1}{1+r}\cdot\frac{\Omega_c}{\Omega_i^1} < 0. \qquad \text{(A1.5d)}$$

For the induction argument, suppose that for some period t+1 = T - n, the derivatives of the policy function and of the value function (using v^n instead of v^1) satisfy the inequality restrictions in (A1.4a-e) and (A1.5a-d). Then for period t, which is T-t = n+1 periods away from the terminal date,

$$v_{kk}^{n+1} = -\alpha(1-\alpha)\cdot H_t^{\alpha}\cdot k_t^{-\alpha-1} + \left[-\frac{i_{t+1}}{k_{t+1}} + \frac{\partial i^*_{t+1}}{\partial k_{t+1}}\right]\cdot\Omega_k$$

$$< -\alpha(1-\alpha)\cdot H_t^{\alpha}\cdot k_t^{-\alpha-1} < 0,$$

using (A1.5a), proving $v_{kk}^{n+1}<0$ for all n. In the limit, at least the weak inequality $v_{kk}\leq 0$ must apply in the infinite horizon problem. But if $v_{kk}\leq 0$, then (A1.2a) implies $\frac{\partial i^*_t}{\partial k_t} \leq \frac{i_t}{k_t}$ and (A1.3a) implies $v_{kk}\leq -\alpha(1-\alpha)\cdot H_{t+1}^{\alpha}\cdot k_{t+1}^{-\alpha-1}<0$. Thus, the inequality is strict. This argument also proves that the second order condition for optimality is satisfied and that the solution for i_t is unique.

Similarly, $v_{kH}^{n}>0$ implies $\partial i^{*}{}_{t}/\partial H_t>0$ in (A1.2b), which implies $v_{kH}^{n+1}>0$ in (A1.3b). Hence, $v_{kH}\geq 0$ and $\partial i^{*}/\partial H_t\geq 0$ apply in the infinite horizon problem. But then

$$v_{kH} = \alpha(1-\alpha)\cdot H_t^{\alpha-1}\cdot k_t^{-\alpha} + \frac{\partial i^{*}{}_{t+1}}{\partial H_{t+1}} \cdot \Omega_k \geq \alpha(1-\alpha)\cdot H_t^{\alpha-1}\cdot k_t^{-\alpha} >0$$

is strictly positive, and $\partial i^{*}/\partial H_t>0$. Thus, higher human capital unambiguously raises investment.

Regarding x and π, (A1.5c,d) combined with (A1.3c,d) imply that $v_{kx}^{2}>0$ $v_{k\pi}^{2}<0$, so that the induction can be started at n=2. If $v_{kx}^{n+1}>0$ and $v_{k\pi}^{n+1}<0$ for some n≥1, (A1.2c,d) imply $\partial i^{*}{}_{t}/\partial x_t>0$, and $\partial i^{*}{}_{t}/\partial \pi_t<0$ for t=T-n, which implies $v_{kx}^{n}>0$ and $v_{k\pi}^{n}<0$. Hence, the inequalities $v_{kx}>0$, $v_{k\pi}<0$, $\partial i^{*}/\partial x_t>0$, and $\partial i^{*}/\partial \pi_t<0$ apply in the infinite horizon problem.

The sign of $\partial i^{*}/\partial k_t$ is generally indeterminate, because of two offsetting effects. A higher k reduces the marginal return to new investment (see negative the v_{kk} term in (A1.5a)) but it also reduces the cost of installing new investment (see the positive Ω_{cc} term in (A1.5a)). The ratio of investment to capital, i_t/k_t, however, is unambiguously declining in k_t. Also, since $\Omega_{cc}>0$. (A1.2a) and the definition of Ω_i imply that

$$\frac{\partial i^{*}{}_{t}}{\partial k_t} = \frac{\Omega_{cc}}{\Omega_i} \cdot \frac{i_t}{k_t} - (1-\delta) \cdot(1-\frac{\Omega_{cc}}{\Omega_i}) > -(1-\delta)$$

and therefore

$$\frac{\partial k_{t+1}{}^{*}}{\partial k_t} = \frac{1}{1+n} \cdot \left[\frac{\partial i^{*}}{\partial k_t} + 1-\delta\right] >0. \qquad \text{(A1.6)}$$

Overall, we have shown that $k_{t+1} = k^{*}(k_t,H_t,x_t,\pi_t,\xi_t)$ and $K_{t+1} = N_{t+1}\cdot k_{t+1}$ are increasing in k_t, H_t, and x_t, and decreasing in π_t, as claimed in Section 2.2. The function $i_t = i^{*}(k_t,H_t,x_t,\pi_t,\xi_t) = k^{*}(k_t,H_t,x_t,\pi_t,\xi_t) - (1-\delta)\cdot k_t$, has the same properties, except that $\partial i^{*}/\partial k_t$ can be positive or negative.

The Investment-Output Ratio

With regard to the investment-output ratio (I/Y), we have

$$(I_t/Y_t)^* = \frac{k^*(K_t,N_t,H_t,x_t,\pi_t,\xi_t)-(1-\delta)\cdot K_t/N_t]}{H_t{}^{\alpha}\cdot(K_t/N_t)^{1-\alpha}}$$

$$\equiv i^+(K_t/N_t,H_t,x_t,\pi_t,\xi_t), \qquad (A1.7)$$

The derivatives of $i^+(\cdot)$ with respect to x_t and π_t have the same sign as $\partial i^*/\partial x_t>0$ and $\partial i^*/\partial\pi_t<0$, respectively, while

$$\frac{\partial i^+}{\partial k_t} = \frac{1}{y_t}\cdot\Big[\frac{\partial i^*}{\partial k_t} -(1-\alpha)\cdot\frac{i_t}{k_t}\Big] \text{ and} \qquad (A1.8a)$$

$$\frac{\partial i^+}{\partial H_t} = \frac{1}{y_t}\cdot\Big[\frac{\partial i^*}{\partial H_t} - \alpha \cdot \frac{i_t}{H_t}\Big]. \qquad (A1.8b)$$

have ambiguous signs. If eq. (4) is used to substitute K_t by Y_t,

$$(I/Y)^* = i^+(y_t{}^{1/(1-\alpha)}\cdot H_t{}^{-\alpha/(1-\alpha)},H_t,x_t,\pi_t,\xi_t) = i^*(y_t,H_t,x_t,\pi_t,\xi_t),$$

which is equation (8) in the text, the derivatives of $(I/Y)^*$ with respect to x_t and π_t have the same signs as before, positive and negative, respectively. (Note that the i^* function in the text has different arguments than the i^* function in (A1.8a,b) and the appendix above; we use $(I/Y)^*$ and $(I/N)^*$ below to prevent ambiguities.)

The derivatives with respect to initial output and human capital,

$$\frac{\partial(I/Y)^*}{\partial y_t} = \frac{k_t}{(1-\alpha)\cdot y_t{}^2}\cdot\Big[\frac{\partial(I/N)^*}{\partial k_t} -(1-\alpha)\cdot(I/K)_t\Big] \text{ and} \qquad (A1.9a)$$

$$\frac{\partial(I/Y)^{**}}{\partial H_t} = \frac{1}{y_t}\cdot\Big[\frac{\partial(I/N)^*}{\partial H_t} - \frac{\alpha}{1-\alpha}\cdot(y_t/H_t)^{1/(1-\alpha)}\cdot\frac{\partial(I/N)^*}{\partial k_t}\Big] \qquad (A1.9b)$$

have ambiguous signs. But unless $\partial(I/N)^*/\partial k_t$ takes a large positive value, the fact that $\partial(I/N)^*/\partial H_t>0$ suggests a positive sign in (A1.9b).

Human Capital Accumulation

This section of the appendix explains why the regression model (8) is consistent with both exogenous and endogenous human capital accumulation. This issue deserves comment because productivity and its determinants are, at best, imperfectly measured and because the exact interpretation of the proxies for human capital, such as schooling variables, depends on the model of human capital accumulation.

Suppose human capital is produced according to a production function

$$H_{t+1} = H_t^{\beta}\cdot h(x_t) + (1-\delta_h)\cdot H_t,$$

where $0\leq\beta\leq1$ and $0\leq\delta_h\leq1$. If $\beta<1$ and x_t is stationary, human capital will converge to a stochastic steady state. In this case, a country's mean level of human capital is a weighted average of past investments. Hence x_t and H_t in (8) can be proxied by current and past schooling rates and trade variables.[1]

If $\beta=1$, the long run growth rate of the economy is endogenously given by $g_H = h(x_t)-\delta$. Then H_t does not converge to a steady state and, because $g_H(\cdot)$ does not depend on H_t in this case, the economy's optimal K_{t+1} depends on $K_t/(N_t\cdot H_t)$, x_t, π_t, and ξ_t, but not on H_t separately. Using the production function as before to replace K_t, the investment share of output can be written as

$$\left(\frac{I_t}{Y_t}\right) = i^*\left(\frac{Y_t}{N_t\cdot H_t},x_t,\pi_t,\xi_t\right).$$

Although K_t/N_t and Y_t/N_t do not converge to steady states in this model, I_t/Y_t and $Y_t/(N_t\cdot H_t)$ do. Further, the balanced growth prediction implies that $K_t/(N_t\cdot H_t)$ and $Y_t/(N_t\cdot H_t)$ might show little sample variation. Instead of trying to find proxies for H_t one might therefore omit these regressors and subsume them into the error term. The above regression specification reduces to $(I_t/Y_t) = i^*(x_t,\pi_t,\xi_t)$ in this case. Then schooling variables should be interpreted as proxies for x_t.

[1] A potential empirical concern is that an investment model that uses past schooling as proxy for H_t could suffer from an omitted variables bias because some components of H_t are not measured. This would seem especially problematic if the political variables are correlated with output, because output depends on the true H_t, i.e., is correlated with the unobserved components of H_t. Nonetheless, if output is included as a regressor, as in (8), the coefficient on π_t will be consistent provided π_t is conditionally (conditional on Y_t/N_t) uncorrelated with H_t. Only the coefficients on output and on the proxies for human capital would be biased.

Overall, both of the specifications with endogenous growth (with and without $Y_t/(N_t \cdot H_t)$) are restricted versions of equation (8). Without making judgments about the nature of human capital accumulation, we estimate eq.(8) without restrictions and let the data determine the significance of Y_t/N_t and/or H_t. This approach yields consistent coefficient estimates whether or not growth is endogenous.

A2. The Optimal Oil Production and Exploration Policy

To derive the properties of the optimal plan for oil exploration, production, and investment in oil production capital, we divide the problem into three parts--the three steps below. As in the previous section, we can re-write the integral $\int V(\cdot)\, dG$ in (11) as an integral over the marginal distributions of the innovations to p and π,

$$\int V dG = (1-\pi_t)\cdot\int V(R_{t+1},H_{t+1},K^{\Theta}_{t+1},f^p(p_t,\varepsilon^p_{t+1}),f^{\pi}(\pi_t,\varepsilon^{\pi}_{t+1}),0) \cdot dG^{\varepsilon}(\varepsilon^p_{t+1},\varepsilon^{\pi}_{t+1}).$$

Step 1: Exploitation of a fixed reserve

First, consider the optimization problem of a firm with fixed initial reserves and capital equipment, R_0 and K^{Θ}_0. With some abuse of notation, let $R_{t+1} = R_t - Z_t$ be the remaining reserves of the firm in period t+1 -- assuming reserves are never replenished -- and let K^{Θ}_t be the production equipment of the firm in period t. Let $z_t = Z_t/R_t$ and $k_t = K^{\Theta}_t/R_t$ be the production-reserve and capital-reserve ratios of the firm.

Regarding production, we assume that oil is produced according to a Cobb-Douglas production function with constant returns, using capital K^{Θ}_t, reserves R_t, and labor N^{Θ}_t, and produced materials Y^{Θ}_t,

$$Z_t = (N^{\Theta}_t)^{\eta}\cdot(K^{\Theta}_t)^{\gamma}\cdot(Y^{\Theta}_t)^{\mu}\cdot R_t^{1-\eta-\gamma-\mu},$$

where $0<\eta$, $0<\gamma$, $0<\mu$ and $\eta+\gamma+\mu<1$. Materials and labor are variable within the period, while capital and reserves are predetermined. Labor is assumed to be supplied at a fixed wage rate w; materials have a unit cost. Total variable cost are therefore $w\cdot N^{\Theta}_t+Y^{\Theta}_t$. Note that w is not necessarily the local wage rate. We assume that oil production requires specialized, skilled workers who are internationally mobile.

Cost minimization implies $Y^{\Theta}_t/N^{\Theta}_t = \mu\cdot w/\eta$, which yields the input requirements

$$N^{\Theta}_t = Z_t^{1/(\eta+\mu)}\cdot(K^{\Theta}_t)^{-\gamma/(\eta+\mu)}\cdot R_t^{-(1-\eta-\gamma-\mu)/(\eta+\mu)}\cdot(\mu\cdot w/\eta)^{-\mu/(\eta+\mu)},$$

$$Y^{\Theta}_t = Z_t^{1/(\eta+\mu)}\cdot(K^{\Theta}_t)^{-\gamma/(\eta+\mu)}\cdot R_t^{-(1-\eta-\gamma-\mu)/(\eta+\mu)}\cdot(\mu\cdot w/\eta)^{\eta/(\eta+\mu)}.$$

for producing Z_t at given K^{Θ}_t and R_t. Variable cost per unit production are then

$$(w\cdot N^{\Theta}_t+Y^{\Theta}_t)/Z_t = \chi\cdot Z_t^{1/(\eta+\mu)}\cdot(K^{\Theta}_t)^{-\gamma/(\eta+\mu)}\cdot R_t^{-(1-\gamma-\eta-\mu)/(\eta+\mu)}/Z_t$$

$$= \chi\cdot z_t^{1/(\eta+\mu)-1}\cdot k_t^{-\gamma/(\eta+\mu)}$$

where $\chi = w^{\eta/(\eta+\mu)}\cdot[(\mu/\eta)^{-\mu/(\eta+\mu)}+(\mu/\eta)^{\eta/(\eta+\mu)}]$. This is the cost function in the text, if we interpret $\beta = 1/(\eta+\mu)-1>0$ and $\nu = \gamma/(\eta+\mu)>0$. Note that $\beta-\nu = 1/(\eta+\mu)\cdot[1-\eta-\mu-\gamma]>0$. With this cost function, the profit function of the production firm is

$$PR_t = p_t\cdot Z_t - \chi\cdot Z_t^{\beta+1}\cdot K^{\Theta}{}_t^{-\nu}/R_t^{\beta-\nu} + (1-\delta)\cdot K^{\Theta}_t - K^{\Theta}_{t+1}$$

$$= (p_t - \chi\cdot z_t^{\beta}\cdot k_t^{-\nu})\cdot z_t\cdot R_t + (1-\delta)\cdot K^{\Theta}_t - K^{\Theta}_{t+1} \qquad \text{(A2.1)}$$

Given the initial capital and reserves, R_t and K^{Θ}_t, firms maximize their value

$$V^{\Theta}(R_t,K^{\Theta}_t,p_t,\pi_t,\xi_t) = \max\ PR_t(R_t,R_{t+1},K^{\Theta}_t,K^{\Theta}_{t+1}) + \frac{1-\pi_t}{1+r}$$

$$\cdot\int V^{\Theta}(R_{t+1},K^{\Theta}_{t+1},f^p(p^F_t,\varepsilon^p_{t+1}),f^{\pi}(\pi_t,\varepsilon^{\pi}_{t+1}),0)\ \cdot dG^{\varepsilon}(\varepsilon^p_{t+1},\varepsilon^{\pi}_{t+1}).$$

The first order conditions for R_{t+1} and K^{Θ}_{t+1} are then

$$p_t - \chi\cdot(1+\beta)\cdot k_t^{-\nu}\cdot z_t^{\beta} = \frac{1-\pi_t}{1+r}\cdot\int V^{\Theta}_R\ dG^{\varepsilon}, \qquad \text{(A2.2a)}$$

and

$$1 = \frac{1-\pi_t}{1+r}\cdot\int V^{\Theta}_K\ dG^{\varepsilon}, \qquad \text{(A2.2b)}$$

which are equivalent to (12a,b) in the text. The envelope theorem implies that

$$V^{\Theta}_R = \frac{\partial PR_{t+1}}{\partial R_{t+1}}(R_{t+1},R_{t+2},K^{\Theta}_{t+1},K^{\Theta}_{t+2}) = (\beta-\nu)\cdot\chi\cdot z_{t+1}{}^{\beta+1}\cdot k_{t+1}{}^{-\nu} \quad \text{(A2.3a)}$$

and

$$V^{\Theta}_K = \frac{\partial PR_{t+1}}{\partial K^{\Theta}_{t+1}}(R_{t+1},R_{t+2},K^{\Theta}_{t+1},K^{\Theta}_{t+2}) = \nu\cdot\chi\cdot z_{t+1}{}^{\beta+1}\cdot k_{t+1}{}^{-\nu-1} + 1-\delta$$

$$= \nu/(\beta-\nu)\cdot k_{t+1}{}^{-1}\cdot V^{\Theta}_R + 1-\delta \quad \text{(A2.3b)}$$

Note that since PR_t is linearly homogeneous in $(R_t,R_{t+1},K^{\Theta}_t,K^{\Theta}_{t+1})$, the first order conditions are homogenous of degree zero in these variables. The optimal policy functions, and hence the value function, are therefore linearly homogenous in (R_t,K^{Θ}_t); in short, profits, decisions and the value function are proportional to reserves R_t. To exploit this property, it is useful to define the value per unit reserves v^{Θ} by

$$V^{\Theta}(R_t,K^{\Theta}_t,p_t,\pi_t,1) = R_t\cdot V^{\Theta}(1,K^{\Theta}_t/R_t,p_t,\pi_t,1) \equiv R_t\cdot v^{\Theta}(k_t,p_t,\pi_t).$$

We may substitute $V^{\Theta}_R = v^{\Theta}(\cdot)-k_t\cdot v^{\Theta}_k(\cdot)$ and $V^{\Theta}_K = v^{\Theta}_k(\cdot)$ in (A2.2a,b) and (A2.3a,b). These equations determine z_t and k_{t+1} as functions of k_t, p_t, and π_t. Using (A2.3b), we can rewrite (A2.2b) as

$$\frac{\beta-\nu}{\nu}\cdot\Big[1-\frac{1-\delta}{1+r}\cdot(1-\pi_t)\Big]\cdot k_{t+1} = \frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}dG^{\varepsilon} - k_{t+1}$$

$$= p_t - \chi\cdot(1+\beta)\cdot k_t{}^{-\nu}\cdot z_t{}^{\beta} \quad \text{(A2.4)}$$

Taking the total differential of (A2.2a) and (A2.4), we find

$$dp_t + \nu\cdot(1+\beta)\cdot\chi\cdot z_t{}^{\beta}\cdot k_t{}^{-\nu-1}\cdot dk_t - \beta\cdot(1+\beta)\cdot\chi\cdot z_t{}^{\beta-1}\cdot k_t{}^{-\nu}\cdot dz_t$$
$$= -[\frac{1}{1+r}\cdot\int\int v^{\Theta}dG^{\varepsilon}]d\pi_t + [\frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}_{\pi}\cdot f^{\pi}_{\pi}dG^{\varepsilon}]\cdot d\pi_t$$
$$+ [\frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}_k dG^{\varepsilon}-1]\cdot dk_{t+1} + [\frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}_p\cdot f^{p}_{p}dG^{\varepsilon}]\cdot dp_t,$$

$$= -(\Omega_{\pi\Theta}-\Omega_{\pi})\cdot d\pi_t + \Omega_p\cdot dp_t,$$

where

$$\Omega_p = \frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}_p\cdot f^{p}_{p}dG^{\varepsilon},$$

$$\Omega_{\pi} = \frac{1-\pi_t}{1+r}\cdot\int v^{\Theta}_{\pi}\cdot f^{\pi}_{\pi}dG^{\varepsilon},$$

$$\Omega_{\pi 0} = \frac{1}{1+r}\cdot\int v^{\Theta}dG^{\varepsilon} > 0$$

and where we exploit that the coefficient on dk_{t+1} is zero due to (A2.2b). Hence,

$$dz_t = \frac{1}{\chi_z}\cdot\Big[(1-\Omega_p)\cdot dp_t + (\Omega_{\pi 0}-\Omega_\pi)\cdot d\pi_t + \chi_k\cdot dk_t\Big], \qquad \text{(A2.5a)}$$

where $\chi_k = \nu\cdot(1+\beta)\cdot\chi\cdot z_t^{\beta}\cdot k_t^{-\nu-1} > 0,$

$\chi_z = \beta\cdot(1+\beta)\cdot\chi\cdot z_{t+1}^{\beta-1}\cdot k_{t+1}^{-\nu} > 0.$

Similarly,

$$dp_t + \nu\cdot(1+\beta)\cdot\chi\cdot z_t^{\beta}\cdot k_t^{-\nu-1}\cdot dk_t - \beta\cdot(1+\beta)\cdot\chi\cdot z_t^{\beta-1}\cdot k_t^{-\nu}\cdot dz_t$$
$$= \frac{\beta-\nu}{\nu}\cdot\Big(1-\frac{1-\delta}{1+r}\cdot(1-\pi_t)\Big)\cdot dk_{t+1} + \frac{\beta-\nu}{\nu}\cdot\frac{1-\delta}{1+r}\cdot k_{t+1}\cdot d\pi_t$$

=> $$dk_{t+1} = \frac{1}{\kappa}\cdot[\Omega_p\cdot dp_t - (\chi_\pi+\Omega_{\pi 0}-\Omega_\pi)\cdot d\pi_t] \qquad \text{(A2.5b)}$$

where $\kappa = \frac{\beta-\nu}{\nu}\cdot\Big[1-\frac{1-\delta}{1+r}\cdot(1-\pi_t)\Big] > 0,$

$\chi_\pi = \frac{\beta-\nu}{\nu}\cdot\frac{1-\delta}{1+r}\cdot k_{t+1} > 0.$

To determine the signs of Ω_p and Ω_π, note that (A2.3b) implies

$$v^0 = V^0_R + k_{t+1}\cdot V^0_K = \frac{\beta}{\beta-\nu}\cdot V^0_R + (1-\delta)\cdot k_{t+1}$$

=> $$v^0_p = \frac{dv^0}{dp_{t+1}} = \frac{\beta}{\beta-\nu}\cdot\frac{dV^0_R}{dp_{t+1}}$$
$$= \beta\cdot(\beta+1)\cdot\chi\cdot z_{t+1}^{\beta}\cdot k_{t+1}^{-\nu}\cdot\frac{\partial z^*_{t+1}}{\partial p_{t+1}} = z_{t+1}\cdot(1-\Omega_p) \qquad \text{(A2.6a)}$$

=> $$v^0_\pi = \frac{dv^0}{d\pi_{t+1}} = \beta\cdot(\beta+1)\cdot\chi\cdot z_{t+1}^{\beta}\cdot k_{t+1}^{-\nu}\cdot\frac{\partial z^*_{t+1}}{\partial \pi_{t+1}} = z_{t+1}\cdot(\Omega_{\pi 0}-\Omega_\pi) \qquad \text{(A2.6b)}$$

Now we can use a limit and induction argument analogous to the section on aggregate investment. In the final period of a finite horizon problem (t=T, n=1 periods from the end) $v^0_p=0 \Rightarrow \Omega_p^1=0 \Rightarrow \partial z^*_t/\partial p_t=1/\chi_z>0$. In period t=T-1 (n=2), $\Omega_p^1=0$ implies $v^0_p=z_{t+1}\le 1 \Rightarrow \Omega_p^2 \le \frac{1-\pi_t}{1+r}\cdot\int f^p_p dG^\varepsilon < 1/(1+r)$, provided $0\le f^p_p\le 1$. For the induction, assume that $0\le\Omega_p^n<1/(1+r)$ in some period t+1 = T-n. Then

$$\Omega_p^{n+1} = \frac{1-\pi_t}{1+r}\cdot\int (1-\Omega_p^n)\cdot z_{t+1}\cdot f^p_p dG^\varepsilon$$

also satisfies $0\le\Omega_p^t<1/(1+r)$, provided $0\le f^p_p\le 1$. Thus, $0<\frac{r}{1+r}/\chi_z\le(1-\Omega_P)/\chi_z=\partial z^*_t/\partial p_t<1/\chi_z$ applies for all t in a finite horizon problem, which implies $0<\partial z^*_t/\partial p_t\le 1/\chi_z$ for the infinite horizon problem.

With regard to $\partial z^*_t/\partial\pi_t$, the general conditions for $\partial z^*_t/\partial\pi_t>0$ are more complicated, because $\Omega_{\pi 0}$ may vary over time. But if f^π_π is sufficiently small or r and π_t sufficiently large, we have $0 \le \Omega_\pi < \Omega_{\pi\Theta}$, which implies $\partial z^*_t/\partial\pi_t > 0$; this is assumed throughout the paper.

Separately, it is instructive to evaluate v^Θ_p and v^Θ_π at the mean values of the steady state distribution of z_t, π_t, and p_t. We have

$$\Omega_p = \frac{1-\pi_t}{1+r} \cdot \int z_{t+1} \cdot (1-\Omega_p) \cdot f^p{}_p dG^\varepsilon$$

$$\Rightarrow \bar{\Omega}_p \approx \frac{1-\bar{\pi}}{1+r} \cdot \bar{z}(1-\bar{\Omega}_p) \cdot \bar{f}^p{}_p \Rightarrow \bar{\Omega}_p \approx \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^p{}_p / \left[1+ \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^p{}_p\right]$$

$$\Rightarrow \frac{\partial z^*_t}{\partial p_t} \approx \frac{1}{\chi_z} / \left[1 + \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^p{}_p\right] > 0$$

and

$$\Omega_\pi = \frac{1-\pi_t}{1+r} \cdot \int z_{t+1} \cdot (\Omega_{\pi\Theta} - \Omega_\pi) \cdot f^\pi{}_\pi dG^\varepsilon$$

$$\Rightarrow \bar{\Omega}_\pi \approx \frac{1-\bar{\pi}}{1+r} \cdot \bar{z}(\bar{\Omega}_{\pi\Theta} - \bar{\Omega}_\pi) \cdot \bar{f}^\pi{}_\pi \Rightarrow \bar{\Omega}_\pi \approx \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^\pi{}_\pi / \left[1+ \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^\pi{}_\pi\right] \cdot \bar{\Omega}_{\pi\Theta}$$

$$\Rightarrow \frac{\partial z^*_t}{\partial \pi_t} \approx \frac{\bar{\Omega}_{\pi\Theta}}{\chi_z} / \left[1 + \frac{1-\bar{\pi}}{1+r} \cdot \bar{z} \cdot \bar{f}^\pi{}_\pi\right] > 0$$

which shows that the average values of Ω_π and $\Omega_{\pi\Theta}$ satisfy $0 < \bar{\Omega}_\pi < \bar{\Omega}_{\pi\Theta}$ and that $\bar{\Omega}_p$, $\partial z^*_t/\partial p_t$, and $\partial z^*_t/\partial\pi_t$ are approximately equal to strictly positive quantities. If the stochastic disturbances $\varepsilon^p{}_t$ and $\varepsilon^\pi{}_t$ have sufficiently small variances, the realizations of $\partial z^*_t/\partial p_t$ and $\partial z^*_t/\partial\pi_t$ should also be positive.

Overall, equation (A2.5b) shows that, under the conditions for $\Omega_p > 0$ and $\chi_\pi + \Omega_{\pi\Theta} - \Omega_\pi > 0$ stated above, the ratio of capital to reserves is an increasing function of p_t and a decreasing function of π_t. Equation (A2.5a) shows that, under the stated assumptions, z_t is an increasing function of p_t, k_t, and π_t. The economic intuition is discussed in the text.

Step 2: The value of newly discovered reserves

Now we consider the valuation of new reserves. Step 1 above has shown that a firm with reserves R_t and production equipment $K^\Theta{}_t$ has a value $V^\Theta(R_t, K^\Theta{}_t, p_t, \pi_t, \xi_t)$. Now suppose the firm can buy new reserves Θ_t at time t The new reserves will become productive at time t+1, i.e.,

$$R_{t+1} = R_t - Z_t + \Theta_t. \qquad \text{(A2.7)}$$

By construction of the value function, the marginal value q_t of the new reserves is

$$q_t = \frac{1-\pi_t}{1+r} \cdot \int V^{\Theta}_R \, dG^{\varepsilon}. \qquad \text{(A2.8)}$$

We will argue that because of linear homogeneity, q_t is an equilibrium price of reserves in a competitive reserve market that does not depend on the level of reserves and the level of new discoveries. To see this, note that according to (A2.5b), $k_{t+1}=k^*_{t+1}(p_t,\pi_t)$ is a function of p_t and π_t; it does not depend on k_t (reflecting the absence of adjustment cost) nor on R_t (reflecting linear homogeneity). Hence,

$$V^{\Theta}_R = v^{\Theta}(k^*_{t+1}(p_t,\pi_t),p_{t+1},\pi_{t+1})-k^*_{t+1}(p_t,\pi_t)\cdot v^{\Theta}_k(k^*_{t+1}(p_t,\pi_t),p_{t+1},\pi_{t+1})$$

is a function of current and future oil prices and political risk. The relevant integral over future prices and political risk,

$$q_t = \frac{1-\pi_t}{1+r} \cdot \int (v^{\Theta}-k_{t+1}{}^*\cdot v^{\Theta}_k) \, dG^{\varepsilon} \equiv q_t{}^*(p_t,\pi_t), \qquad \text{(A2.8)}$$

is then a function of current oil prices and current political risk. Intuitively, the value of reserves is the expected discounted value of a firm with one unit of reserves (v^{Θ}) and an optimal equipment-to-reserves ratio, minus the value of the equipment. (Note that $\frac{1-\pi_t}{1+r} \cdot \int k_{t+1}{}^*\cdot v^{\Theta}_k \, dG^{\varepsilon} = k^*_{t+1}$ according to (A2.2b)). To compute the derivatives of q^*_t, note that (A2.4) and (A2.5a) imply

$$q_t = p_t - \chi\cdot(1+\beta)\cdot k_t{}^{-\nu}\cdot z_t{}^{\beta},$$

$$\Rightarrow \quad \frac{dq^*_t}{dp_t} = 1-\chi\cdot\beta\cdot(1+\beta)\cdot k_t{}^{-\nu}\cdot z_t{}^{\beta-1}.\frac{dz_t}{dp_t} = 1 - \chi_z\cdot\frac{1}{\chi_z}\cdot(1-\Omega_p) = \Omega_p > 0, \qquad \text{(A2.9a)}$$

$$\text{and} \quad \frac{dq^*_t}{d\pi_t} = -\chi\cdot\beta\cdot(1+\beta)\cdot k_t{}^{-\nu}\cdot z_t{}^{\beta-1}.\frac{dz_t}{d\pi_t} = -\Omega_{\pi\Theta} + \Omega_{\pi} < 0, \qquad \text{(A2.9b)}$$

where the signs apply under the assumptions stated in Step 1. Overall, higher political risk reduces the value of reserves while higher current oil prices (signaling higher future prices) raise the value of reserves.

Step 3: Oil exploration

Now consider the activities of an oil exploration firm. The firm owns the country's hidden reserves, incurs cost to discover the reserves, and sells newly discovered (previously hidden) reserves $\Theta_t = H_t - H_{t+1}$ at the competitive price $q_t = q^*(p_t,\pi_t)$ to production firms. Since discoveries are a non-linear function of wells drilled, we have to assume that there is a single firm; the competitiveness assumption is still justifiable because of international competition.

The profit function of the exploration firm is then

$$PR^X_t = q^*(p_t,\pi_t)\cdot(H_t - H_{t+1}) - c\Big(F^{-1}(\Gamma - H_{t+1}) - F^{-1}(\Gamma - H_t)\Big), \qquad \text{(A2.10a)}$$

as function of hidden reserves, or

$$PR^X_t = q^*(p_t,\pi_t)\cdot[F(D_{t+1}) - F(D_t)] - c(D_{t+1} - D_t), \qquad \text{(A2.10b)}$$

as a function of cumulative wells drilled. The implied dynamic programming problem is

$$V^X(H_t,p_t,\pi_t,\xi_t;\Gamma) = \max_{H_{t+1}} \Big\{PR^X_t + \frac{1-\pi_t}{1+r} \cdot \int V^X(H_{t+1}, f^p(p^F_t,\varepsilon^p_{t+1}), f^\pi(\pi_t,\varepsilon^\pi_{t+1}), 0;\Gamma)\cdot dG^\varepsilon(\varepsilon^p_{t+1},\varepsilon^\pi_{t+1})\Big\}. \qquad \text{(A2.11a)}$$

The first order conditions for H_{t+1} is then

$$q^*(p_t,\pi_t) = c'\cdot(F^{-1})'(\Gamma - H_{t+1}) + \frac{1-\pi_t}{1+r} \cdot \int V^X_H \, dG^\varepsilon \qquad \text{(A2.11b)}$$

a simplified version of (12c). Intuitively, the price of reserves must compensate for the drilling cost and for the decline of hidden reserves. Taking the total differential in terms of drilling cost, one finds

$$\begin{aligned} q_p\cdot dp_t + q_\pi\cdot d\pi_t = & -\frac{c'(\Delta D_{t+1})\cdot F''(D_{t+1})}{F'(D_{t+1})^2}\cdot dD_{t+1} + \frac{c''(\Delta D_{t+1})}{F'(D_{t+1})}\cdot(dD_{t+1} - dD_t) \\ & + \Big[-\frac{1}{1+r}\cdot\int V^X_H dG^\varepsilon\Big] d\pi_t + \Big[\frac{1-\pi_t}{1+r} \cdot\int V^X_{H\pi} f^\pi_\pi dG^\varepsilon\Big]\cdot d\pi_{t+1} \\ & - \Big[\frac{1-\pi_t}{1+r} \cdot\int V^X_{HH} dG^\varepsilon\Big]\cdot F'(D_{t+1})\cdot dD_{t+1} \\ & + \Big[\frac{1-\pi_t}{1+r} \cdot\int V^X_{Hp} f^p_p dG^\varepsilon\Big]\cdot dp_{t+1} \end{aligned}$$

$$=> \quad [c'\cdot(-F'')/F''^2 + c''/F' - \Omega_{HH}\cdot F']\cdot dD_{t+1}$$

$$= [\Omega_H-\Omega_{H\pi}+q_\pi]\cdot d\pi_t + [-\Omega_{Hp}+q_p]\cdot dp_t + c''/F'\cdot dD_t \qquad \text{(A2.12)}$$

where
$$\Omega_H = \frac{1}{1+r}\cdot\int V^X{}_H dG^\varepsilon$$
$$\Omega_{HH} = \frac{1-\pi_t}{1+r}\cdot\int V^X{}_{HH} dG^\varepsilon$$
$$\Omega_{H\pi} = \frac{1-\pi_t}{1+r}\cdot\int V^X{}_{H\pi} f^\pi{}_\pi dG^\varepsilon$$
$$\Omega_{Hp} = \frac{1-\pi_t}{1+r}\cdot\int V^X{}_{Hp} f^p{}_p dG^\varepsilon.$$

Note immediately that if r is sufficiently high (and/or π_t high), the Ω-expressions will be small. Since $c'\cdot(-F'')/F''^2 + c''/F'>0$, $q_\pi<0$, $q_p>0$, and $c''/F'>0$, drilling D_{t+1} will be a decreasing function of political risk and an increasing function of the oil price; D_{t+1} will also be an increasing function of D_t, and ΔD_{t+1} is a declining function of D_t.

In general, the envelope theorem implies

$$V^X{}_H = \frac{\partial PR^X{}_{t+1}}{\partial H_{t+1}} = q_{t+1} - c'(\Delta D_{t+2})/F'(D_{t+1}) \qquad \text{(A2.13)}$$

Because of free disposal, we have $V^X\geq 0$ and $V^X{}_H\geq 0$, hence $c'(\Delta D_{t+2})/F'(D_{t+1}) \leq q_{t+1}$ for all t. Also,

$$V^X{}_{HH} = c'{}_{t+1}\cdot F''{}_{t+1}/F'{}_{t+1}{}^2\cdot\frac{dD_{t+1}}{dH_{t+1}} - c''/F'{}_{t+1}\cdot[\frac{dD_{t+2}}{dH_{t+1}} - \frac{dD_{t+1}}{dH_{t+1}}]$$
$$= \frac{c'{}_{t+1}\cdot(-F''{}_{t+1})}{F_{t+1}'^3} + \frac{c''{}_{t+1}}{F'{}_{t+1}{}^2}\cdot\frac{d\Delta D_{t+2}}{dD_{t+1}} \qquad \text{(A2.14a)}$$
$$V^X{}_{Hp} = q_{P,t+1} - c''{}_{t+1}/F'{}_{t+1}\cdot\frac{dD_{t+2}}{dp_{t+1}} \qquad \text{(A2.14b)}$$
$$V^X{}_{H\pi} = q_{\pi,t+1} - c''{}_{t+1}/F'{}_{t+1}\cdot\frac{dD_{t+2}}{d\pi_{t+1}} \qquad \text{(A2.14c)}$$

and the derivatives of the optimal drilling policy are

$$\frac{dD^*{}_{t+1}}{dD_t} = \frac{1}{\Omega^*}\cdot(c''\cdot F'), \qquad \text{(A2.15a)}$$
$$\frac{dD^*{}_{t+1}}{dp_t} = \frac{1}{\Omega^*}\cdot(q_p-\Omega_{Hp}), \qquad \text{(A2.15b)}$$
$$\frac{dD^*{}_{t+1}}{d\pi_t} = \frac{1}{\Omega^*}\cdot(q_\pi+\Omega_H-\Omega_{H\pi}) \qquad \text{(A2.15c)}$$

and
$$\frac{d\Delta D^*{}_{t+1}}{dD_t} = \frac{1}{\Omega^*}\cdot[-c'\cdot(-F'')/F''^2+\Omega_{HH}\cdot F']. \qquad \text{(A2.15d)}$$

where $\Omega^* = c'\cdot(-F'')/F''^2+c''/F'-\Omega_{HH}\cdot F'$.

The second order condition for the optimal choice of H_{t+1} requires that $\Omega^*>0$, hence $dD^*_{t+1}/dD_t>0$.

To compute the impact of changing oil prices, one may use a limit & induction argument as above. Since $q_P\geq 0$, $dD^*_{t+1}/dp_t\geq 0$ applies in the terminal period T of a finite horizon problem. If $dD^*_{t+2}/dp_{t+1}\geq 0$ applies in some period t+1, (A2.14b) and (A2.13b) imply that

$$\Omega_{Hp}{}^t = \frac{1-\pi_t}{1+r}\cdot\int q_{P,t+1}f^p{}_p dG^\varepsilon - \frac{1-\pi_t}{1+r}\cdot\int c''_{t+1}/F'_{t+1}\cdot\frac{dD_{t+2}}{dp_{t+1}}f^p{}_p dG^\varepsilon \quad \text{(A2.16)}$$
$$\leq \frac{1}{1+r}\cdot\int q_{P,t+1}f^p{}_p dG^\varepsilon < q_{P,t}$$

in period t, provided

$$\frac{1}{1+r}\cdot\int q_{P,t+1}f^p{}_p dG^\varepsilon < q_{P,t}. \qquad (*)$$

Then $\Omega_{Hp}{}^t<q_{P,t}$, which implies $dD^*_{t+1}/dp_t>0$ in period t according to (A2.15b), proving the induction. Since $f^P{}_p\leq 1$ and $r>0$, condition (*) is only a mild restriction on the stochastic process of reserve prices. Taking $T\to\infty$ proves the argument for the infinite horizon. Thus, high oil prices increase drilling because they raise the current value of reserves ($q_{p,t}>0$) by more than the future value ($q_{P,t+1}f^p{}_p$).

The effect of political risk is more ambiguous. One the one hand, if $q_\pi+\Omega_H>0$, then one can establish $dD^*_{t+1}/d\pi_t<0$ by an argument analogous to the argument regarding oil prices and under a condition analogous to (*) (exploiting (A2.13a, A2.14a, A2.15a)). On the other hand, the sign of $q_\pi+\Omega_H$ is determined by two offsetting effects. First, political risk decreases the price of known reserves, $q_\pi<0$, which reduces exploration. Second, political risk creates an incentive to drill now, to discover and sell the reserves ahead of a potential expropriation. The economic intuition for the second argument is similar to the arguments in the production setting. The analogy suggests that a more elaborate modeling of the exploration process that takes into account cost of drilling equipment and time lags would reduce the incentive to drill, i.e., suggesting $dD^*_{t+1}/d\pi_t<0$; this a conjecture to be examined empirically.

Finally, consider the effect of past drilling on current drilling rates, $d\Delta D^*_{t+1}/dD_t$. To show that $d\Delta D^*_{t+1}/dD_t<0$, we have to show that $\Omega_{HH}\cdot F' < c'\cdot(-F'')/F''^2$, which is a slightly stronger condition than $\Omega^*>0$. Again, a limit and induction argument applies. Since $-c'\cdot(-F'')/F''^2<0$, $d\Delta D^*_{t+1}/dD_t<0$ applies in the terminal period T of a finite horizon problem. If $dD^*_{t+2}/dp_{t+1}\leq 0$ applies in some period t+1, (A2.14a) implies $V^X_{HH} \leq c'_{t+1}\cdot(-F''_{t+1})/F_{t+1}'^3$ and hence $\Omega_{HH}{}^t\leq \frac{1-\pi_t}{1+r}\cdot\int V^X_{HH}dG^\varepsilon\leq c'_{t+1}\cdot(-F''_{t+1})/F_{t+1}'^3$, which implies $d\Delta D^*_{t+1}/dD_t\leq 0$, proving the induction. Taking the limit, $d\Delta D^*_{t+1}/dD_t\leq 0$ must hold for the infinite horizon problem. To prove the strict inequality $d\Delta D^*_{t+1}/dD_t<0$, note that $\Omega_{HH}{}^t\leq \frac{1-\pi_t}{1+r}\cdot\int V^X_{HH}dG^\varepsilon<\frac{1}{1+r}\cdot c'_{t+1}\cdot(-F''_{t+1})/F_{t+1}'^3$, hence $d\Delta D^*_{t+1}/dD_t\leq -\frac{r}{1+r}\cdot c'_{t+1}\cdot(-F''_{t+1})/F_{t+1}'^3/\Omega^*<0$ for r>0. Hence, the current drilling rate ΔD_{t+1} is unambiguously a declining function of cumulative past drilling, D_t, or equivalently, an increasing function of the remaining reserves, H_t.

A3. The Optimal Forestry Policy

The forestry cost function in the text is motivated as follows. Let

$$Z_t = z_t\cdot F_t = A\cdot(N^F{}_t{}^\varphi\cdot K^F{}_t{}^{1-\varphi})^\gamma\cdot Y^F{}_t{}^\delta\cdot F_t{}^{1-\gamma-\delta} \quad (A3.1)$$

be the production function of harvested biomass, which is a function of forestry labor N^F (measured in human capital efficiency units), capital used in forestry K^F, other goods used in forestry Y^F, and the stock of forests F_t. The contribution of forest stocks can be interpreted as a congestion effect in this context. That is, the production function can be interpreted as representing production with constant returns to scale in N^F, K^F, and Y^F combined with a congestion effect that reduces the efficiency of harvesting when the forest area is small. The total cost of harvesting Z_t units is then

$$Cost_t = MPN_t \cdot N^F_t + MPK_t \cdot K^F_t + Y^F_t. \qquad (A3.2)$$

This reflects a unit price of output, Y^F_t, and economy-wide wage and capital rental rates, MPN and MPK, respectively, that equal the economy-wide marginal products of capital and labor from equation (4). Cost minimization for given Z_t implies

$$MPN_t = \frac{\varphi \cdot \gamma \cdot Z_t / N^F_t}{\delta \cdot Z_t / Y^F_t} = \alpha \cdot \frac{Y_t}{N_t \cdot H_t} \text{ and } MPK_t = \frac{(1-\varphi) \cdot \gamma \cdot Z_t / K^F_t}{\delta \cdot Z_t / Y^F_t} = (1-\alpha) \cdot \frac{Y_t}{K_t}$$

Substituting into (A3.2), cost minimization implies that

$$Cost_t = \frac{\varphi \cdot \gamma}{\delta} \cdot Y^F_t + \frac{(1-\varphi) \cdot \gamma}{\delta} \cdot Y^F_t + Y^F_t = \frac{\delta + \gamma}{\delta} \cdot Y^F_t. \qquad (A3.3)$$

These conditions imply that the inputs to forestry production are used in proportions that depend on the national output-capital and output-labor ratios and on the parameters of the forestry production function,

$$N^F_t / Y^F_t = (\gamma/\delta) \cdot (\varphi/\alpha) \cdot (N_t \cdot H_t)/Y_t, \text{ and } K^F_t / Y^F_t = (\gamma/\delta) \cdot (1-\varphi)/(1-\alpha) \cdot K_t / Y_t.$$

Substituting these ratios into (A3.1), the required input of goods for a given production level Z_t is given by

$$Z_t = A \cdot (\{(\gamma/\delta) \cdot (\varphi/\alpha) \cdot Y^F_t \cdot N_t \cdot H_t / Y_t\}^{\varphi}$$
$$\cdot \{(\gamma/\delta) \cdot ((1-\varphi)/(1-\alpha)) \cdot Y^F_t \cdot K_t / Y_t\}^{1-\varphi})^{\gamma} \cdot Y^F{}_t^{\delta} \cdot F_t^{1-\gamma-\delta}$$
$$= A^* \cdot Y^F{}_t^{\delta+\gamma} \cdot F_t^{1-\gamma-\delta} \cdot [K_t/(N_t \cdot H_t)]^{(\alpha-\varphi) \cdot \gamma},$$

where $A^* = A \cdot (\gamma/\delta)^{\gamma} \cdot (\varphi/\alpha)^{\varphi\gamma} \cdot [(1-\varphi)/(1-\alpha)]^{(1-\varphi)\gamma}$. Hence,

$$Y^F_t = (A^*)^{-1/(\delta+\gamma)} \cdot Z_t^{1/(\delta+\gamma)} \cdot F_t^{1-1/(\delta+\gamma)} \cdot [K_t/(N_t \cdot H_t)]^{-(\alpha-\varphi) \cdot \gamma/(\delta+\gamma)}$$

Substituting into (A3.3), the total cost of producing Z_t is

$$Cost_t = (\delta+\gamma)/\delta \cdot (A^*)^{-1/(\delta+\gamma)} \cdot Z_t^{1/(\delta+\gamma)} \cdot F_t^{1-1/(\delta+\gamma)} \cdot [K_t/(N_t \cdot H_t)]^{-(\alpha-\varphi) \cdot \gamma/(\delta+\gamma)}$$

The aggregate capital-labor ratio $K_t/(N_t \cdot H_t)$ matters only if $\alpha \neq \varphi$, i.e., if the capital intensities differ. If $\alpha = \varphi$, the “c” in equation (16) can be interpreted as the constant $c = (\delta+\gamma)/\delta \cdot (A^*)^{-1/(\delta+\gamma)}$. The exponent $1/(\delta+\gamma) > 1$ corresponds to $1+\beta$ in (16). If $\alpha \neq \varphi$, the “c” in equation (16) depends on the aggregate capital-labor ratio, which according to Section 2.2 depends on the political and other variables determining aggregate investment. It seems reasonable to assume that the capital intensity for forest harvesting is similar to that of the economy as a whole. Hence, we assume $\alpha \approx \varphi$ and treat c as constant in Section 2.4.

The optimal harvesting policy is derived as followed. The partial derivatives of the optimal policy function F^* are obtained by taking the total differential of (16). As before, we can write the integral $\int V_F \, dG$ in (16) as an integral over the marginal distributions of the innovations to p^F and π,

$$\int V_F dG = (1-\pi_t)\cdot\int V_F(f^p(p^F{}_t,\varepsilon^p{}_{t+1}),F_t,\bar{F},f^\pi(\pi_t,\varepsilon^\pi{}_{t+1}),1)\cdot dG^\varepsilon(\varepsilon^p{}_{t+1},\varepsilon^\pi{}_{t+1})$$

where $p^F{}_{t+1}=f^p(p^F{}_t,\varepsilon^p{}_{t+1})$ reflects the dependence of forestry prices on lagged prices and a stochastic component $\varepsilon^p{}_{t+1}$. For convenience, we write the production choice in terms of the scaled production variable

$$z_t = Z_t/F_t = 1 + g(f_t) - F_{t+1}/F_t,$$

where $$f_t = F_t/\bar{F}.$$

Since unit cost and the natural growth rate for biomass depend on Z_t, F_t, and $\bar{F}$ only through their respective ratios, the value function must be homogenous of degree one in F_t and $\bar{F}$. That is,

$$V(p^F{}_{t+1},F_t,\bar{F},\pi_{t+1},\xi_{t+1}) = \bar{F}\cdot V(p^F{}_{t+1},F_t/\bar{F},1,\pi_{t+1},\xi_{t+1})$$

and $$V_F(p^F{}_{t+1},F_t,\bar{F},\pi_{t+1},\xi_{t+1}) = V_F(p^F{}_{t+1},f_t,1,\pi_{t+1},\xi_{t+1})$$

Equation (16) then reduces to

$$p^F{}_t - c\cdot(1+\beta)\cdot z_t{}^\beta = \frac{1-\pi_t}{1+r}\cdot\int V_F(p^F{}_{t+1},f_{t+1},1,\pi_{t+1},\xi_{t+1})\cdot dG^\varepsilon$$

and the total differential is

$$dp^F{}_t - c\cdot\beta\cdot(1+\beta)\cdot z_t{}^{\beta-1}\cdot dz_t = -[\frac{1}{1+r}\cdot\int V_F dG^\varepsilon]d\pi_t + [\frac{1-\pi_t}{1+r}\cdot\int V_{F\pi}\cdot f^\pi{}_\pi dG^\varepsilon]\cdot d\pi_t$$
$$+ [\frac{1-\pi_t}{1+r}\cdot\int V_{Fp}\cdot f^p{}_p dG^\varepsilon]\cdot dp^F{}_t + [\frac{1-\pi_t}{1+r}\cdot\int V_{FF}dG^\varepsilon]\cdot df_{t+1} \qquad \text{(A3.4)}$$

Since $f_{t+1} = (1+g(f_t)-z_t)\cdot f_t$, we have

$$df_{t+1} = [1+g(f_t)+g'(f_t)\cdot f_t - z_t]\cdot df_t - f_t\cdot dz_t.$$

The partial derivatives of the optimal policy function are then

$$\frac{\partial z^*_t}{\partial f_t} = \frac{1}{\Omega_z}\cdot\frac{1-\pi_t}{1+r}\cdot\int(-V_{FF})dG^\varepsilon\cdot[1+g(f_t)+g'(f_t)\cdot f_t - z_t] \qquad \text{(A3.5a)}$$

$$\frac{\partial z^*_t}{\partial p^F_t} = \frac{1}{\Omega_z}\cdot\left[1+\frac{1-\pi_t}{1+r}\cdot\int(-V_{Fp})\cdot f^p{}_p dG^\varepsilon\right] \qquad \text{(A3.5b)}$$

$$\frac{\partial z^*_t}{\partial \pi_t} = \frac{1}{\Omega_z}\cdot\left[\frac{1}{1+r}\cdot\int V_F dG^\varepsilon + \frac{1-\pi_t}{1+r}\cdot\int(-V_{F\pi})\cdot f^\pi{}_\pi dG^\varepsilon\right] \qquad \text{(A3.5c)}$$

where $\quad \Omega_z = c\cdot\beta\cdot(1+\beta)\cdot z_t^{\beta-1} + \frac{1-\pi_t}{1+r}\cdot\int(-V_{FF})dG^\varepsilon\cdot f_t.$

To compute the derivatives of the value function, note that the envelope theorem implies that

$$V_F(\cdot) = \frac{\partial PR_{t+1}(F_t,F_{t+1})}{\partial F_{t+1}} = [1+g(f_{t+1})+g'(f_{t+1})\cdot f_{t+1}]$$
$$\cdot[p^F{}_{t+1} - c\cdot z_{t+1}{}^\beta\cdot(1+\beta)] + c\cdot\beta\cdot z_{t+1}{}^{1+\beta}$$

can be written as a function of $p^F{}_{t+1}$, f_t, and z_{t+1}. Hence,

$$V_{FF}(\cdot) = [2\cdot g'(f_{t+1})+g''(f_{t+1})\cdot f_{t+1}]/\bar{F}\cdot[p^F{}_{t+1} - c\cdot z_{t+1}{}^\beta\cdot(1+\beta)]$$
$$- \Omega_{zz}/\bar{F}\cdot\frac{\partial z^*{}_{t+1}}{\partial f_{t+1}} \qquad \text{(A3.6a)}$$

$$V_{Fp}(\cdot) = [1+g(f_{t+1})+g'(f_{t+1})\cdot f_{t+1}] - \Omega_{zz}\cdot\frac{\partial z^*{}_{t+1}}{\partial p^F{}_{t+1}} \qquad \text{(A3.6b)}$$

$$V_{F\pi}(\cdot) = -\Omega_{zz}\cdot\frac{\partial z^*{}_{t+1}}{\partial \pi_{t+1}} \qquad \text{(A3.6c)}$$

where $\quad \Omega_{zz} = c\cdot z_{t+1}{}^{\beta-1}\cdot(1+\beta)\cdot[1+g(f_{t+1})+g'(f_{t+1})\cdot f_{t+1} - z_{t+1}].$

To determine the signs of the partial derivatives of $V(\cdot)$, we proceed as in the previous section, showing sign restrictions for the analogous finite horizon problem by induction and taking the limit $T\to\infty$. In the last period of an infinite horizon problem, $z_T = [p^F{}_T/(c\cdot(1+\beta))]^{1/\beta}$ depends only on $p^F{}_T$. Hence $V_F{}^1(p^F{}_T, f_T, 1, \pi_T, 1) = [p^F{}_T/(1+\beta)]^{(1+1/\beta)}\cdot\beta/c^\beta$ is positive and increasing in $p^F{}_T$, $V_{Fp}{}^1>0$ (recall that the superscript of V denote the decision horizon); $V_F{}^1$ does not depend on f_T and π_T. For period t=T-1, $V_F{}^1>0$ implies $p^F{}_t - c\cdot z_t{}^\beta\cdot(1+\beta)>0$; hence (A3.6a) and $g'<0$, $2\cdot g'(f_t)+g''(f_t)\cdot f_t<0$ imply

$$V_{FF}{}^2 = [2\cdot g'(f_t)+g''(f_t)\cdot f_t]/\bar{F}\cdot[p^F{}_t - c\cdot z_t{}^\beta\cdot(1+\beta)] < 0.$$

For the induction, suppose $V_{FF}{}^n<0$ for some period t+1 = T-n. Then

$$\Omega_z{}^n = c\cdot\beta\cdot(1+\beta)\cdot z_{t+1}{}^{\beta-1} + \frac{1-\pi_t}{1+r}\cdot\int(-V_{FF}{}^n)dG^\varepsilon\cdot f_t > 0,$$

and in period $t = T - (n+1)$, $1+g(f_t)+g'(f_t)\cdot f_t \geq 1 \geq z_t$ implies

$$\frac{\partial z^*_t}{\partial f_t} = \frac{1}{\Omega_z{}^n} \cdot \frac{1-\pi_t}{1+r} \cdot \int (-V_{FF}{}^n) dG^\varepsilon \cdot [1+g(f_t)+g'(f_t)\cdot f_t - z_t] > 0,$$

and $$\Omega_{zz}{}^{n+1} = c\cdot z_t{}^{\beta-1}\cdot(1+\beta)\cdot[1+g(f_t)+g'(f_t)\cdot f_t - z_t] > 0.$$

Moreover, $V_F>0$ implies $p^F{}_t - c\cdot z_t{}^\beta\cdot(1+\beta) > 0$ and

$$V_{FF}{}^{n+1}(\cdot) = [2\cdot g'(f_t)+g''(f_t)\cdot f_t]/\bar{F}\cdot[p^F{}_t - c\cdot z_t{}^\beta\cdot(1+\beta)] - \Omega_{zz}{}^{n+1}/\bar{F}\cdot\frac{\partial z^*_t}{\partial f_t} < 0,$$

which proves that $V_{FF}{}^n<0$ for all n. This shows that in the limit, $V_{FF}(\cdot)\leq 0$ and $\partial z^*/\partial f_t \geq 0$. But if $\partial z^*/\partial f_t \geq 0$, (A3.6a) implies the strict inequality $V_{FF}(\cdot)<0$.

For $\partial z^*/\partial \pi_t$, we start with $V_{F\pi}{}^1=0$; (A3.5c) at t=T-1 then implies $\partial z^*_t/\partial \pi_t = \frac{1}{1+r}\cdot\int V_F dG^\varepsilon/\Omega_z > 0$; (A28c) at t=T-2 then implies $V_{F\pi}{}^2<0$, which reinforces the argument for $\partial z^*_t/\partial \pi_t>0$ in (A3.5c). Induction and limit arguments analogous to the ones above show that $\partial z^*/\partial \pi_t>0$ and $V_{F\pi}<0$ apply for the infinite horizon problem.

For $\partial z^*/\partial \pi_t$, the argument is somewhat more complicated, because higher current prices tend to raise z_t while the expectation of higher future prices tends to lower z_t. In any case, if the persistence in prices, $f^\pi{}_\pi$, is sufficiently low, (A13b) implies $\partial z^*/\partial \pi_t>0$; this is assumed.

Overall, we have shown that $z_t{}^*(F_t/\bar{F}, p^F{}_t, \pi_t)$ depends positively on all three arguments. Through z_t, $F_{t+1} = F_t(1+g(F_t/\bar{F})-z_t)$ and $(F_{t+1}-F_t)/F_t = g(F_t/\bar{F})-z_t$ therefore depend negatively on $p^F{}_t$ and π_t. Since $g'<0$ and $\partial z^*/\partial f_t>0$, $(F_{t+1}-F_t)/F_t$ depends negatively on F_t and $(F_{t+1}-F_t)/F_t$ and F_{t+1} both depend positively on $\bar{F}$. Since $1+g+g'\cdot f > 1 \geq z_t$, we also have $\partial F_{t+1}/\partial F_t>0$.

Supplement to the Appendix to:

Ownership Risk, Investment, and the Use of Natural Resources

by Henning Bohn and Robert T. Deacon
Department of Economics
University of California at Santa Barbara
Santa Barbara, CA 93106

November 1998

The appendix enclosed with the previous version of the paper remains unchanged, except that we have taken the discussion of endogenous and exogenous growth out of Section 2.1 and moved it into the appendix. To avoid duplication, we are only enclosing the new section of the appendix, as shown below. The complete appendix is avaialble upon request.

Human Capital Accumulation

This section of the appendix explains why the regression model (8) is consistent with both exogenous and endogenous human capital accumulation. This issue deserves comment because productivity and its determinants are, at best, imperfectly measured and because the exact interpretation of the proxies for human capital, such as schooling variables, depends on the model of human capital accumulation.

Suppose human capital is produced according to a production function

$$H_{t+1} = H_t^{\beta} \cdot h(x_t) + (1-\delta_h) \cdot H_t,$$

where $0 \leq \beta \leq 1$ and $0 \leq \delta_h \leq 1$. If $\beta<1$ and x_t is stationary, human capital will converge to a stochastic steady state. In this case, a country's mean level of human capital is a weighted average of past investments. Hence x_t and H_t

in (8) can be proxied by current and past schooling rates and trade variables.[2]

If β=1, the long run growth rate of the economy is endogenously given by $g_H = h(x_t)-\delta$. Then H_t does not converge to a steady state and, because $g_H(\cdot)$ does not depend on H_t in this case, the economy's optimal K_{t+1} depends on $K_t/(N_t \cdot H_t)$, x_t, π_t, and ξ_t, but not on H_t separately. Using the production function as before to replace K_t, the investment share of output can be written as

$$(\frac{I_t}{Y_t}) = i^*(\frac{Y_t}{N_t \cdot H_t}, x_t, \pi_t, \xi_t).$$

Although K_t/N_t and Y_t/N_t do not converge to steady states in this model, I_t/Y_t and $Y_t/(N_t \cdot H_t)$ do. Further, the balanced growth prediction implies that $K_t/(N_t \cdot H_t)$ and $Y_t/(N_t \cdot H_t)$ might show little sample variation. Instead of trying to find proxies for H_t one might therefore omit these regressors and subsume them into the error term. The above regression specification reduces to $(I_t/Y_t) = i^*(x_t, \pi_t, \xi_t)$ in this case. Then schooling variables should be interpreted as proxies for x_t.

Overall, both of the specifications with endogenous growth (with and without $Y_t/(N_t \cdot H_t)$) are restricted versions of equation (8). Without making judgments about the nature of human capital accumulation, we estimate eq.(8) without restrictions and let the data determine the significance of Y_t/N_t and/or H_t. This approach yields consistent coefficient estimates whether or not growth is endogenous.

---

[2] A potential empirical concern is that an investment model that uses past schooling as proxy for H_t could suffer from an omitted variables bias because some components of H_t are not measured. This would seem especially problematic if the political variables are correlated with output, because output depends on the true H_t, i.e., is correlated with the unobserved components of H_t. Nonetheless, if output is included as a regressor, as in (8), the coefficient on π_t will be consistent provided π_t is conditionally (conditional on Y_t/N_t) uncorrelated with H_t. Only the coefficients on output and on the proxies for human capital would be biased.