# Low Altruism, Austerity, and Aversion to Default:
# Are Countries Converging to the Natural Debt Limit?

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**Abstract**

Democracies around the world are making promises to the old at the expense of future generations. I interpret this as reflecting *low altruism*—a discount rate on children's utility greater than the world interest rate—and I examine the implications in a small open economy with overlapping generations. A focus is on the public sector: The model includes public capital in production and public education as determinant of human capital. I examine to what extent both are crowded out by spending on debt and retiree entitlements. In the model, altruism towards children determines bequests, government debt, and the time-path of consumption. Altruism towards parents influences incentives to default. If altruism is low, voters demand fiscal policies that extract substantial resources from future generations. Public debt rises until debt service requires maximum taxes forever, and an era of austerity ensues: investment in human capital declines to a lower bound, and reduced human capital discourages investment in private and public capital. The threat of default enters as a constraint that may protect future generations.

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## 1. Introduction

This paper is about the efficient exploitation of future generations, and why low altruism must lead to either persistent austerity, a debt crisis, or austerity followed by a debt crisis.

For the last several decades, public debts and unfunded pension obligations have been rising in many countries, along with current account deficits and increasing foreign debts. The debt crisis in southern Europe and the growing concerns about U.S. debt have focused attention on these long-simmering problems.

The paper interprets the growth in public debts as symptom of low intergenerational altruism operating in a political/financial setting that makes government default extremely costly.[1] The paper shows that the fate of an open economy with prohibitive costs of default is rather unpleasant: After a phase of growing debts, taxes hit an upper limit, and a phase of public sector austerity starts. Public spending on education contracts and converges to a lower bound. Reduced human capital discourages investment in private and public capital. Public debt converges to a natural debt limit. International openness matters because it blocks the crowding out mechanism that under autarchy would raise interest rates.

Debt is limited when default is a credible threat. Default in effect allows the young to revolt against austerity. Hence the paper also examines conditions for default. Altruism is again a key factor: Children's altruism toward parents discourages default. Thus altruism matters in two ways, as determinant of public debt and as a limiting factor.

The assumption of cross-country differences in altruism parallels Song et al. (2012). In addition, I allow altruism to vary over time.[2] Whereas Song et al.'s main calibration assumes that the impact of low altruism on debt is contained by a rising excess burden of taxation, I examine the consequences of low altruism without such a containment mechanism. Debt and

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[1] For instance, Greece has been threatened by expulsion from the European union and/or from Eurozone. More generally, international law holds a country's population unconditionally responsible for external obligations incurred by prior governments.

[2] In many countries, debt-GDP ratios have increased since the 1960s, suggesting that altruism may have declined over time. One may speculate if social changes have reduced the linkages between current voters and the next generation (e.g., more zero-child families, single parenthood, increased divorce rates), or if international financial openness has exposed long-standing differences in time preference. Regardless of the causes of low altruism, the objective here is to examine the ramifications.

taxes are bounded, as in Trabandt and Uhlig (2012); but instead of a Laffer curve, I model labor income taxes as non-linear with participation constraint (simple Mirrleesian). This yields a separation between tax revenues and tax distortions. Tax rates and the size of the public sector are determined jointly, following Pigovian logic, and rising debts are financed efficiently. Importantly, low altruism is powerful even at the tax limit, which is reached after a finite number of periods. Thereafter, low altruism triggers cutbacks in public spending—an era of austerity—as debt converges asymptotically to the natural debt limit.

The analysis is presented in a small open overlapping-generations economy. There are three generations: retired old, a working-age (middle) generation, and a young generation that needs education to be productive in the next period.[3] The middle generation controls the government and faces the problem of succession planning—how to ensure the generation's well-being in retirement, when the next generation is in control. Government succession creates time-inconsistency problems if the next generation may default or impose capital levies. These well-known problems have long served as justification for institutions that make default costly. The resulting focus on strong commitments has a downside: it allows governments controlled by selfish voters to extract resources from future generations. This is a problem if voters discount the welfare of future generations at a higher rate than the market interest rate. Then the fiscal dynamics are analogous to the classic savings problem:[4] the government keeps borrowing. In many countries, government commitments have in fact expanded greatly. Domestic commitments include not only government bonds but also vast promises to pay for public pensions, government employee pensions, and retiree health care; in most countries, these are largely unfunded.

In the model, public debt allows the old to leave negative bequests and to burden future generations, as in Cukierman and Meltzer (1989) and Bohn (1992). As in Breyer (1994), altruism is the key determinant of equilibrium debt. A maintained assumption is that

[3] There is no intra-generational heterogeneity. The time scale is generational (about 20 years) and hence not suitable for studying short-run issues, such as a sudden debt crisis.

[4] See Ljungqvist and Sargent (2004; ch.16-17) for a textbook exposition of the savings problem.

governments represent their voters. This differs from the political economy literature that attributes rising public debt to political frictions (e.g., Barseghyan and Battaglini, 2012; Aguiar and Amador, 2010). The policy and welfare implications are quite different, because if voters are to blame for rising debts, political reforms are unlikely to help.

A major simplifying assumption is the absence of economic uncertainty. This is to focus on the growth paths around which real economies fluctuate. With perfect foresight, default is a threat that keeps the economy on the edge of defaulting, but defaults do not occur in equilibrium.

The paper is organized as follows. Section 2 presents some illustrative data. Section 3 presents the model. Section 4 describes the equilibrium path with prohibitive default cost. Section 5 examines the threat of default. Section 6 concludes.

## 2. Evidence on International Creditor and Debtors

Cross-country differences in altruism are a plausible motive for debt accumulation. This section briefly presents some data on international credit. Table 1 lists the top-10 international creditors and debtors as ranked by their Net Investment Position in 2005, before the financial crisis. Of the 111 countries for which IMF data provide data, 22 had positive net positions and 89 had negative positions. The top-10 creditors account for 96% of total positive positions and the top-10 debtors account for 74% of total negative positions (see col.1).[5] Col.2 shows net investment positions in percent of own GDP.

Table 1 shows that international credit is highly concentrated. Japan, Germany, and China account for more than 50% of world credit—more than 60% if Hong Kong were subsumed into China. Special cases—financial centers and oil exporters—account for most of the remainder.[6] International net borrowing is more widely distributed. The United States are

[5] Two caveats: (a) There is a substantial statistical discrepancy: the sum of positive positions is about 30% less than the sum of negative positions. This might suggest significant underreporting of assets, as one might expect if capital flight is an issue. (b) Data for Saudi Arabia are incomplete for 2005. Because results without this major creditor would be biased, 2007 data were used.

[6] Financial centers such as Switzerland, Singapore, and Hong Kong are special because their assets may include substantial deposits by foreign nationals. For oil exporters, consumption smoothing over uneven resource flows provides special motives for lending.

clearly the largest debtor (30% of total) but below average as share of GDP. These data are consistent with the notion that most of the world borrows from a few patient lenders.

The current account balances in Column 3 suggests that international credit is growing. All 10 top creditors in 2005 had current account surpluses for 2006-10, and 9 of the top-10 debtors had current account deficits. This is difficult to reconcile with the long-run sustainability requirement that debtors run surpluses to stabilize their net debt positions (Bohn 1998). However, many countries have only opened their capital account in the last 20-30 years, and debt accumulation is a long-term process that evolves over generations.[7] Hence the data are consistent with the notion that the world is in a transitional state, where after capital account opening, countries that place relatively low welfare weights on future generations borrow from countries that value future generations more highly.

Table 2 displays fiscal data for the same international creditors and debtors. The main insight is that Japan is an outlier: The biggest international lender also has the highest public debt/GDP.[8] Excluding Japan, the international creditors have somewhat lower public debt ratios than the borrowers and a positive average budget balance. Since these differences are to some extent driven by oil country surpluses, they should be considered illustrative apart from flagging Japan as outlier. Pension promises (col.3) are also comparable, with Japan looking relatively stingy.[9]

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[7] For earlier periods, the Feldstein-Horioka (1980) evidence applies.

[8]. Singapore, Hong Kong, and Switzerland had budget surpluses, and deficits in China, Germany, and Belgium were small. However, Singapore and Belgium had higher debt than most of the international debtors.

[9] Japan may deserve an interpretation in the context of the model below: The country's international creditor position suggests strong intergenerational altruism. In the model, governments backed by retirement savers will always—unless default is a threat—to create enough claims against future governments to ensure that their generation will not be bequest-constrained in retirement. In a country with long life expectancy and relatively stingy pensions, this requires high public debt. The main caveat concerns default. In the model, default incentives depend on children's altruism toward their parents. For Japan, the respect for elders enshrined in Shinto culture suggests a high debt-tolerance. Thus Japan's fiscal data are consistent with an altruistic model.

## 3. The Model

This section lays out an overlapping generations model that is tractable yet sufficiently rich to study the ramifications of debt constraints.

### *3.1. Population, Government, and Production*

The economy is populated by overlapping generations that each live for three periods. The life cycle consists of young/student-age, middle/working-age, and retirement/old-age. Each worker has one student-age child. Students are economically inactive except that they consume public education. Workers supply labor, consume, and save. Retirees live off their savings. There is two-sided altruism that may trigger bequests or gifts. The model abstracts from population growth and from economic uncertainty.[10] Each generation is a continuum with mass one, so per-capita units can be interpreted as aggregates. Generations are dated by their work period: generation t is young in period t-1, working in period t, and retired in period t+1.

Individuals are price-takers on markets but recognize the aggregate consequences of their voting choices. The essence of democratic government is the power to set policy without regard to promises made by prior governments. Assume the working generation has the majority of votes and controls all government decisions in a period. This provides a simple modeling of government turnover and the resulting time-consistency problems.[11] This includes the power to default on debt and to impose capital levies.

Public debt is motivated— historically and in the model—by a government role in production: Governments provide public capital and public education. Without government

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[10] Population growth could be added easily but would clutter the notation. Young-age consumption and wages could also be added, e.g., as in Bohn (2006), but also complicate the analysis. Economic uncertainty would raise challenging issues of international and intergenerational risk sharing and it would distract from the main points; adding stochastic shocks under the assumption of no risk sharing would be straightforward but not insightful.

[11] For example, suppose the young are ages 0-29, the middle 30-59, and the old are 60 and up. Only a fraction of the young can vote and the retirement period can be viewed as fractional, too, in the sense retirees are "alive" only for part of the period. For example, if the voting age is 18 and life expectancy is 75, 40% of the young can vote, only 50% of the old would be alive and voting, and the middle generation represents 30 of 57 voting cohorts, which is a majority. The example is not unrealistic: U.S. population shares in 2009 were 41% young, 41% middle, and 18% old. Excluding ages 0-17 from voting, the voting shares are 22% young, 54% middle, and 24% old. The interpretation of old age as a fractional period follows Bohn (2001), which provided further discussion.

investment, laissez-faire would be production-efficient and hence the young would have no reason to accept institutional arrangements that make default costly. The investment motive for debt is well recognized with regard to physical "infrastructure" capital—even stringent balanced budget rules make exceptions for investment (e.g. in U.S. states and in Germany). Boldrin and Montes (2005) argue that the same reasoning applies to education and may justify the concept of retiree "entitlements"—claims on government that future generations are expected to honor just like public debt.[12]

The problems of government succession and time-inconsistency are as old as human civilization. The maintained assumption here is that successful cultures have found commitment devices that make government defaults and capital levies costly—essentially as a precondition for development. Many such devices may fall outside the fixed-preferences paradigm of economies, e.g., the practice of teaching children to keep promises and to respect their elders. Costs of default are therefore modeled in two ways: as loss of utility (detailed later) and as output costs.[13]

Specifically, assume aggregate market-economy output is produced from labor, private capital, and public capital:

$$Y_t = F(L_t, K_t, K_t^G) \cdot (1 - \Lambda_t^Y). \tag{1}$$

Labor inputs $L_t = l_t H_t$ are a product of work time $l_t$ and per-capita human capital $H_t$. Private capital is augmented by gross investment $I_t \geq 0$ and depreciates between periods at rate $\rho \in [0,1]$, so $K_{t+1} = I_t + (1-\rho)K_t$. Public capital is provided by government investments $I_t^G \geq 0$ and accumulates according to $K_{t+1}^G = I_t^G + (1-\rho)K_t^G$, with same $\rho$-value for simplicity. The factor $\Lambda_t^Y \in [0,1)$ captures output costs of default.

[12] A multiplicity of "good reasons" for public debt also motives why it is difficult to limit such debt once mechanisms required supporting commitment have been established.

[13] The latter is included because it is common in the literature, motivated e.g. by trade sanctions or breakdowns in cooperation a la Cole-Kehoe (1998). A utility cost may be interpreted as resulting from preferences over intangibles ("respect") that parents produce at zero cost and give to their children if and only if they do not default. The objective here is not to explain why default is costly, but to explain why very high default costs—too much commitment—can be problematic, too.

Human capital requires public spending on education in the previous period, $G_{t-1}^h$. To allow secular economic growth, define $h_t = H_t / \Gamma_t$ and assume that $G_{t-1}^h = \Gamma_{t-1}\chi(h_t)$ shifts over time by a productivity index $\Gamma_t = (1+\gamma_t)\Gamma_{t-1}$. The cost function $\chi$ is assumed increasing and strictly convex.[14] Growth rates $\gamma_t \geq 0$ are exogenous.

Public capital poses challenges for optimal policy: it may be a source of pure profits, and non-excludability may cause congestion externalities. For clarity, suppose

$$F(L,K,K^G) = L^{\varphi_L} K^{\varphi_K} (K^G)^{\varphi_G}$$

has Cobb-Douglas shape with factor shares $\varphi_L, \varphi_K, \varphi_G > 0$. Empirical evidence suggests non-decreasing overall returns to scale ($\varphi_L + \varphi_K + \varphi_G \geq 1$), non-increasing returns to private factors ($\varphi_L + \varphi_K \leq 1$), and decreasing returns to overall capital ($\varphi_K + \varphi_G < 1$). To model congestion externalities, suppose an individual firm that uses private inputs $(\tilde{L}_t, \tilde{K}_t)$ can appropriate a fraction $\tilde{K}_t^G / K_t^G = (\tilde{L}_t / L_t)^{\zeta_L} (\tilde{K}_t / K_t)^{\zeta_K}$ of public capital, where $\zeta_L, \zeta_K \geq 0$. Assuming competitive factor markets, the firm-level marginal products of labor and capital are then

$$\tfrac{dY_t}{d\tilde{L}_t} = (1-\Lambda_t^Y)\cdot [F_L + F_G \cdot \zeta_L \, \tilde{K}_t^G / K_t^G] = (\varphi_L + \zeta_L \varphi_G)\cdot Y_t / \tilde{L}_t \equiv \tilde{w}_t \qquad (2)$$

and

$$\tfrac{dY_t}{d\tilde{K}_t} = (1-\Lambda_t^Y)\cdot [F_K + F_G \cdot \zeta_K \, \tilde{K}_t^G / K_t^G] = (\varphi_K + \zeta_K \varphi_G)\cdot Y_t / \tilde{K}_t \equiv \tilde{r}_t . \qquad (3)$$

They define competitive wage and rental rates $(\tilde{w}_t, \tilde{r}_t)$.

Three observations follow. First, because the aggregate marginal products $\frac{dY_t}{dL_t} = \varphi_L \frac{Y_t}{L_t}$ and $\frac{dY_t}{dK_t} = \varphi_K \frac{Y_t}{K_t}$ differ from (2-3), the congestion externality provides Pigovian motives for taxing capital and labor—at rates $\xi_L = \frac{\zeta_L \varphi_G}{\varphi_L + \zeta_L \varphi_G}$ and $\xi_K = \frac{\zeta_K \varphi_G}{\varphi_K + \zeta_K \varphi_G}$, respectively. Second, if $(\varphi_L + \zeta_L \varphi_G) + (\varphi_K + \zeta_K \varphi_G) < 1$, firms earn pure profits even with Pigovian taxes. Third, if $\varphi_L + \varphi_K + \varphi_G > 1$, the government must have access to additional revenues (apart from Pigovian taxes and profit taxes) to finance public capital. To avoid complications arising from pure profits and from reliance on other revenues, the main analysis focuses on a benchmark

[14] As technical conditions to ensure (a) positive production even when a government fails to invest and (b) a bounded human capital/productivity ratio, assume moreover there are values $0 < h_0 < h_\infty < \infty$ such that $\chi(h) = \chi'(h) = 0$ for $h \leq h_0$, and (b) $\chi'(h) \uparrow \infty$ as $h \uparrow h_\infty$. Then $h_t / \Gamma_t \in [h_0, h_\infty]$ for any $G_{t-1}^h$.

case: $\varphi_L + \varphi_K + \varphi_G = 1$ and $\zeta_L + \zeta_K = 1$. Under these assumptions, there are no pure profits and Pigovian taxes cover the cost of public capital.[15]

*3.2. Work and Savings*

Working-age individuals divide a unit of time into work time $l_t$ supplied to the labor market, and time $1 - l_t$ outside the domestic market economy. This could be work in a shadow economy, work abroad, or time for leisure—called shadow labor for brevity. Workers labor supply in efficiency unit is $L_t = l_t H_t$, each earning the market wage $\tilde{w}_t$.

Shadow labor produces non-market output under a production function $H_t s(1 - l_t)$, so total labor income is

$$y_t^1 = \tilde{w}_t l_t H_t + s(1 - l_t) H_t. \tag{4}$$

The function $s(\cdot)$ is assumed increasing and concave, with $s'(0) = \infty$ to ensure $l_t < 1$, and with possible jump at $l_t = 0$ if there is an extensive margin for non-market work. Let $\bar{s} H_t$ with $\bar{s} \equiv s(1) \geq \lim_{x \uparrow 1} s(x)$ denotes maximum non-market income. Implicit in (4) is the assumption that human capital has benefits outside the market economy. This is plausible not only for a domestic shadow economy, but also for working abroad and for enjoying leisure.[16] It implies that the social value of education is greater—perhaps much greater—than its marginal contribution to taxable output.

Market labor is subject to income taxes. While linear taxes are common in the literature, they are restrictive because they link revenues to tax distortions. Hence I consider a Mirrleesian specification, i.e., an arbitrary tax function $T_t = T(\tilde{w}_t L_t)$ subject to incentive

[15] This benchmark specification is consistent with empirical evidence (e.g., Aschauer 1989) yet simple. Increasing returns to scale would not alter the analysis because human capital is bounded, provided returns to scale are decreasing in $(K_t, K_t^G)$. Decreasing returns to $(L_t, K_t)$ without congestion externality or with $\zeta_L + \zeta_K < 1$ would imply pure profits. Then firm ownership would have to be modeled in some detail, which is a distraction because efficient policy would tax pure profits at 100%—and then the allocation is the same as if $\zeta_K = 1 - \zeta_L$, as assumed. The Cobb-Douglas shape is purely for convenience; a seemingly restrictive implication is that $F(L, K, 0) = 0$, which makes public capital essential to produce. However, since a non-market sector is added below, a general specification with $F(L, K, 0) > 0$ would yield similar results. Pigovian taxes also helps to sidestep Chamley/Judd-type questions about the optimality of zero marginal tax rates. Positive marginal taxes, as observed empirically, are efficient here and hence viable even with tax competition.

[16] Having $H_t$ enter linearly in shadow production is for convenience, to allow balanced growth. One could assume that shadow labor benefits from public infrastructure, but this is arguably less important (especially if non-market work is abroad) and multiple factors would clutter the model. For many countries, work abroad may be the most relevant outside opportunity. Then $\bar{s}$ would be the foreign wage net of migration costs.

constraints. Mirrleesian taxes are simple here because workers are identical and will choose the same income.[17] The constraints are that individuals could earn $\bar{s}H_t$ in the shadow economy and that they set $l_t$ to maximize their disposable income $y_t^d \equiv y_t^1 - T(\tilde{w}_t L_t)$. With identical agents, the problem can be treated as letting the government pick $(T_t, l_t)$ subject to the participation constraint $T_t \le y_t^1 - \bar{s}H_t$ (or equivalently $y_t^d \ge \bar{s}H_t$) and the individual optimality condition

$$(1-\tau_t^l)\tilde{w}_t = s'(1-l_t), \tag{5}$$

where $\tau_t^l = \frac{dT}{d(\tilde{w}L)}$ is the marginal tax rate at the government-selected income.

Generation-t has a working-age budget equation

$$c_t^1 = y_t^d - a_{t+1} + b_t^+ - b_t^-, \tag{6}$$

where $a_{t+1}$ are net savings, $b_t^+ \ge 0$ receipts of bequests, and $b_t^- \ge 0$ gifts to parents. Savings can be invested in domestic capital $k_{t+1} \ge 0$, government debt $d_{t+1} \ge 0$, and net foreign assets $a_{t+1}^f = a_{t+1}^{f+} - a_{t+1}^{f-}$, where $a_{t+1}^{f+} \ge 0$ denotes gross assets and $a_{t+1}^{f-} \ge 0$ denotes liabilities (if any). With returns denoted $R_{t+1}^k$, $R_{t+1}^d$, $R_{t+1}^{f+}$ and $R_{t+1}^{f-}$, savings $a_{t+1} = k_{t+1} + d_{t+1} + a_{t+1}^{f+} - a_{t+1}^{f-}$ translate into retirement assets

$$A_{t+1}^2 \equiv R_{t+1}^k k_{t+1} + R_{t+1}^d d_{t+1} + R_{t+1}^{f+} a_{t+1}^{f+} - R_{t+1}^{f-} a_{t+1}^{f-}, \tag{7}$$

Retirement assets pay for consumption, after adjustment for bequests and gifts:

$$\lambda_{t+1} c_{t+1}^2 = A_{t+1}^2 - b_{t+1}^+ + b_{t+1}^-. \tag{8}$$

Public pensions are not modeled explicitly and simply treated as a form of government debt. To avoid tax distortions, a government would naturally demand working-age "contributions" to obtain pension entitlements, as in a Bismarckian system. Any other payroll taxes would be included in $T_t$.; and any other retirement-age entitlement can be interpreted as created via transfers. The present value of entitlements is equivalent to a government bond and subsumed into debt holdings $d_{t+1}$. (This follows generational accounting, as in Auerbach et al. 1989.)

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[17] If individual productivity were heterogeneous, as in the original Mirrlees model, a modest link between revenues and labor supply distortions may reappear, but it would be a distraction and not necessarily of first-order importance.

A central assumption is that the economy is small and open, where small means that the gross return on international borrowing and lending is exogenous (denoted $\overline{R}_{t+1}$) and open means that the government cannot restrict access to world markets (i.e., $R^{f+}_{t+1} = \overline{R}_{t+1} \forall t$). To ensure well-defined budget constraints, assume $\overline{R}_{t+1} > 1+\gamma$, so the present value of domestic output is finite, and that foreign lenders impose a No-Ponzi condition. Then capital, bonds, and international loans must offer the same equilibrium returns; and $\overline{R}_{t+1}$ determines incentives to savings.[18]

### *3.3. Preferences*

Preferences deserve some elaboration to be precise about the roles of altruism, aversion to default, and longevity. Let generation t have utility over own consumption

$$v_t = u(c^1_t) + \upsilon\lambda_{t+1}u(c^2_{t+1}), \tag{9}$$

where $u(c) = \frac{1}{1-\eta}c^{1-\eta}$ is increasing and concave with curvature $\eta > 0$.[19] The weight on old-age consumption $c^2_{t+1}$ consists of a discount factor $\upsilon \in (0,1)$ and a longevity factor $\lambda_{t+1} > 0$ that captures the need to stretch old-age resource over a longer period as life expectancy increases over time. Old age spending is $\lambda_{t+1}c^2_{t+1}$; so $c^2_{t+1}$ should be interpreted as consumption flow.

The overall utility function of generation t includes a general specification for two-sided altruism and utility costs:

$$V_t = v_t + \alpha_t \cdot U^-_{t-1} + \beta_t \cdot U^+_{t+1} - \Lambda^U_t \tag{10}$$

where $U^-_{t-1} = v_{t-1} + \alpha_{t-1} \cdot U^-_{t-2}$ with weights $\alpha_t \geq 0$ captures altruism towards preceding generations; and $U^+_{t+1} = v_{t+1} + \beta_{t+1} \cdot U^+_{t+2}$ with weights $0 < \beta_t < 1$ captures altruism towards descendents. The variable $\Lambda^U_t \geq 0$ is a utility cost of defaults; $\Lambda^U_t = 0$ except in default.[20]

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[18] Smallness implies that governments cannot manipulate interest rates to relax implementability conditions. This rules out the time-consistency issues raised by Lucas and Stokey (1983). Even "large" countries like the U.S. are arguably "small" in this sense, because their ability to influence world interest rates is very limited, especially for the long-term real rates relevant for retirement savings. Openness implicitly assumes that the government cannot tax savings abroad. Hence the Bulow-Rogoff (1989) argument applies and rules out reputational arguments to sustain foreign debt.

[19] As usual, $\eta = 1$ means logarithmic utility. Power utility is needed only to obtain balanced growth. As extension, one could include preferences over public goods consumption, say additively for comparison to Song et al (2012), but government consumption is not essential here and hence omitted.

[20] The disutility of default is distinct here from general altruism towards parents; a question below will be to what extent altruism can substitute for default cost, or reduce the default costs required to sustain a government commitment.

These preferences reduce to Abel's (1987) specification, if the weights $(\alpha_t, \beta_t, \lambda_t)$ are constant and if $\Lambda_t^U = 0$. Utility is defined recursively so that if altruism changes over time, preferences over distant generations are consistent with the preferences of the generations in between. Assume $\alpha_t \leq 1$ so there is generational conflict, though possibly small if $\alpha_t \beta_{t-1} < 1$ is near one. (In the excluded case $1/\alpha_t = \beta_t$, each generation would act like a social planner and there would be no conflict.)

Maximizing (9-10) subject to the constraints (6-8), one obtains first-order conditions for optimal savings, gifts, and bequests:

$$\overline{R}_{t+1} \upsilon u'(c_{t+1}^2) - u'(c_t^1) = 0 \quad (11)$$

$$\alpha_t \upsilon u'(c_t^2) - u'(c_t^1) + \mu_{b_t^- \geq 0} = 0 \quad (12)$$

and

$$\beta_t u'(c_{t+1}^1) - \upsilon u'(c_{t+1}^2) + \mu_{b_{t+1}^+ \geq 0} = 0 \quad (13)$$

where $\mu_{b_t^- \geq 0} \geq 0$ and $\mu_{b_{t+1}^+ \geq 0} \geq 0$ are Kuhn-Tucker multipliers.[21] Let

$$MRS_t \equiv \upsilon u'(c_t^2) / u'(c_t^1)$$

denote the marginal rate of substitution across generations. From (12-13), $\beta_{t-1} \leq MRS_t \leq 1/\alpha_t$ is bounded below by bequests and above by gifts. Thus gifts and bequests are mutually exclusive, and both are inactive when $\beta_{t-1} < MRS_t < 1/\alpha_t$.

Equations (11-12) provide an intuition about default: A unit reduction in parental consumption reduces childrens' utility by $\alpha_t MRS_t$. This suggests that the gain from defaulting on a unit of domestic debt (held by parents) is $1 - \alpha_t MRS_t \leq 1 - \alpha_t \beta_{t-1}$, i.e., reduced by at least $\alpha_t \beta_{t-1}$ relative to the gain from defaulting on foreigners. Gains are reduced if parents are so impoverished by default that $MRS_t > \beta_{t-1}$, and gains vanish if default would activate the gift motive (at $MRS_t = 1/\alpha_t$). This suggests that two-sided altruism is important for understanding government defaults.

The model assumes a representative agent per cohort and disregards cross-sectional differences, notably in family structures and in wealth. This is in part for tractability and in

[21] Throughout, let $\mu_{x \geq 0}$ denote the Kuhn-Tucker multiplier on any inequality condition $x \geq 0$. That is, $\mu_{x \geq 0} \geq 0$, $x \cdot \mu_{x \geq 0} = 0$, and, $x > 0$ implies $\mu_{x \geq 0} = 0$. A constraint is called *binding* if the associated multiplier is non-zero.

part because the literature on bequests suggests that links between having children and individual bequests are weak (Kopczuk and Lupton 2007). The objective here is not to study individual bequests but the role of altruism for government policy—altruism towards the next representative agent. (As I show below, individual bequests are not reliable indicators of bequest motives when policy is endogenous. Instead, the basic observable implication of an altruistic bequest motive is restraint in government borrowing.)

The representative agent setting has one significant limitation: it cannot capture an unequal distribution of wealth. This is a quantitatively important (see Levine 2012), notably for calibrating asset positions and aggregate bequests. To capture wealth inequality simply, assume the domestic household sector described above *excludes* a small slice of "rich" households (a.k.a. "top 1%" in American politics, formally measure zero) that hold a significant share of national net worth but do not supply labor. The rich do not complicate the model because they act like foreign investors and can be subsumed into the foreign sector, except when needed for calibrating national accounts.

*3.4. Low Altruism, World Interest Rates, and a Simplification*

Conditions (11-13) imply that

$$\overline{R}_{t+1}\beta_t \cdot u'(c_{t+1}^1) = u'(c_t^1)$$

whenever the bequest motive is active (i.e., when $\mu_{b_{t+1}^+ \geq 0} = 0$). Hence dynastic paths of consumption depend critically on the relationship between world interest rates and altruism towards children. Accounting for productivity growth, one may write

$$\frac{u'(c_t^1/\Gamma_t)}{u'(c_{t+1}^1/\Gamma_{t+1})} = \overline{R}_{t+1}\beta_t(1+\gamma_{t+1})^{-\eta} = \overline{R}_{t+1}\tilde{\beta}_t, \tag{14a}$$

where $\tilde{\beta}_t = \beta_t(1+\gamma_{t+1})^{-\eta}$. If $\beta_t$ is low enough that $\tilde{\beta}_t < 1/\overline{R}_{t+1}$, (14a) implies consumption growth less than productivity growth, suggesting that dynastic parents will want to extract resources from their children. To be specific, let the following set of parameters define low altruism:

*Definition (general version):* ***Low Altruism*** *applies if*

$$\beta_t \leq (1+\gamma_{t+1})^{\eta}/\overline{R}_{t+1} \textit{ for all t, and } \prod_{s=0}^{t-1}\left(\frac{\overline{R}_{s+1}}{(1+\gamma_{t+1})^{\eta}}\beta_s\right) \to 0 \textit{ as } t \to \infty. \tag{14b}$$

In this definition, $\beta_t \leq (1+\gamma_{t+1})^{\eta} / \overline{R}_{t+1}$ is not required in all periods, but sufficiently often that along a balanced-growth path, the current generation assigns vanishing weights to the marginal utilities of generations far ahead.

With economic growth, low altruism includes cases with $\beta_t > 1/\overline{R}_{t+1}$, which means that parents want their children to be better off than themselves, but by factor less than productivity growth. A desire to extract resources from future generations can then be interpreted as an attempt to reduce consumption inequality, in the spirit of progressive taxation. Thus low altruism does not require extreme or unreasonable selfishness.

Openness is important to generate conditions of low altruism. In closed-economy dynastic models, $\beta_t$ determines capital investment. Then the relevant return in (14a) would be an endogenous return to capital, and $R_t^k \approx 1/\tilde{\beta}_t$ should hold the long run—precluding the conditions in (14b). This is different if capital is internationally mobile and $\beta$-values differ across countries. Then countries with high $\beta$-values, which have low autarchy returns to capital, will accumulate foreign assets and eventually dominate the determination of $\overline{R}_t$ (Song et al. 2012). All other countries eventually face a situation with $\tilde{\beta} < 1/\overline{R}_t$, so the conditions in (14b) apply.

As shown in Table 1, a small number of countries account for most of international lending, whereas borrowing is more dispersed.[22] This is consistent with cross-country heterogeneity in altruism. However, I focus on a single country and use cross-country comparisons only to explain why low altruism is empirically relevant.

The preferences with time-varying parameters, as in (10), provide an alternative motivation for low altruism. Social changes may strengthen or weaken generational linkages over time. Hence a country may have low altruism going forward but not throughout their history. And this may occur in a closed economy if altruism remains unchanged for fraction of

[22] In the 2005 IMF data used for Table 1, there are 83 countries with net investment positions of -10% of GDP or less (i.e., borrowing 10% or more) and only 19 countries with net investment positions of +10% of GDP or more.

the population that can serve as lenders (e.g., the rich). Then openness is merely a simplifying assumption that ensures exogenous interest rates.

Time-dependence is instructive for the motivation, but a notational nuisance for general equilibrium analysis. To simplify the presentation, I will treat all parameters and all exogenous variables (notably productivity growth and world interest rates) as constants in the following, keeping in mind that initial conditions may come from an era with different parameters and that time-variation could be modeled with extra notation. Specifically, assume

$$(\alpha_t, \beta_t, \lambda_t, \gamma_t, \overline{R}_t) = (\alpha, \beta, \lambda, \gamma, \overline{R}) \forall t \geq 0, \qquad (*)$$

where time-subscripts are omitted to denote constants. (Footnotes will identify instances where time-dependence provides additional insights; time-subscripts are retained when useful to clarify the timing of decisions.) To avoid a proliferation of cases, I focus on two scenarios for the parental altruism parameter:

**Balanced-growth altruism**: $\beta = (1+\gamma)^{\eta} / \overline{R}$

**Low altruism** (simple version): $\beta < (1+\gamma)^{\eta} / \overline{R}$

In both cases, assume $1/\alpha > (1+\gamma)^{\eta} / \overline{R}$, so the gift motive is inactive along a balanced growth path—though still important for policy and potentially active outside balanced growth.

## 4. The Equilibrium Path with Government Commitment

For this section, assume that the government faces prohibitive costs of default, can make credible promises about marginal tax rates that influence capital investment, and enforces financial contracts with foreigners. With these assumptions, the equilibrium path will be efficient. It provides a benchmark for studying default in Section 5.

### *4.1. Financial Markets and Equilibrium Conditions*

Since retirement savers can earn $\overline{R}$ abroad, capital investment is zero unless $R^k_{t+1} \geq \overline{R}$, and government debt must offer a return $R^d_{t+1} = \overline{R}$.

In the market for capital, let $Q_t$ be the period-t price of $K_{t+1}$. Note that $Q_t \leq 1$ because investment has unit cost and is irreversible, and that $I_t > 0$ requires $Q_t = 1$. Capital ownership yields pre-tax income $\tilde{r}_{t+1}K_{t+1}$ and a claim on depreciated capital $(1-\rho)K_{t+1}$. Let taxes on capital be divided into an income tax at rate $\tau^k_{t+1}$ and a potential capital levy on $(1-\rho)K_{t+1}$ at rate $\delta^k_{t+1}$, where $\tau^k_{t+1},\delta^k_{t+1} \in [0,1]$. Then a unit of capital has after-tax payoffs

$$R^K_{t+1} = (1-\tau^k_{t+1})\tilde{r}_{t+1} + (1-\delta^k_{t+1})(1-\rho)Q_{t+1}. \tag{15a}$$

Note that $R^K_{t+1}$ is decreasing in $\tau^k_{t+1}$ and increasing in the marginal product $\tilde{r}_{t+1} = F_K(L_{t+1},K_{t+1},K^G_{t+1})$. The latter depends on the labor tax rate $\tau^l_{t+1}$, because $\tau^l_{t+1}$ influences optimal labor supply $L_{t+1}$ through (5). Hence time-consistency generally requires commitments about tax rates on capital *and* labor. (To anticipate, optimal rates will be Pigovian, $(\tau^l_{t+1},\tau^k_{t+1}) = (\xi_L,\xi_K)$.)

Capital income taxes and capital levies are worth distinguishing because seizing and liquidating physical capital assets is more difficult than taxing cash incomes. (Also, government may not be an efficient manager of capital.) To capture this distinction in the model, assume capital seized by the government is devalued by $\Lambda^K \in (0,1)$, so revenues are only $\delta^k_{t+1}(1-\rho)(1-\Lambda^K)Q_{t+1}$. Then income taxes are more efficient than a capital levy, so governments will always promise $\delta^k_{t+1} = 0$.

Because gross investment is non-negative, there are two scenarios for capital market equilibrium: If

$$R^K_{t+1} = (1-\tau^k_{t+1})F_K(L_{t+1},(1-\rho)K_t,G^K_{t+1}) + (1-\rho)Q_{t+1} < \overline{R}_{t+1}, \tag{15b}$$

new investments would earn a below-market return. Then $I_t = 0$ and existing capital trades at price $Q_t = \overline{R}_{t+1}/R^K_{t+1} < 1$. At this price, savers earn a market return $R^k_{t+1} = R^K_{t+1}/Q_t = \overline{R}_{t+1}$ on their (financial) investments. If (15b) does not apply, $I_t$ is determined implicitly by the first order condition

$$(1-\tau^k_{t+1})F_K(L_{t+1},I_t+(1-\rho)K_t,G^K_{t+1}) + (1-\rho) = \overline{R}_{t+1}, \tag{16}$$

and $Q_t = 1$. In both cases, capital market equilibrium requires $k_{t+1} = Q_t K_{t+1}$.

For most of the analysis, the irreversibility of capital investments is a distraction—interesting only in Section 5. Intuitively, (15b) tends to apply if $\tau^k_{t+1}$ is near one (because $(1-\rho)Q_{t+1} < 1 < \overline{R}_{t+1}$) and if replacement investment is small (small γ, high ρ). To avoid case distinctions, let $I_t > 0 \forall t$ apply at Pigovian tax rates, so (16) will hold throughout this section.[23]

In the market for government debt, let $q_t$ denote the period-t price of coupon bonds that promise a payoff $\overline{R}_{t+1}$. Since opportunistic default is ruled out, the government can issue any amount $D_{t+1}$ at unit price (up to a natural debt limit to be determined in equilibrium). Let $d^f_{t+1} \geq 0$ denote foreign purchases. Equilibrium requires $d_{t+1} + d^f_{t+1} = q_t D_{t+1}$.

The government budget equation is then

$$I^G_t + G^h_t + \overline{R}D_t = T_t + \tau^k_t \tilde{r}_t K_t + D_{t+1}. \quad (17)$$

Spending on investment, education, and initial debt are financed by taxes and new debt.[24]

Mirrleesian taxes have an advantage over linear taxes in this context because promises about $\tau^l_t$ to not constrain the *level* of labor taxes $T_t$ in (17). This differs from linear tax models in which tax revenues and tax rates are tightly linked by a Laffer curve. In reality, governments have wide discretion over current revenues. Hence the model assumes that $T_t$ is set by the period-t government. This does not preclude commitments about tax rates and is consistent with Pigovian taxes.[25]

Individuals must borrow when the capital implied by (16) exceeds assets $a_{t+1}$. This motivates the foreign liabilities $a^{f-}_{t+1}$ in (7).[26] Since loan contracts are enforced, individuals can borrow at rate $R^{f-}_{t+1} = \overline{R}$ up to the value of their capital and other assets, so they are never liquidity-constrained. The combined net foreign asset position of individuals and government is denoted $A^f_{t+1} \equiv a^{f+}_{t+1} - a^{f-}_{t+1} - d^f_{t+1}$. Generation-t retirement assets can be written as

[23] This assumption sidesteps another complication, which is that $Q_{t+1}$ could be reduced by prohibitively high taxes announced for period after t+1. This is uninteresting, but when Q<1 occurs along the optimal path, elaborate additional notation would be needed to formalize multi-period commitments. Along paths with Q=1, no formal treatment is needed if one interprets a commitment not to impose a capital levy in (t+1) broadly as promise not to act in a way that will devalue old capital.

[24] In reality, there is also government consumption, which this is omitted here to avoid clutter. Recall that $D_t$ includes public pensions and other retiree entitlements. Transfers and other government spending for the benefit of working-age households could be interpreted as negative taxes.

[25] The assumption of identical workers helps to clarify the distinction between tax rates and revenues. In an extended model with cross-sectional distribution of labor incomes, commitment can be limited similarly to a single promise about an appropriately weighted average marginal tax rate (to preclude distortions in the marginal product of capital), without otherwise restricting labor income taxes.

[26] In reality, foreign liabilities are of course mostly foreign claims against corporations—debt, equity, or direct investments—but since capital structure is uninteresting here, corporations are subsumed into the “household” sector.

$$A_{t+1}^2 = \overline{R}_{t+1} a_{t+1} = \overline{R}_{t+1}(K_{t+1} + D_{t+1} + A_{t+1}^f). \quad (18)$$

The composition of net foreign assets is for now irrelevant.

Equation (18) provides an intuition why debt matters, and helps clarify its definition. Retirees must rely on assets to finance their consumption: $\lambda_{t+1} c_{t+1}^2 \le A_{t+1}^2$ follows from (8), because bequests are non-negative and gifts are zero except in undesirable scenarios (when retirees are so poor that $MRS_t = 1/\alpha_t$). While capital and external assets are real, government debt can be created by political fiat—simply by cutting taxes in the previous period, when generation-t was working-age and in control of government. A key role of government is for the generation in power to distribute enough assets to its members that their retirement is secure when the next generation takes over.

Assets in this context are not only real assets, $\overline{R}_{t+1}(K_{t+1} + A_{t+1}^f)$, but any claims on public funds (a.k.a. entitlements) that future governments *must* pay in the sense that non-payment would trigger prohibitive default costs. The government debt is the sum of all such claims, i.e., it is effectively defined by the scope of default costs.[27]

### *4.2. Optimal Policy*

Successive governments have discretion over spending, taxes, and debt. Every period, the government takes as given the state of the economy as defined by capital stocks $(H_t, K_t, K_t^G)$, assets positions $(a_t, D_t, A_t^f)$ and promised tax rates $(\tau_t^l, \tau_t^k)$, and it makes choices about current policy $(I_t^G, G_t^h, T_t, D_{t+1}, K_{t+1}^G, H_{t+1})$ and about tax rates $(\tau_{t+1}^l, \tau_{t+1}^k)$ for the next period.

The period-t government maximizes generation-t's utility $V_t$. If one omits lagged terms and default costs, this reduces to maximizing $V_t^1 = v_t + \alpha_t \lambda_t \upsilon u(c_t^2) + \beta_t \cdot U_{t+1}^+$. The utility of period-t retirees, $V_{t-1}$, reduces similarly to $V_t^2 = \lambda_t \upsilon u(c_t^2) + \beta_{t-1} \cdot U_t^+$. These value functions can be combined and written recursively as

$$V_t^2 = (1 - \alpha_t \beta_{t-1}) \lambda_t \upsilon u(c_t^2) + \beta_{t-1} \cdot V_t^1, \quad (19)$$

and $$V_t^1 = u(c_t^1) + \alpha_t \lambda_t \upsilon u(c_t^2) + V_{t+1}^2 = \tilde{v}_t + \beta_t \cdot V_{t+1}^1 \quad (20)$$

where $$\tilde{v}_t = u(c_t^1) + \alpha_t \lambda_t \upsilon u(c_t^2) + (1 - \alpha_{t+1} \beta_t) \lambda_{t+1} \upsilon u(c_{t+1}^2).$$

[27] See Bohn (1992) for a discussion of measurement issues and implications for government accounting.

The term $(1-\alpha_t\beta_{t-1})\lambda_t \upsilon u(c_t^2)$ in (19) captures the generational conflict: Period-t retirees put a higher weight on their own consumption than the period-t government, which is controlled by their children. Otherwise their objectives coincide.

Because (20) is recursive, generation t has the same preferences as the period-(t+1) government over policy in future periods t+j, for all j>1. The equilibrium path with sequential policies can therefore be computed by solving the infinite-horizon planning problem at t=0. However, the generational conflict between government and retirees implies that the solution is not time-consistent unless policy is constrained to satisfy (12). As noted, this can be implemented by giving retirement savers sufficient assets.

The setup is then straightforward. Regarding production, equations (2, 5) and (3, 16) provide implementability conditions for labor and capital:

$$\tfrac{1-\tau_{t+1}^l}{1-\xi_L} F_L(l_{t+1}, \tfrac{K_{t+1}}{H_{t+1}}, \tfrac{K_{t+1}^G}{H_{t+1}}) = (1-\tilde{\tau}_{t+1}^l)\tilde{w}_{t+1} = s'(1-l_{t+1}). \tag{21}$$

$$\tfrac{1-\tau_{t+1}^K}{1-\xi_K} F_K(l_{t+1}, \tfrac{K_{t+1}}{H_{t+1}}, \tfrac{K_{t+1}^G}{H_{t+1}}) = (1-\tilde{\tau}_{t+1}^k)\tilde{r}_{t+1} = \overline{R} - (1-\rho) \equiv \overline{r}. \tag{22}$$

Hence the period-t government controls $(l_{t+1}, K_{t+1})$ by choice of $(K_{t+1}^G, h_{t+1}, \tilde{\tau}_{t+1}^l, \tilde{\tau}_{t+1}^k)$. Using $T_t = \tilde{w}_t l_t H_t + H_t s(1-l_t) - y_t^d = \frac{F_L(\cdot)}{1-\xi_L} L_t + H_t s(1-l_t) - y_t^d$, $\tilde{\tau}_t^k \tilde{r}_t K_t = \frac{F_K(\cdot)}{1-\xi_K} K_t + \overline{r} K_t$, and $\frac{F_K(\cdot)}{1-\xi_K} K_t + \frac{F_L(\cdot)}{1-\xi_L} L_t = F(L_t, K_t, K_t^G)$, the budget equation (17) can be written as

$$K_{t+1}^G + \Gamma_t \chi(\tfrac{H_{t+1}}{\Gamma_{t+1}}) - D_{t+1} = F_t^+ - \overline{R} K_t - y_t^d - \overline{R} D_t, \tag{23}$$

where $\quad F_t^+ \equiv F(l_t H_t, K_t, K_t^G) + (1-\rho)K_t + (1-\rho)K_t^G + H_t s(1-l_t)$

summarizes the economy's productive resources. Given the No-Ponzi condition, this implies

$$\sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i [K_{t+1+i}^G + \Gamma_{t+i} \chi(\tfrac{H_{t+1+i}}{\Gamma_{t+1+i}})] = \sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i [F_{t+i}^+ - \overline{R} K_{t+i} - y_{t+i}^d] - \overline{R} D_t. \tag{24}$$

Regarding consumption, (11) and (13) are satisfied trivially because they are optimal for the governing generation. One can show that (12) is non-binding for all t>0, so $b_t^- = 0$. To impose this constraint, note that (6-8) imply

$$b_t^+ - b_t^- = \sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i [c_{t+i}^1 + \tfrac{\lambda}{R} c_{t+i+1}^2] - \sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i y_{t+i}^d,$$

so $b_t^- = 0$ and $b_t^+ \ge 0$ are equivalent to the constraint set

$$\sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i [c_{t+i}^1 + \tfrac{\lambda}{R} c_{t+i+1}^2] \ge \sum\nolimits_{i\ge 0} (\tfrac{1}{R})^i y_{t+i}^d \quad \forall t \ge 1 \tag{25}$$

In period t=0, the old consume $c_t^2 = \frac{1}{\lambda}(A_0^2 - b_0^+ + b_0^-)$ where $A_0^2 - b_0^+$ is given and $b_0^- \ge 0$ is a generation-0 choice variable.

The period-0 government problem is then to maximize $V_0^1$ by choice of $\{l_t, H_t, K_t, K_t^G\}_{t\ge1}, \{c_t^1, c_t^2, y_t^d\}_{t\ge0}$ subject to (24), (25), and the tax constraints $y_t^d \ge \bar{s}H_t$. The solution is unique (denoted by *) and implies a unique sequence $\{A_t^{f*}\}_{t\ge1}$ of net foreign asset positions. To make the policy time-consistent, the government must issue enough debt that

$$D_t + K_t^* + A_t^{f*} \ge \tfrac{1}{\bar{R}} \lambda_t c_t^{2*} \quad \forall t \ge 1 \tag{26}$$

so each old generation can pay for planned consumption.[28] Apart from this inequality constraint, government debt is arbitrary. From (8), optimal bequests are

$$b_t^{+*} = \bar{R}(D_t + K_t^* + A_t^{f*}) - \lambda_t c_t^{2*}. \tag{27}$$

Any public debt above the amount needed to satisfy (26) is simply bequeathed to the next generation—so Ricardian neutrality applies unless debt is otherwise constrained.

Regardless of parameters, one finds:

***Proposition 1 (Allocations with Commitment):***

a) Production satisfies $F_K = F_G = \bar{r}$, and $F_L = s'(1-l_t)$. These conditions uniquely define optimal factor proportions $(l_{t+i}, \frac{K_{t+i}}{H_{t+i}}, \frac{K_{t+i}^G}{H_{t+i}}) = (l^*, (K/H)^*, (K^G/H)^*)$.

b) Capital and labor income taxes are Pigovian: $\tau_t^l = \xi_L$ and $\tau_{t+1}^k = \xi_K$.

c) Debt $D_t \le \hat{D}_t$ is bounded by the natural debt limit

$$\hat{D}_t = \hat{D}(H_t, l_t, K_t, K_t^G) = K_t^G + \tfrac{1}{\bar{R}}(\hat{\tau}^h + \Delta F_t)\cdot H_t + \tfrac{1+\gamma}{\bar{R}-(1+\gamma)}\pi(\hat{h})\Gamma_t, \tag{28}$$

where, $S^* \equiv F_L l^* + s(1-l^*)$, $\Delta F_t = \frac{1}{H_t}[F_t^+ - \bar{R}K_t - \bar{R}K_t^G] - S^* \le 0$, $\hat{\tau}^h = S^* - \bar{s}$

$\pi(h) = \hat{\tau}^h h - \frac{\bar{R}}{1+\gamma}\chi(h)$, *and* $\hat{h} = \arg\max_h \pi(h)$,

*Proof: This and the following Prop.2-4 follow from first-order condition of maximizing $V_0$. Details are in a technical appendix (to be available online).*

The intuition for production efficiency is as follows. Efficient labor supply requires equating the marginal product of market labor, $F_L(\cdot)$, with the marginal product of non-market

[28] If longevity were treated as time varying, rising longevity would make (26) more difficult to satisfy and (ceteris paribus) requires more public debt. In the popular press, a positive correlation between population aging and rising debt "burden" is often presented as a problem. In this model, this is a feature of optimal, time-consistent retirement financing.

time, $s'(1-l_{t+1})$. In (21), this holds with Pigovian taxes $\tau^l_{t+1}=\xi_L$. Efficient private capital requires similarly that the marginal products $F_K(\cdot)$ and $F_G(\cdot)$ equals their opportunity cost $\bar{r}$. For private capital, this requires $\tau^k_{t+1}=\xi_K$ in (22). Given constant returns to scale, these three conditions uniquely determine factor inputs relative to human capital $(l^*,(K/H)^*,(G^K/H)^*)$.[29] Output, capital, and government capital are then proportional to human capital, so production has $H_t$ as single state variable.

In (28), the $\Delta F$-term allows for starting positions with inefficient production, for completeness. Optimal factor proportions $(l^*,(K/H)^*,(G^K/H)^*)$ imply $\Delta F=0$. Provided $\Delta F=0$, debt-financed public capital investment can be repaid from the resulting Pigovian taxes. Hence the debt limit varies one-for-one with public capital. Thus: Debt does not constrain efficient investment in public capital.

Provided $\Delta F=0$, the debt limit is linear in $H_t$ with slope $\hat{\tau}^h/\bar{R}$. The slope captures the impact of human capital on the tax base. If one writes $y^d_t \geq \bar{s}H_t$ as $T_t \leq y^1_t - \bar{s}H_t$ and subtracts Pigovian taxes on both sides, it implies $T_t-\tau^l_t\tilde{w}_t \leq (S^*-\bar{s})\cdot H_t = \hat{\tau}^h H_t$, which is the maximum revenue the government can extract from workers with human capital $H_t$. One the margin, $\hat{\tau}^h$ can be used to repay debt $\bar{R}D_t$. Discounting by a period, a unit of $H_t$ raises the capacity to issue debt $D_t$ in period t-1 by $\hat{\tau}^h/\bar{R}$.

The term $\pi(h)$ is the difference between the revenue from human capital and the investment cost $\chi(h_t)$. It can be interpreted as "fiscal dividend" from future workers. By construction, $\pi(h)$ positive and increasing for $h<\hat{h}$ and maximized at $\pi(\hat{h})$. That is, $\pi(\hat{h})$ represent the maximum revenue the government can extract from future workers minus the cost of providing them with revenue-maximizing human capital. Debt is bounded by the present value thereof, which is a multiple of $\pi(\hat{h})$.

[29] As noted above, the main analysis assumes constant returns to scale and $\zeta_L+\zeta_K=1$. Without these assumptions, Prop.1 turns out to apply in modified form: $F^+_t$ in (23) would be reduced by any untaxed pure profits and $F_L l^*$ in $S^*$ in (28) would be reduced by a terms proportional to $\varphi_L+\varphi_K+\varphi_G-1$.

For comparison, $S^*$ is the overall or social value of human capital. It includes the output effect $F_L l^*$ and non-market benefits $s(1-l^*)$, Pareto-efficient investment in human capital would equate the social value and the marginal cost, $S^* = \frac{\bar{R}}{1+\gamma}\chi(h^*)$, which defines $h^*$. Because of the outside option $\bar{s}$, $\hat{\tau}^h = S^* - \bar{s} < S^*$ and $h^* > \hat{h}$. Thus: Debt can be a constraint on human capital investment. Equivalently, the human capital $\hat{h}$ that maximizes fiscal dividends is inefficiently low.

*4.2. Policy Regimes and the Transition to Austerity*

Turning to consumption and human capital, there are two distinct policy regimes and at most three ways to combine them.

The distinction depends on the tax constraint $T_t \le y_t^1 - \bar{s}H_t$.[30] Let $\mu_{Tax,t} \ge 0$ denote the Kuhn-Tucker multiplier on this constraint. If the tax constraint ever binds, it turns out to bind in all subsequent periods. Hence there are three cases: always non-binding tax constraints, $\mu_{Tax,t} = 0 \forall t$; always binding tax constraints, $\mu_{Tax,t} > 0 \forall t$; or a regime shift in some period $t_A > 0$, so $\mu_{Tax,t} = 0$ for all $t < t_A$ and $\mu_{Tax,t} > 0$ for all $t \ge t_A$. The two regimes have very different properties, as detailed in the following propositions. For $\mu_{Tax,t} = 0$, a dynastic allocation applies: successive generations use their control over government to ensure $MRS_t = \beta$, i.e., to control the dynastic consumption profile. I call this the *Dynastic Regime*. In the other regime, labeled *Tax-Constrained*, workers pay maximum feasible taxes, and this constrains the public sector. Specifically:

***Proposition 2 (The Dynastic Regime):***

*If $\mu_{Tax,t} = 0$ in any period $t = t_0$, then $\mu_{Tax,t} = 0$ for all periods $t \le t_0$. Moreover:*

a) *The bequest constraint $b_t^+ \ge 0$* does not bind.
b) *Investment in education is socially optimal: $h_t = h^*$.*
c) *The relative consumption of old and working-age generations is determined by the preferences of the old: $MRS_t = \beta$.*

---

[30] Recall that $y_t^d \ge \bar{s}H_t$ and $T_t \le y_t^1 - \bar{s}H_t$ are equivalent. The former is more concise for stating and solving the government's optimization problem, but the latter more intuitive for interpreting optimal tax policy.

d) *Consumption growth across generations depends on altruism:* $u'(c^1_{t-1}) = \beta\overline{R}u'(c^1_t)$.

The intuition is that if taxes are below the upper bound in any future period, earlier generations can cut taxes marginally and issue public debt to relax their bequest constraint. While this is possible, $b^+_t \geq 0$ cannot bind, which means generation t=0 sets the dynastic path of consumption at least until period $t_0$. This implies the MRS in (c) and the consumption trajectory in (d). Moreover, when parents can issue debt, they invest in education to maximize the utility of their children, so $h_t = h^*$ in (b).

A converse of (a) is that if $b^+_t = 0$ binds, all opportunities to tax future generations must have been exhausted, so $T_t = y^1_t - \bar{s}h_t$ must also bind. Specifically:

***Proposition 3 (The Tax-Constrained Regime):***

*If* $\mu_{Tax,t} > 0$ *in any period* $t = t_1$*, then* $\mu_{Tax,t} > 0$ *for all* $t \geq t_1$*. Moreover:*

a) *The bequest constraint* $b^+_t \geq 0$ binds.

b) *Investment in education is less than socially optimal:* $h_t < h^*$.

c) *Consumption growth matches growth of human capital:* $c^1_{t+1}/c^1_t = H_{t+1}/H_t$.

Prop.3 shows that a binding tax constraint has consequences: Public spending is reduced, notably on human capital; and from Prop.1, this implies reduced investment in private and public capital, so output and wages decline proportionally. Without bequests, consumption is determined by each generation's disposable income and allocated over the life cycle. Hence consumption grows with income; but disposable income $y^d_t = \bar{s}H_t$ is minimal, so consumption is growing along the lowest possible growth path.

An economy that starts tax-constrained in t=0 is uninteresting; it would have suboptimal human capital by assumption. Assuming the economy starts in the dynastic regime, the case $\beta = (1+\gamma)^\eta/\overline{R}$ is also uninteresting: consumption growth equals productivity growth, which equals the growth of human capital under efficient investment. Hence the economy follows a balanced growth path that stays in the dynastic regime forever.

Most interesting is the case of low altruism, $\beta < (1+\gamma)^\eta/\overline{R}$. Then the dynastic regime has consumption growth strictly less than productivity growth. This must eventually conflict with the tax constraint because the lower bound on income $y_t^d \geq \bar{s}H_t$ grows with productivity. Hence *with low altruism, the dynastic regime must end in finite time*, at some $t_A > 0$. Prop.2 applies for all $t_0 < t_A$ and Prop.3 for all $t_1 \geq t_A$. This transition has the following properties:

- Taxes reach their upper bound for the first time.
- Public debt reaches a level that debt service requires maximum taxes forever.
- Education spending is cut back to a less than efficient level.
- Investment in private and public capital less than under the previous growth path.

One may call this the start of an *Era of Austerity*. The bad news is that austerity is permanent and gets worse over time. Specifically:

***Proposition 4 (Consequences of Low Altruism):***

*Under Low Altruism, the dynastic regime applies for at most a finite number of periods. Thereafter Prop.3 applies and:*

a) *Public education declines monotonely to a lower bound:* $h_t \to \hat{h} = \arg\max_h \pi(h)$.

b) *Debt converges to the natural debt limit:* $D_t/\Gamma_t \to (\tfrac{\hat{D}}{\Gamma})_\infty$, *where*

$$(\tfrac{\hat{D}}{\Gamma})_\infty = \hat{h}\cdot[(K^G/H)^* + \tfrac{1}{R}\hat{\tau}^h] + \frac{1+\gamma}{R-(1+\gamma)}\pi(\hat{h}).$$

c) *In the special case* $\beta = 0$, $\mu_{Tax,t} > 0$ *applies for all* $t \geq t_A = 1$ *and convergence is immediate:* $D_t = (\tfrac{\hat{D}}{\Gamma})_\infty \Gamma_t$ *and* $h_t = \hat{h}$ *for all* $t \geq 1$.

Human capital declines because when taxes are at their upper limit, cuts in education relax the government budget constraint. In the limit, educational investments are only made to the extent that the government can extract full repayment through taxes; this characterizes the lower bound $\hat{h}$. The prospects for future generations are therefore grim. They pay maximum taxes and they are given declining levels of human capital. Since investments in physical capital decline along with human capital (from Prop.1), output and incomes are also reduced.

Debt converges to the natural debt limit, as in the classic savings problem. However, the path of debt is not necessarily monotonic. As debt approaches its upper bound, the economy contracts due to declining human capital. Hence, while $D_t/H_t$ always rises, $D_t/\Gamma_t$ may decline as it approaches the limit.

The composition of government outlays shifts during the convergence process: Spending for debt service—and spending for retiree entitlements here subsumed into debt—increases, while spending on education and infrastructure decline. More is spent for the old and less for the young. This is consistent with recent trends in countries struggling with debt. In the United States, for example, public education—including universities—has suffered, whereas Social Security and Medicare remain unchanged or were expanded (e.g., in 2005 with the addition of Medicare Part D). [31]

Part (c) provides a new argument regarding bequest motives. Voters without altruistic bequest motive would issue maximum debt and restrict education spending to the revenue-maximizing level. Altruism ($\beta>0$) is essential to postpone austerity and to slow down the growth of debt. A joy-of-giving motive, in contract, would not restrain debt. For given $\beta > 0$, a desire to make bequests would actually lead to increased public debt, to increase funds available for bequests. Hence the observation that a country has debt below the natural debt limit provides evidence that a majority of the country's voters must have Barro-Becker-type altruistic bequest motives.

## 5. Equilibrium when Government Default is a Threat

Now consider default – the possibility that governments can discard promises made by previous governments. Breaking promises cannot be taken lightly because a market economy requires government credibility, notably about taxes that influence investment incentives and about the enforcement of private debts. Hence government debt and defaults on other commitments are examined jointly.

To limit the scope, I will assume sufficient default costs that allocations are not inefficient by assumption. The objective is to examine economies with high debt—fueled by

[31] Technical note: The statements in Prop.1-4 remain valid even if the parameters in (*) are time-varying, except that: the factor proportions in Prop.1(a) and $h^*$ in 2(b) would also vary (e.g., fluctuate with world interest rates); (29) would involve a present value of time-varying fiscal dividends; the bequest constraint in 3(a) would apply infinitely often but not necessarily in all periods, and 3(d) would apply only over intervals with binding bequest constraints; and the existence of limits Prop.4 requires parameters to converge asymptotically.

low altruism—and the role of two-sided altruism, not to rehash the well-known time consistency problems resulting from near-zero credibility.

*5.1. Financial Markets with Default*

To formalize limited commitment, I define default variables (δ) with the defining property that non-zero values trigger certain default costs. The presumption here is that government default is a major all-or-nothing decision, with serious costs; but once default occurs, the government may as well default optimally and on wide range of commitments.

In the market for government debt, let $\delta^{d}_{t+1},\delta^{df}_{t+1}\in[0,1]$ denote ex-post default rates on domestic and foreign-held debt, respectively.[32] Actual payoffs are $R^{d}_{t+1}=\overline{R}(1-\delta^{d}_{t+1})$ and $R^{df}_{t+1}=\overline{R}(1-\delta^{df}_{t+1})$. If promises not to default are credible, such debt will sell at price $q_t=1$. Otherwise, $q_t=1-\min\{\delta^{d}_{t+1},\delta^{df}_{t+1}\}$ is reduced, potentially to zero.

In the market for capital, the structure of taxes is unchanged: there are income taxes on capital and labor and a possible capital levy. A distinction is now needed between actual rates $(\tau^{l}_{t+1},\tau^{k}_{t+1})$ in t+1 and tax rates promised in period t, denoted by $(\tilde{\tau}^{l}_{t+1},\tilde{\tau}^{k}_{t+1})$. Discretion—the option to set actual rates $(\tau^{l}_{t+1},\tau^{k}_{t+1})$ differently than promised—can then be modeled as default choice, denoted $\delta^{\tau l}_{t+1}\equiv(\tau^{l}_{t+1}-\tilde{\tau}^{l}_{t+1})/(1-\tilde{\tau}^{l}_{t+1})$ and $\delta^{\tau k}_{t}\equiv(\tau^{k}_{t+1}-\tilde{\tau}^{k}_{t+1})/(1-\tilde{\tau}^{k}_{t+1})$. (From Prop.1, the optimal promise is $(\tilde{\tau}^{l}_{t+1},\tilde{\tau}^{k}_{t+1})=(\xi_L,\xi_K)$.) The government will again promise $\delta^{k}_{t+1}=0$, so an ex-post levy indicates a default. If promises about future taxes are not credible, the payoffs in (15a) may be less that $R^{K}_{t+1}$, so capital investment may collapse.

In the market for private borrowing, the government may expropriate foreign claims or abrogate them to "help" domestic borrowers. Either way, actual returns can be written as $R^{f-}_{t+1}=\overline{R}\cdot(1-\delta^{xf}_{t+1}-\delta^{af}_{t+1})$, if a share $\delta^{xf}_{t+1}\in[0,1]$ is expropriated by the government and $\delta^{af}_{t+1}\in[0,1-\delta^{xf}_{t+1}]$ is canceled for the benefit of the borrower. When foreign claims are capital assets (foreign direct investment, FDI), one should assume that expropriation incurs the same cost $\Lambda^{K}\in(0,1)$ as a capital levy.

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32 This allows the government to default selectively. Taxes on interest income can be omitted without loss of generality because taxes would proportionally reduce the market price.

In summary, financial markets require a substantial number of government commitments to operate, even in this simple economy. The analysis has identified six relevant items: two promises not to honor debt $(\delta_{t+1}^{d}, \delta_{t+1}^{df})$, two commitments about tax rates that influence capital investment $(\delta_{t+1}^{\tau l}, \delta_{t+1}^{\tau k})$, and two commitments to respect private property rights $(\delta_{t+1}^{xf}, \delta_{t+1}^{af})$. To express these commitment compactly, define the default vector

$$\vec{\delta}_t \equiv (\delta_t^{d}, \delta_t^{df}, \delta_t^{\tau k}, \delta_t^{\tau l}, \delta_t^{k}, \delta_t^{xf}).$$

Default means setting $\vec{\delta}_t \neq 0$. Default costs are generally specified as $\Lambda_t^U = \Lambda_t^Y = 0$ for $\vec{\delta}_t = 0$; and $\Lambda_t^U = \Lambda^U(\Gamma_t) > 0$, $\Lambda_t^Y = \Lambda^Y \geq 0$ for all $\vec{\delta}_t \neq 0$.

Government revenues with default divide into regular tax revenues $T_t + \tilde{\tau}_t^k \tilde{r}_t K_t$, and revenues from defaulting

$$T_t^{\delta} \equiv \delta_t^{d} \overline{R} d_t + \delta_t^{df} \overline{R} d_t^{f} + \delta_t^{\tau k}(1 - \tilde{\tau}_t^{k}) \tilde{r}_t^{k} K_t + \delta_t^{k}(1 - \Lambda^K)(1 - \rho) K_t + \delta_t^{xf} a_t^{f-}. \qquad (29)$$

The budget equation (replacing (17) and (23)) is then

$$K_{t+1}^G + G_t^h - D_{t+1} = F_t^+ - \Lambda_t^Y Y_t - \overline{R} K_t - \overline{R} D_t + T_t^{\delta} - y_t^d = A_t^G - (y_t^d - \overline{s} H_t). \qquad (30)$$

where $$A_t^G = F_t^+ - \Lambda_t^Y Y_t - \overline{R} K_t - \overline{R} D_t - \overline{s} H_t + T_t^{\delta} \qquad (31)$$

summarizes the resources controlled by the period-t government, possibly via default.

Finally, note that public and private foreign debts are essentially equivalent, except for the restriction on $\delta_t^{xf}$ above, and that gross positions merely encourage default. To reduce the state space, one may assume without loss of generality that $d_t^f = a_t^{f-} = 0$ when $A_t^f \geq 0$, that $a_t^{f+} = 0$ when $A_t^f < 0$, and that $a_t^{f-} = \max(0, K_t + D_t - \lambda c_t^2 / \overline{R})$ is zero when $D_t \leq \lambda c_t^2 / \overline{R} - K_t$. With these conventions, $(d_t, d_t^f, a_t^{f+}, a_t^{f-})$ are functions of $(K_t, D_t, A_t^f)$ and can be omitted as state variables. Also, define $A_t^{f+} = \max(0, A_t^f)$ and $A_t^{f-} = \max(0, -A_t^f)$.

*5.2. Optimization Subject to Default*

Under perfect foresight, creditors can anticipate a default. The state of the economy at the start of period t is defined by the real capital stocks $(K_t, K_t^G, H_t)$, financial claims $(D_t, A_t^f)$, and promised tax rates $(\tilde{\tau}_t^l, \tilde{\tau}_t^k)$. To summarize them, define the state vector

$$\vec{X}_t = (K_t, K_t^G, H_t, D_t, A_t^f, \tilde{\tau}_t^l, \tilde{\tau}_t^k).$$

With possible default, each period has three stages: First the generation-t government sets $\vec{\delta}_t$, then generation-(t-1) retirees set bequests $b_t^+$, and finally the generation-t government implements $\vec{X}_{t+1}$. The latter implicitly defines consumption, taxes, and disposable income $(c_t^1, c_t^2, T_t, y_t^d)$. Let $V_{t+1}^1 = V^1(\vec{X}_{t+1})$ be the value function at the start of period-(t+1). Then $\vec{X}_{t+1}$ for given $(b_t^+, \vec{\delta}_t)$ solves

$$\tilde{V}^1(\vec{X}_t, b_t^+, \vec{\delta}_t) \equiv \max_{\vec{X}_{t+1}} \{\tilde{v}_t + \beta_t \cdot V^1(\vec{X}_{t+1})\}$$

and defines conditional values $\tilde{V}^1$; output costs of default are subsumed inside $\tilde{V}^1$ but not utility costs. Retirees' utility can be written in terms of $\tilde{V}^1$ as

$$V_t^2 = (1 - \alpha_t \beta_{t-1}) \lambda_t \upsilon u(c_t^2) + \beta_{t-1} \cdot \tilde{V}^1(\vec{X}_t, b_t^+, \vec{\delta}_t)$$

Optimal bequests are given by $b_t^+(\vec{X}_t, \vec{\delta}_t) = \arg\max_{b_t^+} \{V_t^2(\vec{X}_t, \vec{\delta}_t)\}$. When choosing default, generation t has perfect foresight about this bequest function. Because the cost of default is discontinuous, let $V^{1*}(\vec{X}_t) = \tilde{V}^1(\vec{X}_t, b_t^+(\vec{X}_t, 0), 0)$ denote the no-default value, and define the value conditional on default as $V^{1\delta}(\vec{X}_t) = \sup_{\vec{\delta}_t} \{\tilde{V}^1(\vec{X}_t, b_t^+(\vec{X}_t, \vec{\delta}_t), \vec{\delta}_t) \mid \vec{\delta}_t \neq 0\}$. To complete the value function recursion, define $V^1(\vec{X}_t) = \max\{V^{1*}(\vec{X}_t), V^{1\delta}(\vec{X}_t) - \Lambda^U(\Gamma_t)\}$. As tiebreaker, assume the government does not default if the two values are equal.

The analysis simplifies because $(\vec{X}_t, b_t^+, \vec{\delta}_t)$ operate only through resource constraints: they determining the asset positions $A_t^2$ defined in (18) and $A_t^G$ in (31). With $A_t^1 = \bar{s} H_t$ denoting resources controlled by generation-t individually, one can write

$$\tilde{V}^1(\vec{X}_t, b_t^+, \vec{\delta}_t) = \omega(A_t^1 + b_t^+, A_t^2 - b_t^+, A_t^G). \tag{32}$$

Conditional on default (given $\vec{\delta}_t \neq 0$), the optimal choice of default variables can be found by maximizing this function. Specifically, one finds:

***Proposition 5 (Optimal Default)***

*Conditional on $\vec{\delta}_t \neq 0$, elements of the optimal default vector $\vec{\delta}_t^*$ satisfy:*

*(a) 100% default on foreign liabilities: $\delta_t^{df} = 1$ and $\delta_t^{af} = 1 - \delta_t^{xf}$, with $\delta_t^{xf}$ defined below.*

*(b) 100% default on domestic debt and 100% capital income tax ($\delta_t^d = \delta_t^{\tau k} = 1$), except in a Dynastic Regime with $b_t^+(\vec{X}_t, 0) \geq \bar{R} d_t + \bar{r} K_t$.*

*(c) Pigovian labor income taxes $\tau_t^l = \xi_L$, so $\delta_t^{\tau l} = 0$.*

*(d) In the Dynastic Regime: If* $\alpha\beta > 1 - \Lambda^K$*, no capital levy is imposed (*$\delta_t^k = 0$*) and* $\delta_t^{xf} = \min(1, \delta_t^{xf\max})$ *is limited by* $\delta_t^{xf\max}\overline{R}a_t^{f-} \le (1-\rho)K_t$*, which means foreign capital claims are repudiated for the benefit of the domestic debtor and not expropriated. If* $\alpha\beta < 1 - \Lambda^K$*, then* $\delta_t^k = \delta_t^{xf} = 1$*.*[33]

*(e) In the Tax-Constrained Regime: If* $\alpha\beta > 1 - \Lambda^K$ *and if the shadow value* $\mu_{Tax,t}$ *in the government problem after the defaults in (a-b) satisfies*

$$(1 - \Lambda^K)(1 + \mu_{Tax,t}) \le \alpha MRS_t \tag{33}$$

*in then* $\delta_t^k = 0$ *and* $\delta_t^{xf}$ *is set like in (d). Otherwise capital levies cannot be ruled out.*

*Proof: Follows from first-order condition of maximizing (32); see online appendix for details.*

In Parts (a-b), it is perhaps not surprising that government debt and foreign liabilities are fully canceled, nor is the full taxation of capital income.

In part (b), an exception applies when $b_t^+(\vec{X}_t, 0) \ge \bar{r}K_t + \overline{R}d_t$, which means planned bequests are equal or greater than the resources subject to default. Then default is uninteresting because it would simply reduce bequests. This condition is equivalent to $\lambda c_t^{2*} \le (1-\rho)K_t^* + a_t^{f+}$. This scenarios may be empirically relevant (i) for historical analysis of economies with low life expectancy and a high value of old capital (e.g. farmland), or (ii) for countries with very high net foreign assets (e.g. oil states). It seems unrealistic for modern societies with decades of life expectancy in retirement.

For part (c), note that efficient labor taxes normally do require commitment. (For example, if labor taxes were made discretionary while restricting capital taxes to $\tilde{\tau}_t^k < 1$, the discretionary labor tax would exceed $\xi_L$, because it acts as a restraint on labor supply that benefits the working generation at the expense of capital owners.) The intuition here is that given $\tau_t^k = 1$, there is no capital income left to tax, and hence the working-age generation would only harm itself by distorting labor supply.

Parts (d-e) show that the optimality of capital levies—domestically or as expropriation of foreign capital claims—depends critically on the relationship between altruism ($\alpha\beta$) and

[33] In the non-generic case $\alpha\beta = 1 - \Lambda^K$, any $\delta_t^k \in [0,1]$ and any $\delta_t^{xf} \in [\delta_t^{xf\max}, 1]$ are optimal.

the cost of seizing assets ($\Lambda^K$). Without such altruism (if $\alpha\beta$=0) capital levies are ex-post optimal even if the deadweight loss $\Lambda^K$ is near one. If $\alpha\beta > 1-\Lambda^K$, capital levies are suboptimal under general conditions even if there are no other costs of default. Thus two-sided altruism *per se* can resolve the capital levy problem. To focus on this more interesting case, assume in the following that $\alpha\beta > 1-\Lambda^K$. (The consequences of $\alpha\beta < 1-\Lambda^K$ are straightforward: capital levies and taxes would then be equivalent, so all result about income taxes below would apply to levies, too.) In the Tax Constrained regime, complications arise because $\mu_{Tax,t}$ creates a wedge between the shadow values of resources in the government and individual funds. This wedge creates incentives to impose capital levies even if $\alpha\beta > 1-\Lambda^K$.

Prop.5 has the notable implications about foreign capital claims (FDI): under the same conditions that make domestic capital levies suboptimal, the government should prefer a transfer of FDI to domestic private owners over an outright expropriation. This is broadly consistent with the behavior of defaulting countries: in countries that respect private property, takings of FDI are often forced sales or forced "partnerships" with domestic firms. Expropriations occur when private property is also seized (e.g. in communist revolutions).

In summary, Prop.5 shows that the optimal default problem $\sup\{\tilde{V}^1 \mid \vec{\delta}_t \neq 0\}$ generally has a solution, so $V^{1\delta}(\vec{X}_t) = \tilde{V}^1(\vec{X}_t, b_t^+(\vec{X}_t, \vec{\delta}_t^*), \vec{\delta}_t^*)$ is attained.

A complete analysis of optimal policy is cumbersome, however, because—especially for low default costs—the state vector $\vec{X}_t$ may take values for which capital levies could be optimal and because multiple case-distinctions would be needed to characterize all post-default trajectories. Because the focus here is on high-debt scenarios, I limit the scope by assuming that default costs are high enough that the following benchmark allocations do not trigger default.

The Benchmark is defined as follows: suppose the initial real allocation is efficient ($H_t = h^*\Gamma_t$, $K_t/H_t = (K/H)^*$, $K_t^G/H_t = (K^G/H)^*$), tax rates are Pigovian, the external position is at least balanced ($A_t^f \geq 0$), and initial debt is no greater than public capital plus the cost of public education, $D_t \leq K_t^G + G_t^h$. The latter follows Boldrin-Montes (2005) and is similar to the

"Golden Rule" of borrowing only for capital investment.[34] Let $\vec{X}_t^{BM}$ denote state vectors with these properties and assume the utility costs of default are high enough that

$$V^{1*}(\vec{X}_t^{BM}) \geq V^{1\delta}(\vec{X}_t^{BM}) - \Lambda^U(\Gamma_t) \text{ for all } \vec{X}_t^{BM}. \tag{34}$$

Assumption (34) shifts the focus away from the problems of *low* commitment emphasized in the classic time-consistency literature (e.g., the capital levy problem in Fischer, 1980). If default turns out to be threat even when default cost high enough to support efficient allocations like $\vec{X}_t^{BM}$, problem is arguably too much debt—not a lack of commitment.

In a default, all debts are canceled and capital income is taxed at 100%. Asset positions shift by $\Delta A_t^2 = -\overline{R}D_t - \overline{r}K_t \leq 0$ and $\Delta A_t^G = \overline{R}D_t + \overline{r}K_t + \overline{R}A_t^{f-} - \Lambda^Y Y_t \geq 0$. After such a default, (30-31) imply a non-binding tax constraint because $A_t^G \geq 0$ and any desired $K_t^G + G_t^h$ can be debt-financed. Hence (34) rules out capital levies. Moreover, it ensures that efficient public investments are always feasible.

State vectors that trigger default in the next period are suboptimal. This is because a foreseen default implies $q_t = 0$ and $Q_t < 1$, so bond markets shut down and capital investment is zero. The same allocation could be obtained without default cost by announcing a 100% capital income tax and not issuing debt. Hence finding an optimal policy divides into two parts: determining which state vectors $\vec{X}_{t+1}$ trigger default in the next period, and finding the optimal $\vec{X}_{t+1}$ outside this set. The set not triggering default is defined by:

$$\mathrm{X}_{ND} = \{\vec{X}_{t+1} : V^1(\vec{X}_{t+1}) \geq V^{1\delta}(\vec{X}_{t+1}) - \Lambda^U(\Gamma_{t+1})\}. \tag{35}$$

The condition $\vec{X}_{t+1} \in \mathrm{X}_{ND}$ distinguishes the government problem here from the commitment solution in Section 4. Commitment is unconditional if $\Lambda^U(\Gamma_{t+1}) = \infty$.

Provided the condition in (35) holds, $V^1(\vec{X}_{t+1}) = V^{1*}(\vec{X}_{t+1})$. The generation-t policy problem is then to maximize $\tilde{v}_t + \beta_t \cdot V^1(\vec{X}_{t+1})$ by choice of $\vec{X}_{t+1}$, subject to the same constraints as in Section 4 *and* subject to $\vec{X}_{t+1} \in \mathrm{X}_{ND}$. If optimal choices for all t are within $\mathrm{X}_{ND}$, so (35) never binds, default costs are prohibitive. Then the planning solutions derived in

[34] Allowing debt-financed education ensures that liquidity constraints do not interfere with efficient investment. Since education benefits the next generation, it is arguably consistent with the Golden Rule principle.

Section 4 remain optimal. Notably, low altruism leads to permanent austerity and to debt converging to the natural debt limit. In contrast, if the commitment solution ever leads to $\vec{X}_{t+1} \notin \mathrm{X}_{ND}$, optimal policy must be computed recursively and the equilibrium path will differ.

When the planning solution falls outside $\mathrm{X}_{ND}$, the condition in (35) must bind at some finite date, either in the Dynastic or in the Tax-Constrained regime. Because policy changes at that time are caused by an imminent threat of default, such an event is a debt crisis. At that time, the Lagrangian

$$\mathcal{L} = \tilde{v}_t + \beta \cdot V^1(\vec{X}_{t+1}) + \mu_{ND} \cdot [V^1(\vec{X}_{t+1}) - V^{1\delta}(\vec{X}_{t+1}) + \Lambda^U(\Gamma_{t+1})] + \ldots$$

places weight $\beta + \mu_{ND}$ on the preferences of generation (t+1). Hence $\mu_{ND} > 0$ discourages debt accumulation. Low altruism can be reconciled with balanced growth if $\mu_{ND} = (1+\gamma)^{\eta} / \overline{R} - \beta$ bridges the gap between altruism and world interest rates and breaks the dismal logic of the savings problem.

However, $\mu_{ND} > 0$ places negative weight on $V^{1\delta}(\vec{X}_{t+1})$, and this distorts the choice of $\vec{X}_{t+1}$. Details of the resulting distorted allocation are beyond the scope of this paper. (Instead, the following focuses on conditions for (35) not to bind.) It is clear from Prop.5, however, that because capital income creates incentives to default, the period-t government has an incentive to discourage capital investment—to nudge members of its generation to save less or to hold retirement assets not subject to default (foreign assets). Thus threats of default are an inefficient mechanism to constrain debt accumulation.

Note that an outside "bailout" of an economy with low altruism—say relief from external debt in a crisis—would merely restart the process of debt accumulation. Because an allocation on the edge of default is inefficient, agreements between generations to limit public debt are potentially Pareto improving. It is unclear, however, how to make such agreements credible.[35]

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[35] This is a promising issue for future research, but beyond the scope of this paper. The history of the Eurozone suggests skepticism about outside enforcement.

*5.3. Interpretation of the No-Default Condition*

This section shows how the no-default condition in (35) can be written as an empirically interpretable condition on government debt, external debt, and capital income.

Let $\omega_G = \frac{\partial\omega}{\partial A_t^G} = u'(c_t^1)(1+\mu_{Tax,t})$ and $\omega_2 = \frac{\partial\omega}{\partial A_t^2} = \alpha v u'(c_t^2)$ denote derivatives of the value functions in (32), and define $\varphi \equiv \omega_2/\omega_G = \alpha MRS_t/(1+\mu_{Tax,t}) < 1$. In the following, let superscripts star (*) and δ refer to values obtained without and with default, respectively, and let superscript zero (0) refer to a hypothetical allocation in between (a partial default). Because defaults shift funds from the old and foreigners to the government, $\omega_G^\delta < \omega_G^*$ and $\omega_2^\delta > \omega_2^*$. With this notation:

***Proposition 6 (Conditions for No Default)***

*(a) If* $\bar{R}d_t + \bar{r}K_t > b_t^+(\vec{X}_t, 0) = b_t^{+0}$, *there are values* $\omega_G^0 \in (\omega_G^\delta, \omega_G^*)$, $\omega_2^0 \in (\omega_2^*, \omega_2^\delta)$, *and* $0 < \varphi^0 = \omega_2^0/\omega_G^0 < 1$, *such that no-default condition in (35) holds if and only if*

$$\Delta_t \equiv \bar{R}d_t^f + (1-\varphi^0)\cdot(\bar{r}K_t + \bar{R}d_t - b_t^{+0}) + \varphi^0 \cdot \bar{R}a_t^{f-} \le \Lambda^Y Y_t + \Lambda^U(\Gamma_t)/\omega_G^0. \tag{36}$$

*(b) If* $b_t^{+0} \ge \bar{R}d_t + \bar{r}K_t$, *then* $\bar{R}A_t^{f-} \le \Lambda^Y Y_t + \Lambda^U/\omega_G^0$ *is sufficient for (35).*

*Proof*: Follows from taking partial derivatives in (32) with respect to element of $\vec{\delta}_t^*$, writing $V^{1*}(\vec{X}_{t+1}) - V^{1\delta}(\vec{X}_{t+1})$ as integral over partial derivatives, and invoking the mean-value theorem. QED.

Equation (36) shows how default costs put an upper bound on the resources subject to default—on government debts, foreign liabilities, and capital income. To preclude default, potential gains from default (Δ) must not exceed a combination of output cost and utility costs. The latter are scaled by the shadow value of government funds, $\omega_G^0$. In effect, these costs define a "credibility budget" that limits the country's asset positions.

For completeness, part (b) provides a sufficient condition for the empirically unrealistic cases with $\bar{R}d_t + \bar{r}K_t - b_t^{+0} < 0$. Cases with positive net foreign assets are included in (36) and would have zeros for $\bar{R}d_t^f$ and $\varphi^0 \cdot \bar{R}a_t^{f-}$.

Empirical work suggests that the output costs of defaults are not large (Borensztein and Panizza, 2009). Hence the paper emphasizes utility costs. With economic growth, utility costs will shrink or grow relative to output unless $\Lambda^U(\Gamma_t) = O(\Gamma_t^{1-\eta})$ has the same trend growth as utility.[36] If $\Lambda^U$ were a constant, $\eta > 1$ would imply an exponentially rising ratio of utility cost to GDP and $\eta < 1$ would imply a vanishing ratio of utility cost to GDP.

The asset positions in (36) are weighted. Foreign-held government debt enters with unit weight, the same weight as the output costs on the right. Domestic debt and capital income enter with reduced weight $1-\varphi^0$. Gains from domestic default are reduced by the altruism of the governing working-age generation towards the old ($\varphi^0$ is proportional to $\alpha$). Private foreign debt enters with weight $\varphi^0$, reflecting the governing generation's valuation of debt-relief for the old.[37]

Two alternative version of (36) provide more intuition. From (27), the no-default condition is equivalent to

$$\overline{R}A_t^{f-} + (1-\varphi^0)\cdot[\lambda c_t^2 - (1-\rho)K_t - \overline{R}A_t^{f+}] \le \Lambda^Y Y_t + \Lambda^U(\Gamma_t)/\omega_G^0. \tag{37}$$

This shows how retirement financing differs from other borrowing. Retirement financing creates default incentives only to the extent that retirement consumption $\lambda c_t^2$ exceeds old capital and foreign assets (which are default-protected), and these items are down-weighted by altruism. Other borrowing must be abroad and requires matching default costs, one-for-one. Alternatively,

$$\overline{R}D_t + \overline{r}K_t - b_t^{+0} - \varphi^0\cdot[\lambda c_t^2 - (1-\rho)K_t - \overline{R}A_t^{f+}] \le \Lambda^Y Y_t + \Lambda^U(\Gamma_t)/\omega_G^0. \tag{38}$$

This shows all resources subject to default entering with unit weight. The resulting default incentives are reduced one-for-one by planned bequests, and reduced partially if funds are used for retirement consumption.

The analysis here leaves one important question unresolved: Will countries default as they approach the natural debt limit, and which ones? Forecasts of default are unfortunately difficult because the utility costs of default are likely unobservable until one actually observes

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[36] This is because along a balanced growth path, $c_t^1, Y_t = O(\Gamma_t)$ grow with $\Gamma_t$, $\omega_G = (c_t^1)^{-\eta}(1+\mu_T) = O(\Gamma_t^{-\eta})$, so stationarity of $\Lambda^U(\Gamma_t)/\omega_G/Y_t$ requires $\Lambda^U(\Gamma_t) = O(\omega_G Y_t) = O(\Gamma_t^{1-\eta})$.

[37] Note that shifting from $d_t^f$ to $a_t^{f-}$ would not change default incentives, because for a given overall debt $D_t$, lower $d_t^f$ means higher $d_t$, and a unit increase in $d_t$ and $a_t^{f-}$ creates the same default incentives as a unit of $d_t^f$. This is to clarify that the default incentives in (36) depend on fundamentals and cannot be changed by financial engineering.

a debt crisis. The level of resources subject to default at the start of a debt crisis could be interpreted as a revealed measure of gains from default. (A debt crisis in this context is a sudden restraint in debt accumulation due to a credible threat of default, not necessarily an actual default.) Taxes may be close to the maximum in many developed countries (Trabandt and Uhlig 2012), suggesting a transition to austerity. But in most countries, default does not appear to be an immediate threat, suggesting a strong aversion to default. (Argentina and Greece may be exceptions.)

The main conclusion at this time is therefore that low altruism necessarily implies either: (a) austerity leading to the natural debt limit; (b) austerity leading to a default crisis; or, (c) a default crisis before maximum taxes are reached. Case (a) applies if and only if the No-Default condition covers $(\frac{D}{Y})_\infty$. Case (b) applies if and only if the No-Default condition covers debt at the transition to austerity ($(\frac{D}{Y})_t$ at $t = t_A$) but not $(\frac{D}{Y})_\infty$. Case (c) applies only if the No-Default condition starts binding before $t = t_A$.

## 6. Conclusions

The paper examines public debt and other government obligations in an overlapping generations model with three generations—retired old, working-age, and student-age young. When the welfare of future generations is discounted at a higher rate than the market interest, a scenario labeled "low altruism," generations use their control over government to extract resources from future generations. The intuition is as in the classical savings problem, but across generations and using the powers of government.

In a full commitment version, public debt converges asymptotical to an upper bound, the natural debt limit. The convergence has two stages characterized by different policy regimes. Initially the allocation is *Dynastic*, meaning the old control the intergenerational allocation of consumption, using public debt and taxes. After a finite number of generations, labor income taxes reach an upper bound and limit the growth of debt. This marks the start of a *Tax-Constrained* regime characterized by maximum taxes and reductions in public spending—an era of austerity. Public education, which is altruistically motivated, declines and converges to a lower bound. Thus government spending shifts from spending on the young (education and public capital) to spending on the old (debt service and pensions).

The analysis with commitment presumes that default is prohibitively costly. To explore how threat of defaults may limit debt accumulation, I also examine a model version with limited commitment. I show that default must be costly even without debt to sustain government commitments required for efficient capital investment. Altruism of children towards parents plays a key role in mitigating incentives to default. In summary, low altruism is a serious problem: it necessarily leads to either persistent austerity with maximum taxes and convergence to maximum debt, or to a debt crisis, or to austerity followed by a debt crisis.

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**Table 1: Net Investment Positions and Current Account Balances of the Top-10 International Creditors and Debtors**

| Country\Measure | Net Investment Position in % of world total 2005 | Net Investment Position as % of own GDP 2005 | Average Current Account as % of GDP Av. 2006-10 |
|---|---|---|---|
| **Top Creditors** | | | |
| Japan | 34% | 34% | 3.7% |
| Germany | 12% | 17% | 6.4% |
| Switzerland | 11% | 124% | 10.0% |
| Hong Kong | 10% | 242% | 10.4% |
| China (Mainland) | 9% | 18% | 7.4% |
| Singapore | 8% | 203% | 21.0% |
| Saudi Arabia** | *6%* | *97%* | 20.0% |
| Norway | 4% | 55% | 13.6% |
| Belgium | 3% | 27% | 0.3% |
| Kuwait | 1% | 61% | 36.2% |
| Sum of top 10 | 96 % | | |
| Average* | | **37%** | **6.5%** |
| **Top Debtors** | | | |
| United States | 30.0% | -15% | -4.3% |
| Spain | 9.3% | -45% | -7.6% |
| United Kingdom | 6.8% | -19% | -2.0% |
| Australia | 6.1% | -53% | -4.6% |
| Brazil | 4.9% | -36% | -0.8% |
| Mexico | 4.6% | -35% | -0.9% |
| Italy | 4.1% | -13% | -2.3% |
| Greece | 2.7% | -62% | -12.4% |
| Turkey | 2.7% | -36% | -5.3% |
| South Korea | 2.7% | -20% | 2.1% |
| Sum of top-10 | 74 % | | |
| Average* | | **-21%** | **-3.7%** |

Source: IMF and own calculations. Creditors and debtors are ranked by the country's net investment position as share the world total of positive and negative net investment positions, respectively, as shown in Col.1.

* Averages are weighted by GDP.

** Net Investment for Saudi Arabia was not available for 2005 and estimated from 2007 data (97% of GDP). This is to avoid biasing the results by omitting a significant creditor.

**Table 2: Public Debt, Budget Balances, and Public Pensions of the Top-10 International Creditors and Debtors**

| Country\Measure | Public Debt (Gross) as % of GDP 2005 | Average Budget Balance as % of GDP Av. 2006-10 | Public Pensions Replacement Rate at average earnings 2011 |
|---|---|---|---|
| **Top Creditors** | | | |
| Japan | **186%** | **-5.9%** | 40% |
| Germany | 69% | -1.8% | 56% |
| Switzerland | 70% | 1.0% | 38% |
| Hong Kong | 34% | 3.7% | ** |
| China (Mainland) | 18% | -1.0% | ** |
| Singapore | 93% | 6.4% | ** |
| Saudi Arabia* | 39% | 15.0% | ** |
| Norway | 48% | 15.2% | 52% |
| Belgium | 92% | -2.1% | 52% |
| Kuwait | 14% | 29.3% | ** |
| | | | |
| Average* | 105% | -1.9% | 46% |
| Average* excl. Japan | **50%** | **0.8%** | 54% |
| | | | |
| **Top Debtors** | | | |
| United States | 68% | -7.2% | 37% |
| Spain | 43% | -4.2% | 85% |
| United Kingdom | 42% | -6.2% | 37% |
| Australia | 11% | -1.3% | 15% |
| Brazil | 69% | -2.6% | ** |
| Mexico | 39% | -2.5% | ** |
| Italy | 105% | -3.5% | 72% |
| Greece | 101% | -9.8% | 111% |
| Turkey | 53% | -2.5% | 93% |
| South Korea | 29% | 1.4% | 47% |
| | | | |
| Average* | **62%** | **-5.7%** | 45% |

Source: IMF (debt and budget balance), OECD (pensions), and own calculations. Creditors and debtor are ranked by net investment position as shown in Table 1.

* Averages are weighted by GDP.

** Not available or not OECD.