Intergenerational Risk Sharing and Fiscal Policy

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Submitted September 2007. Revised May 14, 2009. Second revision June 18, 2009.

Abstract: Risk-sharing implications of alternative fiscal policies are compared in a stochastic production economy with overlapping generations. Ex ante efficiency is shown to be achievable with optimal transfers, regardless of distributional concerns. For CRRA preferences, stylized real-world policies (notably safe debt and safe pensions) are found inefficient in the direction of imposing not enough productivity risk on retirees and too much on future generations. Safe transfers can be rationalized as efficient if preferences display age-increasing risk aversion, such as habit formation. The ubiquity of safe transfers suggests that governments treat the young as more risk tolerant than older cohorts.

JEL classification: H55, H60, E62.

Keywords: aggregate risks; optimal risk sharing; intergenerational transfers; overlapping generations; social security; fiscal policy.

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1. Introduction

Overlapping generations (OG) models are widely used for policy analysis. In stochastic OG models, fiscal policy necessarily influences the allocation of risk across generations. Many recent papers on social security reform, for example, have employed stochastic OG models to study policies under uncertainty; similar models have been used to study tax policy and public debt management.[1]

This paper uses an analytical log-linearization approach similar to Campbell (1994) to examine the allocation of aggregate risks in stochastic OG models, particularly the role of fiscal policy. The key questions are under what conditions a fiscal policy improves shares risk, and how to diagnose forms of inefficiency. I show that ex ante efficiency, conditional on initial capital, is a feasible standard for fiscal policy; that the efficiency of a market allocation (with given fiscal policy) can be evaluated by comparing it to a uniquely defined "comparable" efficient allocation; and that in recursive models with balanced growth, efficiency comparisons can be obtained easily from log-linearized policy functions.

The general approach is then applied to study productivity uncertainty in economies with specific functional forms for preferences and technology. I focus on productivity because uncertain productivity growth is a major source of long-run risk and because fiscal policy profoundly influences how productivity shocks are allocated: Fiscal policy has traditionally protected retirees against such risk, notably by promising safe public pensions and supplying safe government bonds.

The main applied finding is that, for empirically plausible parameters, protecting retirees against productivity risk is inefficient in models with standard preference/technology

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[1] Examples are (as drawn from a huge literature, with apologies to those not cited): Abel (2001), Krueger and Kubler (2002), Shiller (2003), and articles in Campbell-Feldstein (2001); for tax policy, Auerbach and Hassett (2002); for public debt management, Gale (1990) and Bohn (2002).

assumptions, notably for power utility (CRRA) with an elasticity of intertemporal substitution less or equal one. This is because market allocations are inefficient in the opposite direction: Retirees bear less productivity risk than workers. Efficient transfers—at any given level of redistribution—should be contingent on productivity. Safe transfers magnify the inefficiency.

Production and capital investment are important in this context because they endogenize the correlation between capital and labor income and because they allow current and future generations to share risks through variations in capital investment. Because capital and labor incomes are naturally correlated, my focus is on aggregate production uncertainty and not on cohort-specific risks.[2] Throughout, I assume two period lived agents, which eliminates private risk sharing, and I abstract from idiosyncratic risks, bequests, and distortionary taxes.[3]

The inefficiency of relatively safe transfers generalizes to models with a stochastic cost of capital (Tobin's-Q) and asset price uncertainty, general production functions, and endogenous labor-leisure choices. Efficient transfers are sensitive to preferences, however, as I show in a habit formation model. Then safe transfers can be efficient, because retirees with established consumption habits are more risk averse than workers. In spirit of a positive theory of intergenerational transfers, the ubiquity of relatively safe transfers is consistent with consumption habits, or more broadly, with preferences that display age-increasing risk aversion.

The sensitivity of optimal policy to preferences suggests that power utility is not an innocuous assumption for fiscal policy research. The assumption of age-independent risk aversion implicitly favors policy alternatives that shift productivity risk to retirees, e.g., social

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[2] This differs from the literature on intergenerational risk sharing in endowment economies; see, e.g., Enders and Lapan (1982), Fischer (1983), Stiglitz (1983), Gordon and Varian (1988), Gale (1990), Rangel and Zeckhauser (2001). (Stiglitz does allow for capital investment, but assumes exogenous factor prices. Gordon and Varian briefly comment on production.) Baxter and Jermann (1997) have shown that capital and labor incomes are highly correlated at long horizons, suggesting that correlated income shocks are empirically important.

[3] With more than two periods, there would be private risk sharing between "middle-aged" and old agents, but still no risk sharing with future generations, which is the key issue. Idiosyncratic risks are assumed to be shared within a cohort. Tax-distortions are omitted to stay within a first-best (at least potentially) setting. Ricardian bequests would

security reforms that replace defined benefits by private accounts holding risky assets.

The case of log-utility combined with 100-percent depreciation of capital—the most tractable and popular OG specification in the literature—turns out to have non-generic properties even within the CRRA/Cobb-Douglas class of models: It is the only specification in this class for which laissez-faire is efficient and policy cannot improve efficiency.

The paper is organized as follows. Section 2 describes the risk-sharing problem, characterizes efficient allocations, and shows how balanced growth yields simple efficiency comparisons. Section 3 examines the CRRA/Cobb-Douglas framework. Section 4 presents a habit model and other extensions. Section 5 concludes.[4]

2. The Risk Sharing Problem

This section presents the general model and explains the efficiency benchmark.

2.1. The Model

Consider an OG economy with two-period lived agents. Generation t consists of N_t individuals who work in period t and are retired in period $t+1$. Individuals have preferences $U_t = U(c_t^1, c_{t+1}^2, l_t)$ over working-age consumption $c_t^1 \geq 0$ and leisure $l_t \in [0,1]$, and over retirement consumption $c_{t+1}^2 \geq 0$. Utility is increasing, strictly concave, and possibly non-separable; but assume $\partial U_t / \partial c_{t+1}^2$ does not depend on l_t. (This allows habit formation and interactions between working-age consumption and leisure, but not a dependence of $\partial U_t / \partial c_{t+1}^2$ on lagged leisure that would needlessly complicate the dynamics.)

Output Y_t is produced with capital K_t and labor L_t. Each worker supplies $1-l_t$ unit of labor, so $L_t = N_t(1-l_t)$ is the aggregate labor supply. The economy's resource constraints are

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assume away the risk sharing problem.

[4] An online appendix available at http://www.econ.ucsb.edu/~bohn/papers/IGRiskApp.pdf provides a notation table (Part A), proofs (Part B), and supplementary materials (Parts C-E).

$$I_t + N_t c_t^1 + N_{t-1} c_t^2 = Y_t = F(K_t, L_t, A_t, z_t^F) \tag{1}$$

and $$K_{t+1} = G(I_t, K_t, z_t^G), \tag{2}$$

where F is increasing, concave in (K_t, L_t), and subject to random shocks (A_t, z_t^F); and G is increasing and concave in (I_t, K_t) with shocks z_t^G. Linear accumulation, $G = I_t + (1-\delta)K_t$, with fixed depreciation rate $\delta \in [0,1]$ is included as special case. Population growth is constant, $N_t / N_{t-1} = \gamma_N$.[5]

The stochastic shocks are divided into stationary disturbances $z_t = (z_t^F, z_t^G)$ and a non-stationary component $A_t = A_{t-1} \cdot a_t$, which is driven by a permanent productivity shock a_t. Permanent productivity shocks capture the intuitive notion that uncertainty grows with the forecast horizon, and they are arguably the most significant source of long-run economic uncertainty.[6] Temporary shocks may be less relevant on a generational time scale because of time-averaging, but some may be large enough to deserve modeling, e.g., major wars, boom periods, or asset market crashes.[7]

Let h_t denoted the state of nature at time t. To be specific about time, assume the economy starts at $t = 1$ with initial capital K_1 divided equally among an initial "old" generation and with shocks drawn from an initial distribution. Let preferences over c_1^2 be defined by

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[5] Non-zero population growth is included for better calibrations below and because its omission would raise questions about the model's relevance to a world with population growth. Demographic shocks are omitted because a stochastic population would complicate the normative analysis (see Bohn 2001).

[6] The risks at stake are huge: An annual productivity growth two percent higher or lower would, for example, raise or reduce the next generation's income by about 60%, and easily make or break social security. Given the controversy about unit roots in GDP, those favoring trend stationarity with occasional trend breaks might question the relevance of unit root shocks. A unit root component is nonetheless appropriate at generational frequencies, even if a stationary trend fits the data over a shorter horizons (say, a few decades), because the likelihood of future trend breaks implies a unit root-like uncertainty in the very long run (keeping in mind that, say, 20 periods in this model are about 600 years). Section 3 will cover temporary as well as permanent productivity shocks.

[7] Shocks to government spending can be subsumed into z_t^F if one interprets F as privately available output, i.e., net of government spending. I do not include government spending explicitly to ensure that there is a well-defined laissez-faire allocation. An asset market crash can be interpreted as a negative shock to the value of existing capital.

$U_0 = U(c_0^1, c_1^2, l_0)$ with given (artificial) values (c_0^1, l_0). Then states can be defined recursively as $h_0 = \{K_1, A_0, N_0, c_0^1, l_0\}$ and $h_t = \{h_{t-1}, a_t, z_t\}$. Dependence on h_t is often suppressed to avoid clutter.

As conceptual benchmark, consider first a market economy without government, the <u>laissez-faire allocation</u>. Let $Q_t = [{}^{\partial G}\!/\!_{\partial I}(I_t, K_t, z_t^G)]^{-1}$ denote the value of capital in terms of consumption (Tobin's-Q). Then retiree consumption is $c_t^2 = R_t \cdot k_{t-1}^1 / Q_{t-1}$, where k_{t-1}^1 is working-age savings of a current retiree, k_{t-1}^1 / Q_{t-1} the capital stock per retiree, and

$$R_t = \tfrac{\partial F}{\partial K}(K_t, L_t, A_t, z_t^F) + Q_t \cdot \tfrac{\partial G}{\partial K}(I_t, K_t, z_t^G) \tag{3}$$

the return on capital. Workers make choices over consumption, savings, and leisure, subject to a given wage rate $w_t = \tfrac{\partial F}{\partial L}(K_t, L_t, A_t, z_t^F)$ and subject to the budget constraint $w_t(1 - l_t) = c_t^1 + k_t^1$. The optimality conditions

$$E_t[\tfrac{\partial U_t}{\partial c_t^1}] = E_t[\tfrac{\partial U_t}{\partial c_{t+1}^2} \cdot R_{t+1} / Q_t] \text{ and } E_t[\tfrac{\partial U_t}{\partial c_t^1} \cdot w_t] = E_t[\tfrac{\partial U_t}{\partial l_t}] \tag{4}$$

show that workers' optimal choices depend on the current wage and on expectations about R_{t+1} / Q_t (where E_t is shorthand for conditioning on h_t).[8]

Secondly, consider <u>market allocations with fiscal transfers</u>. To model fiscal policy parsimoniously, let b_t denote per-capita transfers from the government to retirees, so retiree consumption is

$$c_t^2 = R_t / Q_{t-1} \cdot k_{t-1}^1 + b_t \,. \tag{5}$$

The term "transfer" is used for brevity. The variable b_t is best interpreted broadly as

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[8] These formula simplify in special cases, though sometimes with strong implications. In the widely-used case of Cobb-Douglas production with fixed depreciation, for example, capital income $\tfrac{\partial F}{\partial K} \cdot {}^{k_{t-1}^1}\!/\!_{Q_{t-1}}$ is proportional to labor income and the value of old capital is constant ($Q = 1$, $\tfrac{\partial G}{\partial K} = 1 - \delta$). For $\delta = 1$, retiree consumption is perfectly proportional to labor income. For $\delta < 1$, retiree consumption is necessarily less volatile (proportionally) than labor income. The general setting here avoids such restrictions; Sections 3-4 examine the Cobb-Douglas case and other

encompassing all components of retirees generational account, i.e., all transfers net of taxes.[9] Transfers must be financed by net taxes $b_t \cdot N_{t-1} / N_t = b_t / \gamma_N$ on workers. Workers face the same choice problem as under laissez-faire, but with budget constraint

$$c_t^1 + k_t^1 = w_t(1 - l_t) - b_t / \gamma_N \, . \tag{6}$$

A fiscal policy is generally defined by a sequence of state-contingent transfers $\{b_t(h_t)\}_{t \geq 0}$. Market allocations are defined by sequences of state-contingent consumption, leisure, and capital such that individuals maximize utility subject to (5)-(6), wages and returns are competitive, and markets clear. Policy analysis means comparing market allocations implied by alternative policies. Laissez-faire can be interpreted as special case $b_t(h_t) \equiv 0$ for all h_t.

Note that in case of safe (fixed) transfers to retirees, workers' stochastic labor income is reduced by a constant, which makes their disposable income more volatile. Thus safe transfers to the old create risks for subsequent generations. This illustrates how fiscal policy influences the allocation of risk—almost inevitably, and even without deliberate state-contingencies.

Third, consider Pareto efficient allocations, which are obtained by solving social planning problems at time $t = 0$. The planning problem is to maximize a welfare function

$$W_0 = E_0[\sum_{i=0}^{\infty} (\prod_{t=0}^{i} \omega_t) \cdot N_i \cdot U_i] \tag{7}$$

with given welfare weights $\omega_t > 0$ subject to the resource constraints (1)-(2).[10] Different Pareto-optimal allocations are obtained for different sequences of weights $\{\omega_t\}_{t \geq 0}$. These allocations are efficient in an *ex ante* sense, though conditional on initial conditions h_0; each can be

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applications.

[9] Generational accounting conveniently treats public debt issues and redemptions as transfers, avoiding the need to model the bond market. Hence k_t^1 should be interpreted as purchases of capital, not including claims against government. This accounting simplifies the exposition and, given lump sum taxes, is without loss of generality.

[10] By conditioning on initial resources, transition costs between steady states are included. This is indispensable in a production economy to ensure that comparisons are between feasible allocations. The planning problem is used as device to characterize efficient allocations without meaning to suggest that actual governments act like social

implemented by unique set of state-contingent efficient transfers, denoted $\{b_t^*(h_t \mid \omega)\}_{t\geq 0}$.

The social planner's first order conditions require

$$\omega_t \cdot E_t[\tfrac{\partial U_t}{\partial c_t^1}] = \tfrac{\partial U_{t-1}}{\partial c_t^2} \text{ for all } h_t, \tag{8}$$

and (4). Condition (8) characterizes the division of consumption between retirees and workers in each state of nature. The planner transfers resources across generations until the marginal utility of the old equals the marginal utility of the young times the welfare weight. This condition is similar to efficiency conditions in endowment models, e.g., in Gale (1990) and Stiglitz (1983), but here embedded in a production economy that allows the planner to shift resources over time.

A main question of the paper is how to assess the efficiency of a given (observed) market allocation. A challenge is that there are infinitely many welfare weights for which the given allocation might maximize welfare. However, efficient allocations must satisfy (8) for all h_t and hence in expectation (at t=0). For a given market allocation, the only possible weights for which it might maximize welfare are therefore the weights

$$\tilde{\omega}_t \equiv 1/E_0[\tfrac{\partial U_t}{\partial c_t^1} / \tfrac{\partial U_{t-1}}{\partial c_t^2}] \text{ for all t.} \tag{9}$$

If the efficient allocation with weights $\{\tilde{\omega}_t\}_{t\geq 0}$ exists, it provides a unique benchmark—henceforth called the comparable efficient allocation—to which the market allocation must be compared.[11] The market allocation is efficient if and only if $\{b_t(h_t)\}_{t\geq 0} = \{b_t^*(h_t \mid \tilde{\omega})\}_{t\geq 0} \forall h_t$.

The comparison is also instructive if there is a mismatch, because it reveals in which way the market allocation misallocates risk—which cohorts are exposed too much or too little to which sources of risk, and how much. Similarly, differences between actual and comparable efficient transfers reveal how policy could be improved. Because all comparisons are conditional

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planners. The Appendix (Part C) explains the efficiency standard in more detail, with comparison to alternatives.

[11] Market allocations for which comparable planning solutions do not exist (e.g. with dynamic inefficiency) are uninteresting for risk sharing (see Appendix, Part C, for details).

on welfare weights, they do not involve distributional judgments. This provides the conceptual foundation for studying intergenerational risk sharing.

2.2. Balanced Growth and Log-Linear Approximations

To obtain more specific results, assume balanced growth and a recursive stochastic structure. Balanced growth requires production with constant returns to scale; labor-augmenting productivity growth; and preferences that are either homothetic in consumption or logarithmic.[12] To obtain a recursive structure, let the permanent shock a_t be i.i.d. with mean $E[a_t] = \gamma_a \geq 1$, and let the stationary shocks z_t follow a mean-zero Markov process.

Efficient transfers are then functions of a Markov state vector S_t. Transfers and other growing variables are stationary after dividing by the stochastic trend A_{t-1}. If U_t is time-separable, the state vector for b_t / A_{t-1} consists of the capital-labor ratio $k_{t-1} \equiv K_t /(A_{t-1}N_{t-1})$ and the stochastic shocks $\{a_t, z_t\}$. If U_t is not time-separable, a lagged consumption term $\chi_{t-1} \equiv c^1_{t-1} / A_{t-1}$ must be included, because χ_{t-1} enters into (8) whenever $\frac{\partial^2 U_{t-1}}{\partial c_t^2 \partial c_{t-1}^1} \neq 0$. Moreover, balanced growth implies that $\{c_t^1/A_{t-1}, c_t^2/A_{t-1}, b_t/A_{t-1}\}$ are each linearly homogeneous in $(a_t, k_{t-1}, \chi_{t-1})$ and that $\{l_t, k_t\}$ are homogenous of degree zero in $(a_t, k_{t-1}, \chi_{t-1})$.[13]

Log-linear approximations are insightful to quantify uncertainty in this setting. For any variable x_t, let $\hat{x}_t$ denote the percentage deviation from the deterministic steady state (obtained

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[12] That is, either $U(\lambda c_t^1, \lambda c_{t+1}^2, l_t) = \lambda^{1-\eta} U(c_t^1, c_{t+1}^2, l_t)$ for some $0 < \eta \neq 1$ and all $\lambda > 0$; or (as $\eta \to 1$) $U = \ln(c_t^1) + \rho \ln(c_{t+1}^2) + u(l_t)$ for some $\rho > 0$ and some increasing and concave function *u*. See King-Plosser-Rebelo (1988) for a discussion of balanced growth requirements. With balanced growth, the welfare weights in (9) converge to a constant ($\tilde{\omega}_t \to \omega = 1/\left(\lim_{t\to\infty} E_0[\frac{\partial U_t}{\partial c_t^1} / \frac{\partial U_{t-1}}{\partial c_t^2}]\right)$ as $t \to \infty$). One may therefore restrict attention to stationary problems and market allocations to comparable efficient allocations with constant $\tilde{\omega}_t = \omega$.

[13] Proofs for these properties are omitted because solutions to infinite horizon balanced growth problems are well known from the representative agent literature. The assumption that $\partial U_{t-1} / \partial c_t^2$ does not depend on leisure keeps lagged leisure out of the state vector.

by setting shocks to zero); and let $\hat{x}_t = \sum_{s_t \in S_t} \pi^*_{x,s} \cdot \hat{s}_t$ denote the log-linearized dynamics. The coefficients $\pi^*_{x,s}$ are elasticities that quantify the exposure of x_t to fluctuations in s_t; e.g., $\pi^*_{c1,a}$ is the efficient exposure of worker consumption c^1 to the permanent shock a.

Characterizations of efficient allocations are useful for studying market allocations, because a market allocation cannot be efficient unless it has a Markov structure with the same state variables, the same homogeneity properties, the same deterministic steady state, and the same log-linearization as its comparable efficient allocation. Market allocations with missing or additional state variables are automatically inefficient.

These efficiency requirements imply that fiscal policy must be inefficient unless transfers can be written as a policy function ${}^{b_t}\!/_{A_{t-1}} = b(S_t)$ with $S_t = (a_t, z_t, k_{t-1}, \chi_{t-1})$, where χ_{t-1} must be included if and only if U_t is not time-separable. This reveals the inefficiency of some plausible, perhaps even realistic policies. Notably:

- *Policies that respond to shocks with lags are always inefficient*, except in the sense that shocks are propagated through k_{t-1} and (in case of non-separable utility) χ_{t-1}.
- *Policies that introduce extraneous state variables are always inefficient.*[14]

To be clear about the practical interpretation, a discussion of policy functions implicitly assumes that $b(S_t)$ is observable, i.e., that fiscal institutions, laws, and operating procedures are stable enough for a researcher to ascertain how transfers typically respond to various shocks—enough to estimate or calibrate a stylized policy function. Policy choices in period t are about alternative functions $b(S_{t+1})$ that describe period-$t+1$ transfers (e.g., how period-t workers' retirement benefits depend on period-$t+1$ wages and inflation). In effect, policy choices are contingent

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[14] For example, though the model is non-monetary, one could introduce "money" as a government-defined unit of account with potentially stochastic real value. Efficiency then requires that either fiscal transfers are indexed to the purchasing power of money, which would make money irrelevant; or purchasing power must be a deterministic

plans that determine the risk-exposure of current workers relative to future generations. This paper presumes that such a planning perspective is instructive for thinking about fiscal policy, e.g., about the design of public pension systems, about public debt management, or about alternative systems of taxation.

For market allocations with the correct state vector, risk-sharing properties can be assessed quantitatively by comparing actual and efficient elasticity values. For any variable x_t in a market allocation with policy function $b(S_t)$ let

$$\hat{x}_t = \pi_{x,a} \cdot \hat{a}_t + \pi_{x,z} \cdot \hat{z}_t + \pi_{x,k} \cdot \hat{k}_{t-1} + \pi_{x,\chi} \cdot \hat{\chi}_{t-1} \tag{10}$$

denote the log-linearized dynamics.[15] Applied to $x_t = b_t / A_{t-1}$, and noting that efficiency requires a match of all elasticity values, one finds:

<u>Observation</u>: A market allocation is inefficient unless $\pi_{b,s} = \pi^*_{b,s} \; \forall s \in S_t$.

Thus efficiency imposes rather stringent restrictions on policy. I will call a policy $b(S_t)$ <u>approximately efficient</u> if $\pi_{b,s} = \pi^*_{b,s} \; \forall s \in S_t$.[16]

When policy is *inefficient*—as in most applications below—differences between $\pi_{x,s}$ and $\pi^*_{x,s}$ reveal the direction and first-order magnitude of inefficiencies. Because individuals care about consumption and leisure, I will focus on deviations of consumption and leisure from their efficient paths, i.e., on $\pi_{c1,s}$, $\pi_{c2,s}$, and $\pi_{l,s}$. Elasticities of consumption and leisure with respect

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function of the efficient state vector, which means that efficiency imposes tight restrictions on monetary policy.

[15] Throughout, $\pi_{x,s}$ refers to a generic allocation; $\pi^*_{x,s}$ with stars denotes efficient values. For non-stationary variables, let $\hat{x}_t$ refer to the stationary transformations. If z_t is a vector, let $\pi_{x,z}$ be interpreted as conforming vector. Approximate planning solutions are obtained from (1), (2), (4), and (8). Approximate market solutions are obtained from (1), (2), (4), and (5), noting that (6) is implied by (1) and (5). A caveat is that (5) cannot be log-linearized around zero transfers, except in the laissez-faire case (setting $b_t \equiv 0$).

[16] Higher-order approximations are not worth pursuing because most applications display first-order inefficiencies, which makes higher-order comparisons moot. Note that the linearizations are not subject to Kim and Kim's (1999) critique: because a market allocation with efficient transfers and the comparable planning solution would have identical linearizations, differences in elasticities cannot be attributed to approximation errors.

to shocks $\{\hat{a}_t, \hat{z}_t\}$ reveal to what extent workers and retirees are over- or under-exposed to current shocks. Elasticities with respect to state variables $\{\hat{k}_{t-1}, \hat{\chi}_{t-1}\}$ reveal to what extent workers and retirees are over- or under-exposed to shocks from previous periods that are propagated through the state variables.

Elasticities with respect to the permanent productivity shocks a_t deserve particular attention in this context because balanced growth requires linear homogeneity of consumption and transfers in $\{a_t, k_{t-1}, \chi_{t-1}\}$. This implies:

<u>Observation</u>: Economies with balanced growth that respond inefficiently to permanent productivity shocks necessarily have an inefficient propagation mechanism.[17]

Because *all* shocks are propagated through $\{k_{t-1}, \chi_{t-1}\}$, inefficient propagation means that *all* shocks are allocated inefficiently over time and across generations. This special role of a_t motivates, in part, my focus on productivity shocks in the applications.

3. Application: The Standard Cobb-Douglas/CRRA Model

This section assumes CRRA preferences and Cobb-Douglas production. Both are common assumptions in the OG and macro literatures. One objective is to document that risk sharing is inefficient in a particular direction for a wide range of parameters and policies.

3.1. Direct implications of CRRA preferences

For preferences, assume power utility over consumption

$$U_t = \tfrac{1}{1-1/\varepsilon}\left[(c_t^1)^{1-1/\varepsilon} + \rho(c_{t+1}^2)^{1-1/\varepsilon} - (1+\rho)\right], \tag{11}$$

with time preference $\rho > 0$ and elasticity of intertemporal substitution $\varepsilon > 0$ (EIS for short); the limit $\varepsilon \to 1$ captures log-utility. Because leisure is not valued, $l_t = 0$ is exogenous and $L_t = N_t$.

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[17] Technically, balanced growth implies $\pi_{x,k} + \pi_{x,\chi} = 1 - \pi_{x,a}$ and $\pi^*_{x,k} + \pi^*_{x,\chi} = 1 - \pi^*_{x,a}$ for $x \in \{c^1, c^2, b\}$. Hence $\pi^*_{x,a} \neq \pi^*_{x,a}$ implies $\pi^*_{x,k} \neq \pi^*_{x,k}$ or $\pi^*_{x,\chi} \neq \pi^*_{x,\chi}$, or both.

The role of CRRA is best understood by starting from general time-separable preferences of the form $U_t = u(c_t^1) + \rho \cdot u(c_{t+1}^2)$ and noting the efficiency restrictions they impose on the log-linearized allocation. From (8), one obtains:

$$(-\tfrac{u_{cc}(c^2)c^2}{u_c(c^2)}) \cdot \hat{c}_t^{2*} = (-\tfrac{u_{cc}(c^1)c^1}{u_c(c^1,l)}) \cdot \hat{c}_t^{1*} \qquad (12)$$

where $(-u_{cc}c/u_c)$ can be interpreted as relative risk aversion. Whenever workers and retirees have the same relative risk aversion ($=1/\varepsilon$ in case of power utility), (12) reduces to $\hat{c}_t^{1*} = \hat{c}_t^{2*}$; or in terms of elasticities, to

$$\pi_{c1,s}^* = \pi_{c2,s}^* \; \forall s \in S_t \, . \qquad (13)$$

That is: *Efficiency requires equal responses of worker and retiree consumption to all shocks, i.e., a perfect pooling of all consumption risks across generations.*

For market allocations, any violation of (13) implies inefficiency. Because individuals care about consumption, the difference $\pi_{c1,s} - \pi_{c2,s}$ provides a natural measure of inefficiency (for each *s*); and conveniently, it does not require computing the efficient allocations.

3.2. Direct implications of Cobb-Douglas production with fixed depreciation

Let production be $F(K_t, N_t, A_t, z_t) = K_t^{\alpha}(N_t A_t z_t)^{1-\alpha}$, where $\alpha \in$ (0,1) is the capital share in output and where z_t is now a temporary i.i.d. productivity shock.[18] Assume constant depreciation. Then the marginal products of labor and capital can be written as

$$R_t = \alpha(k_{t-1}/\gamma_N)^{\alpha-1}(a_t z_t)^{1-\alpha} + (1-\delta), \qquad (14)$$

$$w_t = (1-\alpha)A_{t-1}(k_{t-1}/\gamma_N)^{\alpha} \cdot (a_t z_t)^{1-\alpha} \qquad (15)$$

Risks in period-*t* are generated by the permanent (a_t) and temporary (z_t) productivity shocks, which enter symmetrically into both factor returns. Log-linearization yields the elasticities

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[18] Compared to the general setting (1)-(2), the vector z_t is reduced to a scalar, and $z_t^F = z_t$.

$$\pi_{w,s} = 1-\alpha \text{ and } \pi_{R^k,s} = (1-\alpha)(1-v) \qquad \text{for } s \in \{a,z\}, \tag{17}$$

where $v \equiv (1-\delta)/R \geq 0$ is the steady state value of old capital as share of the return R.

Quantitatively, much of the capital stock depreciates within a generation. Some components of aggregate capital are long-lived, however, such as structures and land. Raw land alone constitutes about 27% of U.S. wealth (Federal Reserve Board, 1994), suggesting $v = 0.27$ as lower bound for quantitative analysis (conservative, so not to overstate risk differences).

For all $v > 0$, (17) implies $\pi_{R,s} / \pi_{w,s} = 1 - v < 1$: Wages are more exposed to productivity shocks than the return on capital. This follows necessarily from Cobb-Douglas production and fixed depreciation, and it turns out to hold under more general conditions (see Section 4.3).[19]

3.3. The Equilibrium Allocation of Risk

A comparison of consumption risks—the central issue for efficiency—requires a general equilibrium analysis of how factor income risks translate into consumption. The answers depend in part on policy and in part on workers' savings behavior.

Policy is conveniently parameterized by the steady state level of transfers as share of output, σ_b, and by policy responses to the shocks $(\pi_{b,a}, \pi_{b,z})$. For retiree consumption, the budget equation $c_t^2 = R_t k_{t-1}^1 + b_t$ yields the log-linearization

$$\pi_{c2,s} = (1 - \tfrac{\sigma_b}{\sigma_{c2}})\pi_{R,s} + \tfrac{\sigma_b}{\sigma_{c2}}\pi_{b,s} \qquad \text{for } s \in \{a,z\}, \tag{18}$$

where for any variable *x*, σ_x denote the steady state share of output. For workers, it is instructive to express consumption $c_t^1 = (1-\kappa_t)\cdot y_t^1$ as function of disposable income

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[19] One seemingly counterfactual property should be noted and explained: Because this section abstracts from other shocks, log-returns have smaller variance than log-wages. This is could be rectified easily *without changing (17)* by adding a shock to the value of old capital, e.g. by assuming $K_{t+1} = I_t + (1-\delta+z_t^c)\cdot K_t$. Also, though a full empirical analysis is beyond the scope of this paper, impulse-response functions computed from long run U.S. GDP and stock market data produce point estimates for $\pi_{R,s} / \pi_{w,s}$ substantially less than one, ranging from 0.29 to 0.73 depending on the specification. (See the Appendix, Part D, for documentation.) A model with $\pi_{R,s} / \pi_{w,s} < 1$ for productivity

$y_t^1 \equiv w_t - b_t / \gamma_N = c_t^1 + k_t^1$ and the savings rate $\kappa_t = k_t^1 / y_t^1$. This yields

$$\pi_{y1,s} = (1 + \tfrac{\sigma_b}{\sigma_w - \sigma_b})\pi_{w,s} - \tfrac{\sigma_b}{\sigma_w - \sigma_b}\pi_{b,s} \qquad \text{and} \tag{19}$$

and $$\pi_{c1,s} = \pi_{y1,s} - \tfrac{\sigma_{k1}}{\sigma_w}\pi_{\kappa,s}, \qquad \text{for } s \in \{a, z\}. \tag{20}$$

Inspecting these equations, one finds that policy determines how factor income risks relate to retiree consumption in (18) and to workers' disposable income in (19), whereas savings behavior determines how workers' disposable income relates to their consumption, in (20).

From (18), the exposure of retiree consumption to productivity shocks is a weighted average of the factor income risk and the responsiveness of transfers. If transfers are safe or nearly safe ($\pi_{b,s}$ is small) and typically positive ($\sigma_b > 0$), then $\pi_{c2,s} < \pi_{R,s}$: Safe transfers reduce the impact of productivity shocks on retiree consumption. For workers, if transfers are relatively safe (meaning $\pi_{b,s} < \pi_{w,s}$) and $\sigma_b > 0$, then (19) implies $\pi_{y1,s} > \pi_{w,s}$: Safe transfers magnify the impact of productivity shocks on workers' disposable income.

Overall, safe transfers reinforce the inequality of factor income risks—they reduce the exposure of retiree consumption below $\pi_{R,s}$ while raising the exposure of worker disposable income above $\pi_{w,s}$.[20] These policy implications apply to both permanent and temporary shocks.

Turning to savings—the final step in determining workers' consumption risk—the analysis is cumbersome because permanent and temporary shocks trigger qualitatively different savings responses and because income and substitution effects tend to conflict. (In technical terms, $\pi_{\kappa,s}$ depends on multiple parameters.) To streamline the exposition, I provide intuition for empirically relevant cases, and then present results in two propositions and a figure.

While both productivity shocks increase workers' current and future (retirement) income,

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shocks is therefore consistent with empirical evidence.

[20] To be precise, safety in the sense of reducing risks in both (17) and (18) requires $0 \le \pi_{b,s} < \min(\pi_{w,s}, \pi_{R,s})$ for

a temporary shock tends to raise *current* income more than future income, whereas a permanent shock tends to raise *future* income more than current income. [21] Also, temporary shocks *reduce* the return on capital, whereas permanent shocks *increase* the return on capital. Thus income and substitution effects are conflicting. If the elasticity of intertemporal substitution (ε) is low enough for the income effects to dominate, a positive temporary shock increases the savings rate ($\pi_{\kappa,a} > 0$) whereas a positive permanent shocks reduces the savings rate ($\pi_{\kappa,z} < 0$). The savings-rate responses are reversed if ε is high enough for substitution effects to dominate.

Empirical evidence on intertemporal substitution favors an elasticity of substitution less than one. Ogaki and Reinhart (1998) suggest $\varepsilon \approx 0.4$. Hall (1988) suggests ε near zero. In the finance literature, risk aversion parameters in the 2-4 range are common, which implies an EIS in the 0.25-0.5 range. Given this evidence—and a desire to avoid too many cases—I focus on $\varepsilon \leq 1$. This turns out to be sufficient for income effects to dominate.

Whenever income effects dominate, the impact of a temporary shock on workers' consumption is dampened by a rising savings rate: in (19), $\pi_{\kappa,z} > 0$ implies $\pi_{c1,z} < \pi_{y1,z}$. In economic terms, consumption smoothing over a two-period horizon allows workers to bear more income risk than retirees. For permanent shocks, in contrast, the impact of higher productivity is magnified by fall in savings: $\pi_{\kappa,a} < 0$ implies $\pi_{c1,a} > \pi_{y1,a}$. A longer horizon does not help workers bear permanent risks; indeed, the anticipation of higher future income magnifies the effect of permanent shocks on current consumption.

For permanent shocks, the inequalities above combine to an unambiguous conclusion: <u>Retiree consumption is less exposed to permanent shocks than workers' consumption</u>. To see why, recall that: (i) for Cobb-Douglas production, $\pi_{w,a} = 1 - \alpha \geq \pi_{R,a}$; (ii) for relatively safe

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$s \in \{a, z\}$. Most arguments below will only require $0 \leq \pi_{b,s} \leq \pi_{w,s}$, a weaker notion of safety relative to wages.

transfers, $\pi_{y1,a} \geq \pi_{w,a}$ and $\pi_{w,a} \geq \pi_{c2,a}$; (iii) for $\varepsilon \leq 1$, the savings responses imply $\pi_{c1,a} \geq \pi_{y1,a}$. In combination:

$$\pi_{c1,a} \geq \pi_{y1,a} \geq \pi_{w,a} \geq \pi_{c2,a}\,. \tag{21}$$

The arguments for $v > 0$ and $\varepsilon < 1$ imply that at least two of the inequalities are strict, so $\pi_{c1,a} > \pi_{c2,a}$. From the efficiency condition (13), this documents a first-order inefficiency.

To obtain *equal* risk exposures, $\pi_{c1,a} = \pi_{c2,a}$, one would need equality at all three steps, and this would require $\varepsilon = 1$, and $v = 0$, and either $\sigma_b = 0$ or $\pi_{b,a} = \pi_{w,a}$. The setting $(\varepsilon, v) = (1,0)$ describes log-utility with Cobb-Douglas production and 100% depreciation, a popular set of assumptions in the OG literature. One can show (exploiting a constant savings rate that yields closed form solutions) that $(\varepsilon, v) = (1,0)$ with laissez-faire is indeed ex-ante efficient—exactly efficient, not just approximately. But efficiency fails for all $(\varepsilon, v) \neq (1,0)$, which means that $(\varepsilon, v) = (1,0)$ a very special case.[22]

For temporary productivity shocks, steps (i) and (ii) above apply as well, so $\pi_{y1,z} \geq \pi_{w,z} \geq \pi_{c2,z}$, but (iii) is reversed due to consumption smoothing, so $\pi_{c1,z} \leq \pi_{y1,z}$. The reversal is most relevant if ε and v are near zero and if transfers are small or not-too-safe. One can show, however, that if v exceeds a certain cutoff value, which is

$$v_0 = \tfrac{(\alpha+\sigma_b)}{1-\alpha-\sigma_b}\left[\sqrt{\vartheta^2 + \tfrac{(1-\alpha)}{r\cdot\alpha}} - \vartheta\right], \text{ where } r = \tfrac{R}{\gamma_A \gamma_N} \text{ and } \vartheta = \tfrac{1}{2}\left(1 + \tfrac{(\alpha+\sigma_b)(1-\alpha)}{r\cdot(1-\alpha-\sigma_b)\alpha}\right) \tag{22}$$

then consumption smoothing is never sufficient to overturn the inequalities in (i) and (ii). Then workers are more exposed to both productivity shocks than retirees.

Because v_0 depends on multiple parameters, a specific calibration is useful. A real return

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[21] Sufficient conditions are v>0 and $0 \leq \pi_{b,s} \leq \pi_{w,s}$. The intuition is that capital adjusts gradually.

[22] For (ε,v)=(1,0), wage-indexed transfers would suffice to maintain efficiency (or rather, not upset the efficiency of laissez-faire), but such transfers are inefficient for all other (ε,v). Hence policy results derived with log-utility/full depreciation assumptions provide little guidance (and may be misleading) about optimal policy in general.

on capital of 6% and population-plus-productivity growth of 2% per year over a 30-year generational period suggest $r = (\frac{1.06}{1.02})^{30} \approx 3.17$. Combined with $\alpha = 1/3$ and $\sigma_b \approx 10\%$, one obtains $v_0 \approx 0.26$. This is less than the 27% share of raw land in U.S. wealth, suggesting $v > v_0$ is the empirically relevant case. For reference below, define the

Benchmark Parameters: $(\varepsilon, v) = (0.4, 0.27)$, $\alpha = 1/3$, $r = \frac{R}{\gamma_A \gamma_N} = 3.17$.[23]

Note that the analysis has sidestepped direct comparisons between market and efficient allocations. Direct comparisons turn out to be algebraically messy because shocks are propagated inefficiently and hence risks are spread inefficiently over many generations. One can show that whenever $\pi_{c1,a} < \pi_{c2,a}$, retirees bear less productivity risk than in the efficient allocation ($\pi_{c2,a} < \pi^*_{c2,a}$), and the response of capital investment is too strong ($\pi_{k,a} > \pi^*_{k,a}$). Hence future generations (some or all) bear too much productivity risk.

To summarize the results (with formal proof in the Appendix, Part B), we have:

Proposition 1: Consider OG economies with Cobb-Douglas production and power utility, and consider either laissez-faire or transfers with $0 \le \pi_{b,s} \le 1 - \alpha$ *for* $s \in \{a, z\}$. *Then:*

(a) $\pi_{c2,a} < \pi_{c1,a}$ *for all* $(\varepsilon, v) \in [0,1] \times [0,1]$ *except* $(\varepsilon, v) = (1, 0)$, *so permanent productivity shocks impact retiree consumption less than workers' consumption.*

(b) $\pi_{c2,z} < \pi_{c1,z}$ *for all* $v > v_0$, *so temporary productivity shocks impact retiree consumption less than workers' consumption.*

(c) Economies with $\pi_{c2,a} < \pi_{c1,a}$ *also satisfy* $\pi_{c2,a} < \pi^*_{c2,a}$ *and* $\pi_{k,a} > \pi^*_{k,a}$.

Figure 1 illustrates how productivity risks are allocated in economics with different (ε, v)-combinations, using a log-scale for ε to cover extreme values. Lines "Equal Temp.", which runs from $(0, v_0)$ to (1,0), delineates (ε, v)-combinations that give workers and retirees equal exposure to temporary shocks. The lines “Equal Perm.” delineate (ε,v)-combinations with equal exposure to permanent shocks. The (main) thick lines are for $r = 3.17$, the benchmark value. Dashed lines drawn are for $r = (^{1.05}\!/_{1.03})^{30} \approx 1.8$ to illustrate how the lines and areas vary with the return parameter (all for $\alpha = 1/3$ and $\sigma_b = 0$).

In Area 1 workers are more exposed to both productivity shocks. This covers most of the parameter space in Figure 1, including the Benchmark Parameters and the empirically relevant subset $\{(\varepsilon,v) : \varepsilon \leq 1, v > v_0\}$. In Area 2 (lower left corner) retirees are more exposed to temporary shocks. In Area 3 (upper and lower right corners) retirees are more exposed to permanent shocks.[24] There is no area where retirees are more exposed to both shocks. There is only one point where both generations are equally exposed to both shocks, namely the log-utility/100%-depreciation case at (ε,v)=(1,0). Overall, Figure 1 suggests that the assumptions of Prop.1 are far from necessary,[25] and that the direction of inefficiency emphasized in Prop.1—retirees bearing less productivity risk than workers—is a fairly general finding.

3.4. Implications for Fiscal Policy

Results about efficient policies follow directly from Prop.1:

<u>*Proposition 2*</u>*: Consider OG economies with Cobb-Douglas production and power utility:*

(a) For any $(\varepsilon,v) \in [0,1]\times[0,1]$ *except* $(\varepsilon,v) = (1,0)$, *the approximately efficient policy is strictly more responsive to permanent productivity shocks than the wage,* $\pi^*_{b,a} > \pi_{w,a} = 1-\alpha$.

(b) For any $v > v_0$, *the approximately efficient policy is strictly more responsive to temporary*

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[23] The elasticity $\varepsilon = 0.4$ is Ogaki-Reinhart’s (1998) preferred value. The other parameters were discussed above.

[24] For relatively low r, the Equal Perm. lines of equal exposure to permanent shocks “connect” at high ε-values and indicate that for extremely high ε, retirees are more exposed to permanent shocks. The required values are quite high, however, e.g., ε>22 for r=1.8 and v=0.27.

[25] Notably, Figure 1 indicates that for all ε>1, $\pi_{c2,z} < \pi_{c1,z}$ holds unconditionally and $\pi_{c2,a} < \pi_{c1,a}$ holds for a range of *v* and *r* values. Figure 1 is based on an algebraic linearization that expresses all relevant elasticities as functions of model the parameters $(\varepsilon,\rho,\alpha,\delta,\gamma_A,\gamma_N)$ and policy parameters $(\sigma_b,\pi_{b,a},\pi_{b,z})$.

productivity shocks than the wage, $\pi^*_{b,z} > \pi_{w,z} = 1-\alpha$.

Recall that b_t represents the retirees' generational account. In practice, the main components of retirement-age generational accounts are public pensions, public debt, and capital income taxes (see Auerbach et.al 1999). In the U.S., social security is partially wage-indexed (up to age 60) and amounts to about 10% of GDP (incl. Medicare). Public debt amounts to about 3% of a generation's income and is essentially safe, even accounting for nominal bonds and inflation. U.S. capital income taxes can be approximated (conservatively) by a 25% marginal rate and yield about 3% of a generation's income. While these taxes are risk-sensitive, they enter *negatively* into the generational account and thus reduce retirees' exposure to productivity risk. Assuming a transfers/output share of $\sigma_b \approx 10\%$ (=10% pensions + 3% debt - 3% capital income taxes) and treating social security as 50% wage-indexed, one obtains an elasticity of transfers to productivity shocks of $\pi_{b,s} \approx 0.11$. This value is much smaller than the elasticity of returns, $\pi_{R,s} \approx 0.49$, and the elasticity of wages, $\pi_{w,s} = 0.67$.

For comparison, consider the efficient policy with $\sigma_b \approx 10\%$, the same level of transfers as in the observed policy. Assume the Benchmark Parameters apply. Then efficient transfers have elasticity coefficients $\pi^*_{b,a} \approx 1.28$ and $\pi^*_{b,z} \approx 0.78$, about an order of magnitude higher than the crudely calibrated value of 0.11. (Given the gross discrepancy, a more detailed calibration seems unnecessary.) Efficient transfers would implement equal consumption responses for workers and retirees, which are $\pi^*_{c1,a} = \pi^*_{c2,a} \approx 0.63$ for permanent shocks. For the calibrated U.S. policy, in contrast, one obtains $\pi_{c1,a} \approx 0.78$ and $\pi_{c2,a} \approx 0.42$, which means that workers bear too much risk whereas retirees bear too little risk.

Generational accounts in other countries have the same main components. Though public pensions are wage-indexed in some countries, indexing is typically less than one-for-one. In most developed countries, public debt is essentially safe. Capital income taxes are also common and (entering negatively) they reduce retiree exposure to productivity risk. This suggests that the safety of intergenerational transfers and the resulting inefficiencies are not specific to the U.S.

The widespread use of OG models, Cobb-Douglas production, and CRRA preferences throughout economics suggest that this type of model is considered a plausible representation of real-world economies. Prop.1-2 suggest that researchers who use such models for policy analysis are likely to conclude that retirees don't bear enough productivity risk.

From a positive-theory perspective, the ubiquity of public institutions that promise safety to retirees is puzzling. Politicians should find Pareto efficient policies attractive even if they (or their voters) don't care much about future generations, because more efficient transfers allow current voters to grant themselves more valuable benefits without increasing the burden on future generations (which might lead them to revolt). Policies with much higher responsiveness than 0.11 are also practically feasible, e.g., $\pi_{b,s} = 1-\alpha \approx 0.67$ with fully wage-indexed pensions. Hence lack of feasibility is not a plausible explanation for the observed policies.

The evident political popularity of safe transfers suggests a different interpretation: Something may be missing in the standard OG model. The next section will probe the generality of the above results and examine if alternative model assumptions might help understand the observed policies.

4. Extensions: How robust are the policy conclusions?

This section studies several model extensions to examine if they might rationalize safe transfers.

4.1. Habit Formation

Habit formation makes retirees with established habits naturally more risk-averse than workers. Specifically, let preferences be

$$U_t = \tfrac{1}{1-1/\tilde{\varepsilon}}\left[(c_t^1)^{1-1/\tilde{\varepsilon}} + \rho(c_{t+1}^2 - \tilde{h}c_t^1)^{1-1/\tilde{\varepsilon}} - (1+\rho)\right], \tag{23}$$

where $\tilde{h} \geq 0$ is a habit parameter and $\tilde{\varepsilon} > 0$. Let $\bar{h} = \tilde{h}\frac{\sigma_{c1}}{\sigma_{c2}\gamma_A\gamma_N}$ denote the steady state ratio of habit stock to retirement consumption. One can show that market allocations with habit parameters $(\tilde{\varepsilon}, \tilde{h})$ have the same log-linearized allocations as economies with CRRA preferences and an elasticity $\varepsilon = \varepsilon(\tilde{\varepsilon}, \tilde{h})$, where $\varepsilon < \tilde{\varepsilon}$ for all $\tilde{h} > 0$. Holding ε constant, habit formation does not affect the log-linearized market allocation. It does, however, change the efficient allocations and hence the efficiency benchmark to which a given market allocation is compared. Specifically, one can show (see the Appendix, Part B, for proof):

Proposition 3: Efficient allocations with habit formation satisfy $\pi^*_{c2,a} \leq (1-\bar{h}) \cdot \pi^*_{c1,a}$.

Prop.3 shows that habit formation reduces the efficient exposure of retirees' to productivity shocks by at least the factor $1-\bar{h}$ relative to workers exposure. While the exact ratio of exposures a complicated function of model parameters, the bound $1-\bar{h}$ suggest that habits have a substantial effect on efficient allocations. For the Benchmark Parameters and $\sigma_b = 10\%$, one finds that the calibrated U.S. policy coefficient $\pi_{b,a} = 0.11$ can be rationalized as efficient if $\bar{h} \approx 0.455$.

This result should be interpreted cautiously, however, because habits have significant ramifications for other aspects of efficient policy. Because utility is non-separable, the efficient Markov state vector must include lagged consumption $\chi_{t-1} = c^1_{t-1} / A_{t-1}$; and because χ_{t-1} is not a state variable in the laissez-faire allocation, efficient responses to χ_{t-1} must be imposed via intergenerational transfers that are highly sensitive to lagged consumption; e.g., $\pi^*_{b,\chi} \approx 1.66$ for $\bar{h} = 0.455$. Retirees who had high (or low) working age consumption would be entitled to sharply higher (or lower) transfers—a seemingly inequitable policy, but efficient ex ante. Moreover, the inequality $\pi^*_{c2,a} < \pi^*_{c1,a}$ is consistent not only with habits, but also with other preferences that make retirees more risk averse than workers.[26] The main conclusion is therefore that age-increasing risk aversion—here exemplified by habits—can rationalize safe transfers.

4.2. Labor-Leisure Choices

A variable labor supply gives workers additional flexibility in responding to shocks and might enable them to bear more risk than retirees. Could preferences over leisure overturn the findings of Section 3? The answer turns out to be no, provided one assumes balanced growth, age-independent relative risk-aversion, and an elasticity of intertemporal substitution less or equal one. The main intuition is that productivity shocks make work effort less productive in exactly those states of nature when income is low and more work effort would be required to stabilize income. This discourages work effort in response to low productivity. One can show that for $\varepsilon<1$, efficient risk sharing actually calls for reduced work effort in response to a negative productivity shock and it imposes more productivity risk on retirees than in the fixed-labor model of Section 3. (See the Appendix, Part E for more details.) Thus adding labor-leisure choices shifts the efficiency standard in the opposite direction of what one would need to rationalize safe transfers.

4.3. General production and capital accumulation

This section examines how the relative exposure of returns and wages to productivity shocks depends on assumptions about technology.

First, suppose production is a general function F, as in (1). The log-linearized responses of wages and returns to productivity shocks are

$$\pi_{w,s} = 1-\alpha/\varepsilon_{KL} \text{ and } \pi_{R,s} = (1-\alpha)(1-v)/\varepsilon_{KL} \qquad \text{for } s \in \{a,z\}, \tag{24}$$

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[26] For example, some forms of Epstein-Zin non-expected utility implies age-increasing risk aversion and they could rationalize relatively safe transfers, though with different implications for the propagation of shocks. This section use habits to model age-increasing risk aversion because of other research pointing towards habits (e.g. several papers in the *AER Papers&Proceedings 2007*). The risk-aversion of the young is unfortunately unobservable because preferences over start-of-life risks could only be revealed by portfolio choices made before birth.

where $\varepsilon_{KL} > 0$ the elasticity of factor substitution and α is now the steady state capital share. For $\varepsilon_{KL} < 1$, capital income is relatively more exposed to productivity shocks than for Cobb-Douglas, and for sufficiently low ε_{KL}-values, returns respond more to productivity than wages. However, a reversal of the key inequality $\pi_{c2,a} < \pi_{c1,a}$ would require quite low elasticity values—values that are difficult to reconcile with the empirical stability of capital and labor shares. For the Benchmark Parameters, $\pi_{c2,a} < \pi_{c1,a}$ holds unless $\varepsilon_{KL} < 0.78$.

Second, suppose $K_{t+1} = G(I_t, K_t, z_t^G)$, as in (2), with concave G and with $Q_t = [\partial G / \partial I (I_t, K_t, z_t^G)]^{-1}$ strictly increasing in I_t. Variations in Q warrant attention because they systematically increase the response of capital returns to permanent productivity: A permanent productivity shock tends to increase investment; the resulting increase in Q raises the value of old capital; hence the elasticity $\pi_{R,a}$ is greater than in fixed-Q models (where a enters only through dF / dK). This "valuation channel" is quantitatively limited, however, because concavity in G also acts as adjustment cost that discourages variations in investment. Hence model parameterizations that make Q highly sensitive to investment tend to have a near-zero investment response to a. (Because an algebraic exposition would be lengthy, details are in the Appendix, Part E.) One can show that the inequality $\pi_{c2,a} < \pi_{c1,a}$ remains valid provided the elasticity of substitution between I_t and K_t in G is above a lower bound.

Overall, inefficiency results of Section 3 appears to be robust with respect to reasonably parameterized general specifications for production, capital accumulation, and labor supply. By elimination, age-increasing risk aversion remains as the most plausible positive explanation for observed fiscal policies.

5. Conclusions

The paper has three main conclusions. First, intergenerational risk sharing can be examined without imposing distributional judgments. For any specification of preferences, technology, and a given fiscal policy, there is at most one *comparable* ex ante efficient allocation with the same implicit welfare weights on the various generations. To be efficient, fiscal policy must respond to economic fluctuations in the same way as the comparable efficient allocation.

Secondly, standard models with power utility make commonly observed fiscal policies appear grossly inefficient. Cobb Douglas production implies that returns to capital are less responsive to productivity shocks than wages. Even accounting for consumption-smoothing and other complications, retirees are less-than-efficiently exposed to productivity risk, workers bear systematically more productivity risk than retirees, and too much risk is shifted into the future. This is shown in a basic model—Cobb-Douglas production and fixed labor supply—and turns out generalize to models with labor-leisure choices, a Tobin's-Q setting with stochastic value of capital, and a more general production function.

Given the direction of inefficiency in the market allocation, efficient fiscal policies should shift risk from workers to retirees. It is therefore puzzling that fiscal institutions around the world seem designed to do the opposite by providing relatively safe transfers to retirees. Because standard modeling assumptions imply that retiree transfers are too safe, one must suspect that economists who use such models will tend to find results supportive of policy reforms that impose more risk on retirees.[27]

The third finding is that relatively safe transfers to retirees can be rationalized as efficient if risk aversion increases with age. This is illustrated by a habit formation model. Because nothing else seems to explain observed policies, one may conclude that policy makers around the world seem to treat future generations of workers as if they are more risk tolerant than retirees.

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[27] Many OG models used for policy analysis, e.g., in the social security reform debate, are more elaborate than my two-period model. But larger models are often built around similar preference and technology assumptions, which appear innocuous but are shown in the two-period model to have "predictable" policy implications.

Whether this is right or wrong is an open question. For this paper, a robust conclusion is that preference assumptions seem crucial for evaluating the efficiency of intergenerational risk sharing and for deriving policy recommendations from OG models.

An important question for future research is how the two-period OG results generalize to multi-period models. With many periods, workers near retirement may have a mixture of labor and asset income and they may condition work effort, retirement, and human capital investments on prior earnings and returns. It seems plausible that the effects of temporary economic disturbances could be attenuated by time averaging and by private risk sharing with adjacent cohorts (who would overlap for multiple periods). However, for shocks that are permanent or long-lasting relative to the life cycle (e.g., industrial revolutions or other booms, major crashes, or wars), time averaging and risk sharing with nearby cohorts are unlikely to help. One may suspect therefore that the two-period model is indicative of mechanisms that are also buried—perhaps less transparently—within larger OG models.

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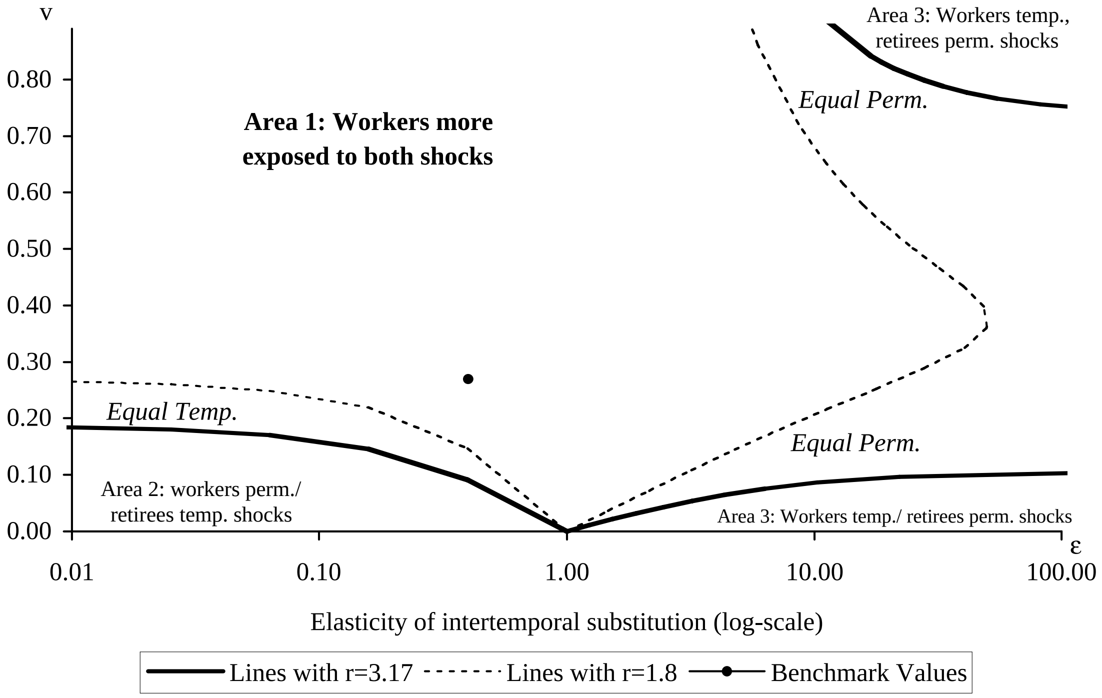
Figure 1: A mapping from parameters to risk-sharing results: Which generation is more exposed to permanent (a) and temporary (z) productivity shocks?



Notes: The lines Equal Perm. and Equal Temp. show combinations of elasticity (ε) and ratio of old capital to returns (*v*) for which both generations are equally exposed to permanent shocks (a) and temporary shocks (z), respectively. Thick lines are for the benchmark return value r=3.17; adjacent dashed lines are for r=1.8 to illustrate how the lines shift with r. Benchmark Values are the point (ε=0.40, *v*=0.27). Areas 1-3 are labeled to indicate which generation is strictly more exposed to permanent (perm.) or temporary (temp.) shocks. Workers are more exposed to temporary shocks everywhere above and to the right of Equal Temp. and more exposed to permanent shocks everywhere to the left and in between the Equal Perm. lines. Retirees are never more exposed to both shocks. Only at (ε=1, v=0) workers and retirees are equally exposed to both shocks. The figure is based on an analytical log-linearization of the CRRA/Cobb-Douglas model of Section 3.