Risk Sharing in a Stochastic Overlapping Generations Economy

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Abstract

This paper examines the impact of government policy on the allocation of aggregate risks in a stochastic OG model with production. The market allocation is generally ex ante inefficient in two ways. The impact of current shocks is neither efficiently shared by the living cohorts nor efficiently shared with future generations. An efficient allocation could be implemented (approximately) through standard policy instruments such as debt and social security. In practice, governments seem to shift risk in the "wrong" direction, however, notably through the issue of safe debt. A social security privatization that replaced a social security system by government debt would likely be efficiency reducing.

1. Introduction

In economies with finitely lived agents, the government has an important role as an institution that can act on behalf of unborn generations. The redistributional effects of government debt and pay-as-you-go social security are well known. In stochastic economies, government policy also affects the allocation of risk; see Enders and Lapan (1982), Smith (1982), Fischer (1983), Stiglitz (1983), Gordon and Varian (1988), and Gale (1990). What are the risk sharing effects of U.S. fiscal policy? What would be the characteristics of an efficiency policy?

In an OG economy, the market allocation of risk is generally ex ante inefficient due to the inability of the unborn to insurance themselves.[1] Government intervention is potentially Pareto-improving. This paper suggests, however, that fiscal policy is less far-sighted in practice: By supplying debt securities and other safe claims to the old, the government seems to protect current generations (voters) by shifting too much risk to future generations.[2]

I examine the allocation of aggregate risks in a Diamond (1965) type overlapping-generations economy with production. The main objectives are to determine how the equilibrium allocation of risk depends on government policy and to compare alternative market allocations to the benchmark of a Pareto-efficient allocation. Production is important in this context, because it places government interventions in an environment in which the labor income of the young and the capital income of the old are naturally

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[1] I focus on ex-ante rather than interim efficiency (Peled, 1982; Wright, 1987) because interim efficiency imposes virtually no constraints on policies that shift risk from current to future generations (see below).

[2] The risks at stake are huge. Just a percent per year higher growth over a generation would make the next generation much better off and substantially reduce the debt-GDP ratio. Risk-shifting also plays a key role in recent social security reform proposals (see Bohn, 1997; Advisory Council, 1997; and below).

correlated and because it allows risk sharing with the unborn through variations in capital investment.

Efficiency requires that the impact of all economic shocks is shared by the young, the old, and the unborn. The old and the young should bear consumption-risk in proportion to their risk tolerances. The risk sharing with the unborn depends on both preferences (willingness to bear risk) and technology (ability to shift risk over time). The market allocation of risk is generally not efficient, except for the special case of log-utility combined with Cobb-Douglas production, 100% depreciation and permanent productivity shocks. For substitution elasticities below one (the empirically relevant range; see Hall 1988), the efficient allocation tends to impose too much risk on the young generation.

Regarding fiscal policy, it is fairly obvious that any efficient allocation could be implemented through state-contingent taxes. This is worth noting but too obvious to be interesting. More challenging are questions about actual policy: Given a simple (realistic) tax system, how do standard tools of fiscal policy affect the allocation of risk? I find that policy tools with similar redistributional properties--debt and social security--have very different risk-shifting effects. If the government operates a wage-indexed social security system, all cohorts share the risk of uncertain future productivity growth. If the government issues safe debt, it provides safety to the old but increases the volatility of after-tax incomes for future generations. Future generations will have to pay a non-contingent debt service out of a stochastic income, implying a relatively high (low) tax rate whenever pre-tax incomes are unexpectedly low (high).

In practice, governments tend to issue substantial amounts of safe debt and only partially wage-index their social security systems. These policies seem to shift risk in the wrong direction, because the resulting supply of safe assets to the old shifts productivity risk from old to young, who already bear too much risk. If the government engages in redistribution, it should do so through risk-sensitive tools such as wage-indexed social security (or nominal debt with productivity-contingent inflation) rather than through safe debt.[3]

The distinction between state-contingent and safe policy tools is also relevant for social security reform. Proposals to replace social security by government bonds or to substitute "individual accounts" for a trust fund holding government debt (e.g., Feldstein, 1996; Advisory Council, 1997) would shift even more productivity risk to future generations.

Methodologically, the paper differs somewhat from the literature. Following business cycle and finance literature, I use log-linearizations to examine assumptions about preferences, technology, and policy that are more general than those that would yield closed form solutions. I do not use calibration, however, but instead derive analytical solutions to the model's log-linearized optimal decision rules. The analytical solutions reveal how preference, technology, and policy parameters interact to generate the model's risk characteristics and aggregate dynamics.[4]

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[3] An extensions section explores complicating factors that might alter the results, but I find that none is sufficient to rationalize safe debt.

[4] The business cycle literature often uses calibration or simulation methods to examine such issues. (See Bohn (1997) or Rios-Rull (1996) for calibrated OG models.) I argue that analytical solutions are more desirable here, because the objective is to derive general conditions under which certain policies are efficiency-increasing. In any case, the analytical formulas here provide an explicit, general mapping from structural parameters to reduced form coefficients (or to population moments) so that readers interested in calibration may insert their own preferred values.

A number of simplifying assumptions are made to avoid distracting technical complications. The assumption of two period lived agents eliminates private risk sharing.[5] For most of the paper, I focus on i.i.d. productivity risk, Cobb-Douglas production, and an inelastic labor supply. A final section explains how the model can be generalized to cover labor-leisure choices, CES-production, and other macro shocks. Survival uncertainty, idiosyncratic risks, demographic uncertainty, bequests, and distortionary taxes are left for future research.[6] Dynamic efficiency is assumed throughout, to rule out bubbles and related issues that would distract from risk sharing.

The paper is organized as follows. Section 2 describes the general risk sharing problem and the efficiency benchmark. Sections 3 set up a more specific model that allows parametric comparisons between alternative allocations. Section 4 explains why the market allocation tends to impose too much risk on the young. Section 5 examines simple policy interventions and shows that policies observed in practice appear to be efficiency-reducing. Section 6 provides extensions. Section 7 concludes.

2. The General Risk Sharing Problem

This section explains the general framework and the efficiency benchmark.

Throughout the paper, I consider an OG model with two-period lived agents. Generation t consists of N_t individuals who work in period t and are retired in period $t+1$. They have preferences $U_t(c^1_t, c^2_{t+1})$ over consumption c^1_t in period t (when young) and c^2_{t+1} in period $t+1$ (when old). Goods are produced with capital K_t and labor. To simplify, I assume that

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[5] With more than two periods, there would be private risk sharing between "middle-aged" and old agents, but still no risk sharing with the unborn, which is one of the key issues.

[6] Idiosyncratic risks could be shared within a cohort. Tax-distortions are potentially important because the government will have to vary tax rates to execute risk sharing contracts on behalf of the unborn. Ricardian bequests would make the model uninteresting.

each young agent supplies one unit of labor, so that N_t is also the labor supply. Aggregate output $F(K_t, N_t, z_t)$ is stochastic, driven by a stochastic process of productivity shocks z_t.[7] The economy's resource constraint is

(2.1) $\qquad K_{t+1} + N_t c^1_t + N_{t-1} c^2_t = F(K_t, N_t, z_t).$

The value of depreciated old capital is subsumed in F( ). Utility is increasing and strictly concave, production satisfies the Inada conditions and constant returns to scale. This structure is sufficient to explain the principal policy and efficiency issues. (Extensions are in Section 6.)

In a market setting without government, agents of generation t divide their period-t wage income into consumption and savings (s_t), $w_t = F_t / N_t = c^1_t + s_t$. Savings are invested in capital, $K_{t+1} = N_t s_t$. In period t+1, each agent consumes the $c^2_{t+1} = R^k_{t+1} \cdot s_t$, where $R^k_{t+1} = F( )/ K_{t+1}$ is the gross return on capital. The consumption of the old (c^2_t) depends on their past savings and on the current return on capital, R^k_t. The consumption of the young (c^1_t) depends on the wage rate w_t and on expectations about the return on savings, R^k_{t+1}. Apart from the stochastic productivity, this is a standard Diamond (1965) economy.

Generally, z_t affects both wages and the returns on capital so that both the young and the old bear productivity risk. Under reasonable assumptions (see below), wages and returns are neither independent nor perfectly correlated.[8] In the absence of perfect correlation, the consumption variance of both cohorts could be reduced if they could pool their resources. But of course, the young cannot trade before birth.

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[7] The process z_t can be vector-valued to avoid stochastic singularities below. The shocks may be temporary or permanent, and the may include shocks to real government spending (viewed as a reduction in privately available resources).

[8] A strength of the production model compared to endowment models (e.g., Gale 1990) is that empirically motivated assumptions about production will provide guidance about the correlation of income across cohorts.

The inability of agents to trade before birth provides an obvious motivation for policy intervention, though perhaps a too obvious one.[9] In practice, governments intervene in economic activity for variety of reasons, e.g., to redistribute resources or to finance real government spending. Such policies will generally have (side-) effects on the allocation of risk without necessarily being motivated by risk sharing considerations. Hence, I will examine policy from two perspectives. First, what are the characteristics of efficient allocations? (Such allocations could be implemented by appropriate state-contingent taxes.) Second, what are the risk-sharing effects of practically relevant policy tools such as social security and government debt? The answer to the first question one will provide an efficiency benchmark for the second one.

The set of Pareto-efficient allocation can be obtained as a solution to the social planning problem of maximizing a welfare function

$$(2.2) \qquad W_0 = E_0[\sum_{t=-1} {}_t U_t]$$

subject to the resource constraint (2.1). The initial capital K_0 and the t=-1 consumption of the old, c^2_{-1}, are given; the ${}_t > 0$ are deterministic welfare weights.

There is some controversy in the literature about ex ante versus interim efficiency (Peled 1982; Wright 1987). If period-t agents have ex ante well-defined preferences over states of nature, Pareto-efficiency involves one representative agent per period (fixed weights ${}_t$), as assumed above. If agents born in different state of nature are considered distinct

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[9] In this simple model, it is obvious that the government, and only the government, can solve the efficiency problem, e.g., though state contingent taxes. In a more realistic multi-period model, all but the youngest agents could share risk privately. The risk of being born in a "bad" state of nature might still raise serious policy issues (especially if most shocks are permanent, making far-ahead hedging valuable), but the analysis would be much more complicated. The strength of the two period model is that it is rich enough to yield interesting positive results in a tractable setting.

agents, (2.2) must be interpreted as double sum over states and periods with birth-contingent agents having distinct welfare weights. The latter (interim efficiency) is a much weaker criterion, because a Pareto improvement would require that no birth-contingent agent in any birth-state is made worse off. By construction, this precludes an analysis of risk sharing issues involving future generations, which are at the heart of this paper. The stronger notion of ex ante efficiency is therefore the appropriate efficiency benchmark.[10]

The social planner's first order conditions provide two necessary conditions for ex ante efficiency,

$$(2.3) \qquad \frac{dU_{t-1}}{dc^2_t} \frac{_{t-1}/N_{t-1}}{_t/N_t} = \frac{dE_tU_t}{dc^1_t}; \text{ and}$$

$$(2.4) \qquad \frac{dE_tU_t}{dc^1_t} = E_t[\frac{dU_t}{dc^2_{t+1}} R^k_{t+1}].$$

Eq. (2.3) is a "<u>distributional optimality condition</u>" linking the consumption of old and young.[11] Eq. (2.4) is an "<u>intertemporal optimality condition</u>" that reveal how the planner allocates resources over time by varying the capital stock. Note that (2.4) is identical to the individual optimality condition for savings. Hence, the planning problem can be

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[10] Since the issue is controversial, I provide some additional motivation in the appendix (TO BE COMPLETED). There are three main reasons to assume fixed welfare weights. First, U.S. policy makers are apparent concerned about risks imposed on future generations (e.g. in the social security reform debate), i.e., they reveal a willingness to make tradeoffs across state of nature. Second, interim efficiency is perhaps best interpreted as a technical "trick" that allows theorists to treat OG economies as if they were Arrow-Debreu economies. At least, this was Wright's original motivation (1987, p.190). Without denying its usefulness elsewhere, it would be counterproductive in this paper: This paper has virtually the opposite objective, to work out the implications of agent's inability to trade before birth. Third, there is an alternative, distributional motivation for fixed weights: Namely, ex ante efficient allocations are the subset of interim efficient allocations chosen by a social planner who does not favor one birth-contingent agent over another. In terms of policy, concern about ex ante efficiency is equivalent to a concern about not being unfair to some future birth-contingent agents. Thus, readers who favor interim efficiency on philosophical grounds may interpret the paper as being about distributional neutrality rather than about efficiency. Regardless of the labeling, I would contend that state-contingent variations in the welfare of future generations are worth studying.

[11] Condition (2.3) is similar to the efficiency condition in endowment models, e.g., in Gale (1990) and Stiglitz (1983). This paper goes beyond the endowment literature by focusing on risk sharing in an explicitly dynamic setting, where individuals earn endogenous factor incomes and the social planner can shift resources over time.

decentralized through state-contingent taxes/transfers. The efficiency condition (2.3), in contrast, is not generally satisfied by the market allocation. For time-separable utility, for example, (2.3) calls for a deterministic, monotone link between the contemporaneous consumption of the young and the old (c^1_t and c^2_t).

The characteristics of the efficient allocation and of the market allocation clearly depend on preferences, the production function, and the stochastic process generating aggregate disturbances (z_t). Starting in the next section, I will impose assumptions (empirically motivated ones) about functional forms and stochastic processes to compare the alternative allocations and to determine which preference, technology, and policy parameters tend to produce smaller or larger inefficiencies.

3. The Allocation of Risk in a Production Economy

This section lays out a simple market model with parametric assumptions about preferences, technology, and shocks. The assumptions are designed to ensure balanced growth and a Markov process for the macroeconomic dynamics, a setting that allows economically insightful comparisons of actual and efficient allocations. To prevent clutter, generalizations are deferred to an extensions section.

Regarding production, I assume that the technology is Cobb-Douglas and that aggregate uncertainty is due to an stochastic process for productivity A_t with i.i.d. growth rate a_t. The output of new goods is

$$Y_t = K_t \ (A_t N_t)^{1-}$$

where   is the capital share. The total resources available for consumption and capital investment are $F(\ ) = Y_t + v K_t$, where v is the value of old capital (net of depreciation). The stochastic component is $z_t = A_t$. The marginal products of labor and capital are

$$(3.1) \qquad w_t = (1-\ ) A_t^{1-} K_t N_t^{-} = (1-\ ) A_t \left(\frac{k_{t-1}}{(1+a_t)(1+n)}\right)$$

$$(3.2) \qquad R^k_t = K_t^{-1} (A_t N_t)^{1-} + v = \left(\frac{k_{t-1}}{(1+a_t)(1+n)}\right)^{-1} + v$$

where $k_{t-1} = K_t/(A_{t-1} N_{t-1})$ is the effective capital-labor ratio (lagged), $1+a_t=A_t/A_{t-1}$ is the productivity growth rate, and $1+n=N_t/N_{t-1}$ is the constant population growth rate.

Since A_t is the only source of disturbances, capital and labor income are deterministicly linked (though not linearly, if v 0). This stochastic singularity is clearly restrictive, though instructive: In contrast to endowment models that usually emphasize relative income risk, the focus here is on other sources of inefficiency--preferences, differential exposure to common shocks, and (below) policy.[12] The Cobb-Douglas assumption is motivated by empirical stability of factor shares. An extension to CES is in Section 6.

In modeling the time series of total factor productivity, it seems reasonable to abstract from short-run autocorrelation, because each period amounts to a generational time unit of about 20-30 years. The assumption of i.i.d. productivity growth implies a random walk in productivity levels. Though this assumption is empirically reasonable,[13] temporary shocks to the productivity level may be of theoretical interest because the young should be able to bear temporary shocks more easily than the old, through consumption-smoothing. Hence, the extensions section will also examine temporary productivity shocks.

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[12] Baxter and Jermann (1997) and Bohn (1998) have shown that capital and labor income are almost perfectly correlated at long horizons, suggesting that an emphasis on relative income risk would be inappropriate. Section 6 will show how the stochastic singularity can be broken by making the value of old capital stochastic without changing the main results.

[13] The question of a unit root in GDP versus trend stationarity with occasional breaks is controversial in the literature. Despite the controversy, the unit root assumption is more appropriate at generational frequencies, because even if a stationary trend fits the data over a shorter horizons (say, a few decades), the likelihood of future trend breaks implies a unit root-like uncertainty in the very long (keeping in mind that, say, 20 periods in this model are about 400-600 years).

For preferences, I assume homothetic utility to obtain balanced growth, but I consider a more general functional form than CRRA, for three reasons. First, intertemporal substitution and risk aversion turn out to play very different conceptual roles in the model: Risk aversion is essentially irrelevant for the market dynamics but crucial for efficiency assessments, while intertemporal substitution is important for both. Second, the equity premium and real rate puzzles (Mehra and Prescott, 1985; Weil 1989) suggest that CRRA utility with plausible intertemporal substitution parameter would imply a risk aversion far too low to match the equity premium. To avoid this problem, I assume preferences that are at least in principle consistent with empirically revealed attitudes towards risk. Finally, the possibility of age-dependent risk-aversion should not be assumed away at the outset. For these reasons, I assume a recursive non-expected utility function

$$(3.3) \qquad U_t = \frac{1}{1-\ _1}\Big[(c^1_t)^{ } + \ \big\{E_t[(c^2_{t+1})^{(1-\ _2)}]\big\}^{ /(1-\ _2)}\Big]^{(1-\ _1)/ }$$

to capture the preferences of generation t, where is the rate of time preference and $E_t[\ ]$ denotes the conditional expectations at time t. This specification is similar to Epstein-Zin (1989) and Weil (1989), but generalized to allow different degrees of risk aversion for old (2) and young (1); the elasticity of intertemporal substitution, 1/(1-), is not necessarily their inverse. For interpreting the solutions, I will often concentrate on the special cases of time-separability (1=1-), age-independent risk aversion (1= 2), or the CRRA case (1= 2=1-).[14]

For positive analysis, the only relevant property of (3.3) is the implied marginal rate of substitution,

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[14] Since CRRA is taken as benchmark for the interpretation, readers uncomfortable with non-expected utility should not worry that the results rely on non-expected utility. The limiting cases =0, 1=1, and/or 2=1 are covered as usual by applying de l'hospital's rule.

$$(3.4) \qquad \mathrm{MRS}(c^1_t, c^2_{t+1}) = \left\{ \frac{c^2_{t+1}}{\{E_t[(c^2_{t+1})^{(1-\gamma_2)}]\}^{1/(1-\gamma_2)}} \right\}^{1-\gamma-\gamma_2} \left\{ \frac{c^1_t}{c^2_{t+1}} \right\}^{1-\gamma}.$$

Note that the MRS depends on intertemporal substitution and on the risk aversion of the old, but not on γ_1. Since the young cannot participate in risk-sharing contracts before birth, their risk aversion is not identifiable from market data.

The operation of this economy is straightforward. Every period, the young divide their given wage income into consumption and savings (s_t), $c^1_t + s_t = w_t$ and the old consume the return on savings, $c^2_{t+1} = R^k_{t+1} \cdot s_t$. Savings and the capital stock are determined by the optimality condition

$$(3.5) \qquad E_t[\mathrm{MRS}(c^1_t, c^2_{t+1})\, R^k_{t+1}] = 1$$

and the equilibrium condition $s_t = K_{t+1}/N_t = k_t A_t$. Since productivity is non-stationary and all choices are proportional to A_{t-1} (due to homothetic utility and constant returns to scale), it is convenient to study productivity-ratios such as c^1_t/A_{t-1}, c^2_t/A_{t-1}, w_t/A_{t-1}, and k_{t-1}, which are stationary. In ratio form, period-t consumer decisions only depend on w_t/A_t and its determinants, k_{t-1} and a_t (see 3.1). The re-scaled economy therefore follows a Markov process with state variables k_{t-1} and a_t.[15]

The special case of log-utility and 100% depreciation deserves a separate comment. With Cobb-Douglas production and full depreciation ($v=0$), the wage income of the young and the capital income of the old are perfectly correlated and proportional to each other,

$$R^k_t\, s_{t-1} = \alpha\, A_t \left(\frac{k_{t-1}}{(1+a_t)(1+n)} \right) (1+n) = \frac{\alpha}{1-\alpha}(1+n)\, w_t.$$

If in addition utility is logarithmic, the savings rate is constant. Then proportional incomes translate into proportional consumption, (2.3) is

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[15] Individual decisions also depend on expectations about R^k_{t+1}, but because of homothetic utility and constant returns to scale, (3.5) reduces to $E_t[\mathrm{MRS}((w_t/A_t - k_{t+1}, R^k_{t+1} \cdot k_{t+1})\, R^k_{t+1}] = 1$, an implicit function for k_{t+1} in terms of w_t/A_t. Thus, decisions depend only on w_t/A_t.

satisfied, and it is straightforward to verify that the market allocation is indeed efficient.

This special case is quite non-generic, however. Even with proportional incomes and equal risk aversion, any variation in savings rates (substitution elasticity 1) would destroy the proportionality in consumption and make the allocation inefficient. The proportionality of incomes is also fragile, e.g., with old capital (v 0), additional shocks (below), or policy interventions (below). The special case is nonetheless instructive because it helps to focus the analysis on sources of inefficiency other than relative income fluctuations.

In general, the optimal decision rules for consumption and capital investment are non-linear and do not have closed form solutions. In the literature, two responses to such problems are common. Either drastic simplifying assumptions are imposed to obtain closed form solutions, or the model is calibrated to obtain numerical solutions. Neither approach is satisfactory here. The only tractable closed form setting, log-utility and 100% depreciation, would be misleading because of its non-generic properties. And calibration is problematic because the objective is to understand the qualitative role of preference, technology, and (below) policy parameters for the allocation of risk, not to calibrate a specific economy.[16]

Hence, I pursue a somewhat different approach. Following the business cycle literature, I log-linearize the relevant constraints and first-order conditions, but I derive <u>analytical</u> rather than numerical solutions to the log-linearized model. The basic linearization is taken around the

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[16] One could of course numerically simulate many alternative parameter settings to obtain a mapping from parameters to outcomes, but given the number of relevant parameters, this would require a large number of simulations, without producing a general mapping of the type obtained below.

deterministic steady state obtained by equating the stochastic shocks to their expected values, $a_t=a$ (see King-Plosser-Rebelo, 1988a, 1988b). This linearization is sufficient for understanding macroeconomic dynamics, but it is not necessarily sufficient for policy issues involving uncertainty, such as questions about precautionary savings and asset pricing.[17] I use a procedure motivated by Campbell and Viceira (1996) as alternative: The budget equations are log-linearized as in King-Plosser-Rebelo, but the Euler equations are evaluated exactly under the assumption of log-normal disturbances. The resulting policy functions are identical to the King-Plosser-Rebelo solution except that they include intercept terms reflecting the mean "displacement" of the stochastic relative to the deterministic steady state. Most results below are about the slope coefficients, however, so that the King-Plosser-Rebelo approach is sufficient and log-normality is not required. The fact that similar results can be obtained with a method that recognizes precautionary savings and the equity premium is still reassuring.

To be precise about the notation, let x (without subscript) be the deterministic steady state of any stationary variable x_t, let $\hat{x}_t = \ln(x_t) - \ln(x)$ be the log-deviation from the steady state, and let

$$\hat{x}_t = \pi_{x0} + \pi_{xk}\hat{k}_{t-1} + \pi_{xa}\hat{a}_t,$$

denote the log-linearized law of motion, where π_{xz} are fixed coefficients that can be interpreted as elasticities. The intercept terms π_{x0} are formally included but set to zero in the King-Plosser-Rebelo approximation. The main focus will be on the coefficients π_{xa} and π_{xk} of the consumption and investment rules under alternative assumptions. These coefficients

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[17] The arguments for social security investments in the stock market (e.g., Social Security Advisory Council 1997) are usually based on a non-trivial equity premium. But the equity premium is zero if one linearizes around deterministic steady state.

reveal to what extent the different cohorts are exposed to risk, either directly (high ϵ_{xa}) or through endogenous fluctuations in capital ($d\hat{x}_t/d\hat{a}_{t-i} = \epsilon_{xk}\,\epsilon_{kk}^{\,i-1}\,\epsilon_{ka}$). Throughout, non-stationary variables such as consumption and wages are scaled by lagged productivity to induce stationarity.[18]

The key elasticity coefficients are as follows.[19] The wage-productivity ratio w_t/A_{t-1} has elasticities $\epsilon_{wk} =$ and $\epsilon_{wa} = 1-$ (see 3.1). The consumption of the old (= their income) has elasticities

(3.6a) $\epsilon_{c2k} = \ + (1-\ )\,v/R^k$, and $\epsilon_{c2a} = (1-\ )(1-v/R^k)$.

With full depreciation (v=0), capital income is proportional to wage income, so that the elasticities are the same as for the wage. Since the income from selling old capital is independent of a_t and linear in k_{t-1} (versus a weight <1 in new production), a high value of old capital (v/R^k) raises ϵ_{c2k} above   while reducing ϵ_{c2a} below 1- .

The young divide their wage income between consumption and savings. If savings respond more to a shock than the wage, consumption must respond less, and vice versa. The relative responses are determined by individuals' willingness to substitute intertemporally. Combined, these considerations imply

(3.6b) $\epsilon_{c1k} = \dfrac{}{\ +(1-\ )\cdot\ }$, $\epsilon_{c1a} = 1-\ + \dfrac{(1-\ )}{\ +(1-\ )\cdot\ }$,

(3.6c) $\epsilon_{kk} = \epsilon_{sk} = \dfrac{}{\ +(1-\ )\cdot\ }$, $\epsilon_{ka} = -\dfrac{}{\ +(1-\ )\cdot\ } = \epsilon_{sa}-1$,

where =s/w is the steady state savings rate and

$$ = 1 + (1-\ )(1-v/R^k)\cdot\left[\frac{1}{1-\ }-1\right]$$

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[18] A scaling by lagged productivity is more convenient that a scaling by current productivity, because a positive shock a_t would produce a negative response in ratios like c^i_t/A_t (due to higher A_t), which is counterintuitive. The elasticities of c^i_t/A_{t-1} with respect to a_t and k_t are the same as the corresponding elasticities of c^i_t itself, so that one can omit the scaling in labeling the elasticity coefficients $\epsilon_{ci,z}$. That is, $\epsilon_{ci,a} = \dfrac{\ln(c^i_t)}{\ln(1+a_t)} = \dfrac{\ln(c^i_t/A_{t-1})}{\ln(1+a_t)} = \dfrac{\ln(c^i_t/A_t)}{\ln(1+a_t)} + 1$.

[19] Details of all derivations are in a technical appendix available from the author. The derivations are generally straightforward, but quite lengthy.

is a constant that is above (below) one if and only if the elasticity of substitution $1/(1-\ )$ is above (below) one. The relation $k_t = s_t/A_t$ ties the dynamics of the capital stock to savings.

For $=0$ (log-utility), $=1$ implies $_{c1a} = \ _{sa} = 1-\ $, so that consumption is proportional to the wage and the savings rate is constant. The capital stock then has a "generational" autocorrelation coefficient $_{kk}= \ $. (The elasticity $_{ka}=-\ $ is negative, because $k_t=s_t/(1+a_t)$ is scaled by current productivity. The K_{t+1}-level is increased by a_t.)

For a higher elasticity of substitution (>0), >1 implies $_{c1k}> \ $, $_{c1a}<1-\ $, $_{kk}< \ $, and $_{ka}>-\ $. Intuitively, savings incentives depend on the expected return on capital (see 3.5). As a log-linear approximation,

$$(3.7) \qquad \hat{R^k}_{t+1} = \ _{Rk}\ (\hat{a}_{t+1} - \hat{k}_t),$$

where $_{Rk} = (1-\ )(1-v/R^k)$. Hence, the expected return $E_t[\hat{R^k}_{t+1}] = -\ _{Rk}\hat{k}_t$ depends negatively on current investment. In equilibrium, an increase in the effective capital labor ratio $k_{t-1}/(1+a_t)$ will always raise k_t and therefore reduce the expected return on savings. Lower interest rates trigger an income and a substitution effect. For >0, the negative substitution effect dominates so that the savings rate declines in response to a higher k_{t-1} and/or lower a_t value. These movements in the savings rate reduce the consumption impact of productivity shocks ($_{c1a}$ as) and they magnify the impact of initial capital ($_{c1k}$).[20]

For <0, the income effect dominates. Then the savings rate rises with $k_{t-1}/(1+a_t)$ so that the consumption impact of productivity shocks is larger than with log utility: $_{c1a}>1-\ $. With incomplete depreciation ($v>0$),

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[20] To gain further intuition, note that reduces $_{kk}$ and $|\ _{ka}|$ in absolute value and thereby reduces the impact of k_{t-1} and a_t on R_{t+1}. That is, increased intertemporal substitution induces savings responses that partially offset the interest rate changes that would otherwise occur.

becomes less sensitive to variations in   because any given change in savings has a smaller impact on interest rates (see 3.7).

If one compares (3.6a) and (3.6b), one finds that an the elasticity of substitution below 1 (<0) is a sufficient condition for ϵ_{c2a} $1-<$ ϵ_{c1a} and ϵ_{c2k} $>$ ϵ_{c1a} (for all v 0). If v>0, $\epsilon_{c2a} <$ ϵ_{c1a} and $\epsilon_{c2k} >$ ϵ_{c1k} apply even for some substitution elasticities $1/(1-$) above 1 (because safe capital tends to affect the c^2-elasticities more than the c^1-elasticities). Under these conditions, the young bear more productivity risk than the old, while they are less exposed to changes in initial capital.

These conditions are likely satisfied empirically. Hall (1988) suggests that $1/(1-$) is near zero, so that $\epsilon_{c1a}>$ ϵ_{c2a} for all v 0. Moreover, if one thinks of generational savings as a repetition of annual savings and uses annual data to calibrate v/R^k, an annual return on capital of about 6% and a depreciation rate of about 5% suggests that v/R^k $0.95/1.06 = 0.90$ is far above zero. Setting, e.g., $=1/3$, $=1$, and $=0.2$, one finds $\epsilon_{c1a} = 0.67$ $>>$ $\epsilon_{c2a} = 0.07$. The young bear vastly more productivity risk than the old.[21]

Finally, note the risk aversion parameters 1 nor 2 do not appear in any of the above formulas. Though risk aversion has some impact through precautionary savings, it is essentially irrelevant for the market allocation.[22] This is intuitively reasonable, because individuals cannot privately share risks across generations. Their exposure to risk is therefore determined by technology (,v) and by the young generation's willingness to substitute intertemporally ( ).

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[21] See Bohn (1998) for a more elaborate calibration. This simple calculation may exaggerate the quantitative difference in risk exposure if the value of old capital is correlated with productivity growth or if productivity shocks have a temporary component; see Bohn (1998) and below. The qualitative point is robust, however.

[22] In the Campbell-Viceira style approximation, one can show that the intercept terms ϵ_{k0}, ϵ_{c10}, and ϵ_{c20} are functions (complicated ones) of 2. Hence, 2 affects the -value at which (3.6b-c) are evaluated. The King-Plosser-Rebelo approach abstracts from this effect by linearizing around a deterministic steady state.

The unequal exposure to productivity risk in the market allocation raises natural questions about the welfare implications. This is examined in the next section.

4. Efficient Allocations in the Parametric Model

To obtain an efficiency benchmark for the market economy, consider the planning problem of Section 2 with the preference and technology assumptions of Section 3. Some insights can be obtained from the distributional efficiency condition (2.3) alone, while others require a complete solution to the social planning problem.

4.1. Distributional efficiency

The market allocation does not generally satisfy one key condition for Pareto-efficiency, the distributional condition (2.3). It can be log-linearly approximated as

$$(4.1a) \qquad _2 \hat{c}^2_t - _1 \hat{c}^1_t = (_2 - 1)(\hat{c}^1_{t-1} + \hat{a}_{t-1}) + (1 -) \frac{1 - - 1}{1 - } \, _{Rk} \hat{k}_t$$

$$+ [1 - 2 + (1 - - 1)]/(1 -) \, _{Rk} \hat{k}_{t-1}.$$

where $(0,1)$ is a constant. (Intercepts are suppressed; $_{Rk}$ is defined as in 3.7; $\hat{c}^1_t$ denotes the log-deviation of c^i_t / A_{t-1} from its steady state.)

This efficiency condition applies for any set of fixed welfare weights. Intuitively, different welfare weights primarily affect the planner's desired level of intergenerational redistribution, but they do not have a direct effect on the efficient allocation of risk.[23] For this reason, one may refer to "the" efficient allocation of risk without conditioning on the welfare weights.

In the special case of CRRA utility, $_1 = _2 = 1 - $, (4.1a) reduces to $\hat{c}^2_t = \hat{c}^1_t$. That is, <u>with CRRA utility, the young and the old should be exposed</u>

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[23] Since the elasticity coefficients are evaluated at a particular steady state, the welfare weights may have an impact by changing the point of evaluation, but this effect is quite indirect.

equally to variations in all state variables. Hence, the finding of unequal elasticities in Section 3 implies that the market allocation is generically inefficient.

If utility is time-separable ($\gamma_1 = 1-\sigma$) but with potentially age-dependent risk aversion, (4.1a) implies

(4.1b) $\hat{c}^2_t = (\gamma_1/\gamma_2)\,\hat{c}^1_t + (1-\gamma_1/\gamma_2)[\hat{c}^1_{t-1}+\hat{a}_{t-1}+\rho_{Rk}/(1-\gamma)\,\hat{k}_{t-1}]$.

Since the bracketed term is known at time t-1, (4.1b) implies that the young and the old should be exposed to current shocks in inverse proportion to their risk aversions: $\epsilon^*_{c2a} = (\gamma_1/\gamma_2)\,\epsilon^*_{c1a}$, where the stars (*) denote efficient elasticities. Thus, an allocation that imposes more productivity risk on the young than on the old can be rationalized by preferences with age-increasing risk aversion ($\gamma_1 < \gamma_2$).

Preferences with exogenous age-dependent risk-aversion are quite non-standard, however. Also, since γ_1 is unobservable from market data, efficiency is untestable if one treated γ_1 as a free parameter. On balance, the point that age-dependent risk aversion might rationalize grossly unequal exposure to shocks is noteworthy because the notion that the young are more risk-tolerant than the old has some plausibility. But a further analysis would not be particularly insightful. For the subsequent analysis, I will therefore assume $\gamma = \gamma_1 = \gamma_2$, i.e., focus on the more standard case of Epstein-Zin utility (or CRRA, if $\gamma = 1-\sigma$).

For Epstein-Zin utility ($\gamma = \gamma_1 = \gamma_2$), (4.1a) can be written as

(4.2) $\hat{c}^2_t - \hat{c}^1_t = h_1\,\hat{k}_t + h_2\,\hat{k}_{t-1}$.

where $h_1 = (1-\gamma)(1-\sigma-\gamma)/\rho_{Rk}/(1-\gamma)$ and $h_2 = \gamma(1-\sigma-\gamma))/\rho_{Rk}/(1-\gamma)$ are non-zero for non-separable preferences ($\gamma \neq 1-\sigma$). If utility is non-separable, (2.3) and (4.2) include intertemporal linkages, which are responsible for the capital terms in (4.2). For $h_1 \neq 0$, efficiency is consistent with unequal

consumption movements $(\hat{c}^2_t\ \hat{c}^1_t)$. The details depend on the dynamics of the capital-labor ratio, i.e., they require a solution to the complete social planning problem.

Even in the CRRA case, a solution of the complete problem is needed to determine <u>by how much</u> consumption should change in response to any particular shock, and to what extent the impact of a shock can be shared with future generations through changes in capital investment.

4.2. The Social Planning Problem

The overall planner's problem is essentially a standard infinite-horizon optimal growth problem, except that there are two goods (c^1, c^2) and a quite general Epstein-Zin type function. Most of the interpretation will focus on CRRA preferences, =1- , but since substitution and risk aversion have fundamentally different economic roles, all formulas are presented for the Epstein-Zin case, so that one can immediately recognize if a formula's economic intuition involves substitution or risk aversion.

As usual, the optimal risk-sharing problem does not have a closed form solution. A natural approach to characterizing the properties of the optimal risk-sharing policy is therefore to log-linearize the economy around its steady state.[24] As in the market case, I will derive approximate solutions analytically.

In addition to the distributional condition (4.2), the planning problem involves (2.1) and (2.4). The resource constraint (2.1) yields the log-linearization

(4.3) $\qquad s_k \hat{k}_t + s_1 \hat{c}^1_t + s_2 \hat{c}^2_t = (\ +s_v)\ \hat{k}_{t-1} + (1-\ -s_k)\ \hat{a}_t$

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[24] Note that the complete set of ex ante efficient allocations includes allocations with unbalanced growth. One could examine unbalanced growth by linearizing around a perfect foresight path, but this generalization would not be insightful. I therefore assume exponential welfare weights, $_t = N_t\ ^t$, to ensure balanced growth. The intercept terms $_{x0}$ are usually suppressed to simplify the notation.

where $s_1=c^1/(Y/N)$, $s_2= 1/(1+n) c^2/(Y/N)$, $s_k=k/y$ and $s_v = v/an \, s_k$ are output shares, $y=Y/(N \, A)$, and $an=(1+n)(1+a)$. Not surprisingly, high productivity growth and a high initial capital stock increase the consumption and investment opportunities.

For the intertemporal efficiency condition (2.4), one obtains

(4.4) $E_t[\hat{c}^2_{t+1}] - (\hat{c}^1_t + \hat{a}_t) = - \,_{Rk}\hat{k}_t/(1-)$.

Intuitively, efficient capital investment is determined by the tradeoff between consumption smoothing (equating the l.h.s. terms in 4.4) and intertemporal substitution ( ). For interpreting the r.h.s. of (4.4), recall from (3.7) that $- \,_{Rk}\hat{k}_t$ is the deviation of the expected return on capital from its steady state.

If one uses (4.2) to eliminate $\hat{c}^1_t$ from (4.3-4), one obtains a pair of expectational difference equations in ($\hat{c}^2_t$, $\hat{k}_t$). For CRRA utility, it is straightforward to show that the system's characteristic roots (μ_1,μ_2) satisfy $0<\mu_1<1<R^k/an<\mu_2$, where $R^k/an>1$ is implied by the social planner's transversality condition. For $1-$, I <u>assume</u> that $\mu_1<1<\mu_2$; this holds in a neighborhood of $=1-$ and seems satisfied for plausible parametrizations. The dynamic system is then saddle-path stable and, when combined with (4.2), it yields decision rules for ($\hat{k}_t$, $\hat{c}^2_t$, $\hat{c}^1_t$) as functions of the Markov state vector ($\hat{k}_{t-1}$, $\hat{a}_t$). Specifically, one finds

(4.5a) $\hat{k}_t = \mu_1 \, \hat{k}_{t-1} + \,^*_{ka} \, \hat{a}_t$

where $^*_{ka} = -(1-1/\mu_2)\dfrac{+s_v}{s_k-s_1 h_1}$ $(-1,0)$ and μ_1 $(0,1)$, and

(4.5b) $\hat{c}^2_t = \,^*_{c2k} \, \hat{k}_{t-1} + \,^*_{c2a} \, \hat{a}_t$

where $^*_{c2a} = 1-(1-1/\mu_2)\dfrac{+s_v}{s_1+s_2}$ $(0,1)$ and $^*_{c2k}$ $(0,1)$.[25]

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[25] Formulas for μ_1, μ_2, $^*_{c2k}$ are not displayed because they are complicated functions of the parameters (see the appendix). For CRRA utility, the coefficients on a_t and k_t are related: $^*_{c2a}=1- \,^*_{c2k}$ and $^*_{ka}=-\mu_1$.

Thus, the efficient capital-productivity ratio is positively autocorrelated ($\mu_1 > 0$) and it depends negatively on a_t. Since $^*_{ka} > -1$, the savings and capital investments level depend positively on productivity, however: $d\ln(K_{t+1})/d\ln(1+a_t) = {}^*_{sa} = 1 + {}^*_{ka} > 0$. For CRRA utility, (4.2) implies $\hat{c}^1_t = \hat{c}^2_t$ so that the elasticities in (4.5b) also apply to $\hat{c}^1_t$. For $1-$, one finds

(4.5c) $^*_{c1k} = {}^*_{c2k} - h_1 \cdot \mu_1 - h_2$ and $^*_{c1a} = {}^*_{c2a} - h_1 \cdot {}_{ka}.$

Since $_{ka} < 0$ and $\mu_1 > 0$, $^*_{c1k} > {}^*_{c2k}$ and $^*_{c1a} < {}^*_{c2a}$ hold iff $> 1-$ (because then $h_1, h_2 < 0$). The high equity premium suggests that the condition $> 1-$ is likely satisfied empirically (see Weil 1989). If one includes CRRA as a lower bound for , the empirically plausible parameter set is $1-$. This implies $^*_{c2a}$ $^*_{c1a}$, a quite strong result: <u>The old should carry at least as much exposure to productivity shocks as the young</u>.

Risks can also be shared with future generations through state-contingent variations in the capital stock. The elasticity $^*_{ka}$ determines the magnitude of these variations, and the root μ_1 determines the time horizon, the rate of decline in the impulse response function. Since $^*_{ka}$ (- 1,0) and μ_1 (0,1), the n-period ahead response of the capital-productivity <u>ratio</u> satisfies $^*_{ka}\, \mu_1^{n-1}$ (-1,0). Hence, the response of consumption levels, $d\ln(c^i_{t+n})/d\ln(1+a_t) = 1 + {}^*_{cik}\, \mu_1^{n-1}\, {}^*_{ka} > 0$, is unambiguously positive at all horizons: Productivity risk should be shared.

In the market allocation, shocks are also propagated through the capital-labor ratio, but the elasticities generally differ from the optimal values. A general comparison depends too many parameters to be insightful here. Recall, however, that for elasticities of substitution near one (0), $_{kk} = - {}_{ka}$ is close to the capital share and virtually unaffected by v. For $v > 0$ and 0, one finds $\mu_1 >$, $^*_{ka} < -$; both are increasing in absolute

value with v. At least for these parameters, the market allocation has a too low autocorrelation of capital.

Overall, the market allocation of risk differs from the efficient allocation in how they allocate risk on the two living generations and in how they share risk with future generations. In the efficient allocation, the old should bear at least as much productivity risk as the young. But the reverse inequality applies in the market allocation.

5. Policy Intervention with Simple Tools

In practice, the government intervenes in the market allocation in many ways, e.g., through government debt, social security, and taxation. In addition to their well-understood distributional effects, such interventions have an impact on the allocation of risk. This section will examine these risk-sharing effects from two perspective: First, what policies would be necessary to implement an efficient allocation of risk? Second, what are the risk sharing implications of policies that are observed in practice? The distinction is important because I will argue that observed government practices are in fact difficult to reconcile with efficiency.

I focus specifically on social security and government debt, because they are most prominent tools of intergenerational redistribution. Since social security is wage-indexed while government debt is a safe claim (or nearly safe), they turn out to have very different risk sharing effects.

5.1. Social Security and Government Debt

This section adds three types of government intervention to the market model of Section 3, namely government debt, social security, and taxes on the young.

In general, government debt can have a variety of effects that depend on its type (state-contingent returns) and its growth path. Since debt is usually considered a safe claim (abstracting from inflation risk), I will assume for now that government debt offers a safe return R^b_{t+1}. Debt D_{t+1} is issued in period t and matures in period t+1. To stay in a balanced growth setting with Markov uncertainty, I assume that the stock of debt is proportional to the growth path $A_t N_t$: $D_{t+1} = d_t A_t N_t$, where $d_t = d(k_t)$ may depend on the capital-labor ratio.[26]

In principle, debt service can be provided by taxes on the young or on the old (τ^1_t and τ^2_t), subject to the budget equation

(5.1) $R^b_t D_t = N_t \tau^1_t + N_{t-1} \tau^2_t + D_{t+1}$.

For this section, I consider taxes on the young only, i.e., set $\tau^1 = R^b_t D_t/N_t - D_{t+1}/N_t$ and $\tau^2_t=0$. (τ^2 is defined for reference below. Government spending could be added, too, but that would only complicate the analysis.) Taxes on the young are a useful benchmark because a tax on wages can be interpreted as a lump-sum tax (given a fixed labor supply) and because government debt would neutral if the debt holders (the old) were responsible for the debt service.

One key feature of safe debt is that along any balanced growth path, the debt service depends negatively on productivity growth:

(5.2) $\tau^1_t/A_{t-1} = R^b_t d(k_{t-1}) - \dfrac{1+a_t}{1+n} d(k_t)$

Intuitively, old debt becomes small relative to current income when growth is high, but it is a heavy burden when growth is slow. That is, <u>debt increases the exposure of taxpayers to productivity risk</u>.

This observation raises some immediate and troubling questions about the efficiency implications: If the young are already too exposed to

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[26] This is not a serious restriction, since the efficient allocation has this structure. The assumption of safe debt is important; modifications are discussed below.

productivity risk without government intervention, as Sections 3-4 suggest, why is safe government debt so widely used in practice? I will return to this question below.

Next, consider social security. Social security is modeled most easily by assuming a wage-indexed, pay-as-you-go (PAYG) system that pays benefits $N_{t-1}\, w_t$ at a fixed replacement rate $\quad$ to the old generation and collects payroll taxes $N_t\, w_t\ _t$ from the young. The PAYG constraint implies that the tax rate $\ _t$ must equal the cost rate $\ /(1+n)$. In the U.S., social security benefits are indexed to the average national wage level at the time of retirement and inflation-indexed thereafter, i.e., partially wage-indexed and partially safe in real terms. Here I assume full wage-indexation, for simplicity and to highlight the contrast between social security and safe debt.[27]

The economy then works as follows. Workers earn a disposable income $w_t\,(1-\ _t)-\ ^1{}_t$ that is either consumed ($c^1{}_t$) or saved, either in capital (equity securities, $s^k{}_t$) or in government bonds ($s^b{}_t$),

(5.3) $\qquad c^1{}_t = w_t\,(1-\ _t) - \ ^1{}_t - s^k{}_t - s^b{}_t.$

The old receive wage-indexed social security benefits with a replacement rate $\quad$ and (below) pay taxes $\ ^2{}_{t+1}$. Their consumption is

(5.4) $\qquad c^2{}_{t+1} = R^k{}_{t+1}\, s^k{}_t + R^b{}_{t+1}\, s^b{}_t + \quad w_{t+1} - \ ^2{}_{t+1}.$

Savings behavior is determined by the optimality conditions

(5.5) $\qquad E_t[\mathrm{MRS}(c^1{}_t, c^2{}_{t+1})\, R^k{}_{t+1}] = E_t[\mathrm{MRS}(c^1{}_t, c^2{}_{t+1})\, R^b{}_{t+1}] = 1.$

In equilibrium, the interest rate on government bonds must clear the bond market, $D_{t+1}=N_t\, s^b{}_t$. The next period's capital stock is $K_{t+1}=N_t\, s^k{}_t$. With the

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[27] A partially wage-indexed system would have intermediate properties that could be inferred from the pure cases of safe debt and fully indexed social security. In an earlier version of this paper, I also added a social security trust fund with debt and equity investments. Since if debt above is interpreted as gross debt minus a trust fund, alternative trust fund investments can be captured implicitly by making alternative assumptions about the return on net debt; see Bohn (1998) for a more detailed analysis of trust fund investments.

above assumptions on policy, the economy scaled by the productivity trend still follows a Markov process with state variables k_{t-1} and a_t. Again, there are no closed form solutions except in uninteresting special cases. Hence, I log-linearize around the steady state and evaluate the allocation of risk by examining the resulting elasticity coefficients.

The formulas for the elasticity coefficients for consumption and investment are shown in Table 1. They reveal that policy affects the allocation of risk in several ways.

First, safe government debt reduces the exposure of the old to productivity risk (d s_d c_{2a}). Intuitively, the income of the old is a linear combination of wage-proportional claims, [/(1-)(1+n)+] w_{t+1}, and claims that do not depend on current productivity, $R^b_{t+1} s^d_t + v s^k_t$. For v>0 and/or d>0, these fixed claims necessarily reduce the elasticity of c^2_{t+1} with respect to productivity shocks.

Second, safe government debt increases the exposure of young to productivity risk (d c_{1a}). In equilibrium, the resources available for consumption plus savings are

(5.6) $c^1_t + s^k_t = w_t (1- \ _t) - \ ^1_t - s^b_t$

$= w_t (1-) - R^b_t A_{t-1}/(1+n) d(k_{t-1}).$

Through taxation, workers are responsible for a predetermined debt service. They are in effect endowed with a leveraged claim on wages, with leverage created by government debt. Hence, government debt magnifies the impact of productivity shocks on consumption c^1 and on capital accumulation ($_{sa}= \ _{ka}+1$). Through capital investment, the impact on future generations is also increased.

Third, the government can control the autocorrelation of the capital-labor ratio by varying the elasticity of debt issue with respect to the

capital stock, $_{dk}$. Since k_t changes in response to productivity shocks, a debt policy that makes debt a function of k_t is a tool to allocate productivity shocks over many generations.

Finally, note that social security does not appear explicitly in the above elasticities. With Cobb-Douglas production, wage-indexed social security benefits are proportional to wages and to gross capital income. For v 0 and d 0, social security simply increases the income of the old by a fixed percentage affecting the allocation of risk. For v>0 and/or d>O, social security increases share of old-age income that is effectively wage-indexed, which raises $_{c2a}$ by reducing s_v/s_2 and s_d/s_2. Apart from the latter effect, social security is essentially neutral with regard to the allocation of risk.[28]

Social security is nonetheless conceptually important because it allows the government to separate distributional and risk-sharing issues. By substituting debt for social security, the government can alter the allocation of risk without necessarily changing the scale of intergenerational redistribution (say, as measured by the steady state generational account balance), and it can change the scale of redistribution without affecting the allocation of risk.

How then should the government operate its debt and social security policy? Since individuals' consumption-savings decisions satisfy the same Euler equation as the social planner, it is sufficient for obtaining an efficient allocation if the government equates the actual dynamics of the capital stock to the efficient dynamics, i.e., if it sets $_{kk} = \mu_1$ and $_{ka} = $ $^*_{ka}$. These efficiency conditions provide two equations for the two policy

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[28] I do not place much weight on v>0, because the impact is limited and likely small: Though social security increases $_{c2a}$ by reducing s_v/s_2, $_{c2a}$ remains below 1- and $_{c1a}$ remains unchanged. When comparing social security to debt, the view that social security is roughly neutral is also useful to avoid exaggerating the differences.

parameters d and d_k. By setting d and d_k appropriately, the government can implement an efficient allocation of risk. Moreover, this is possible without constraining distributional goals if social security is used to control the overall scale of redistribution.

Now recall that the market allocation without government imposed too much productivity risk on the young and too little on the old. Inspecting the formula for c_{1a}, one finds that a reduction in c_{1a} and an increase in c_{2a} both require d<0, i.e., a negative amount of safe debt. Even if one questions the realism of negative debt, it is clear that a positive amount of safe debt shifts productivity risk in the wrong direction. In practice, however, many governments in the world are issuing essentially safe debt. Such policies appear to be suboptimal.[29]

The finding that government debt shifts risk in the wrong direction cannot be dismissed by with reference to distributional objectives because the government could instead make transfers to the old through social security. For v 0, social security is approximately neutral with respect to risk-sharing, so that social security transfers would not disturb the allocation of risk. If v>0, social security would even raise c_{2a} and thereby shift productivity risk in the "right" direction. In either case, if the government wants to make transfers to the old, this could be done more efficiently through social security than through government debt.

The risk sharing argument against safe debt has some immediate implications for social security reform. Several recent proposals to reform social security propose a reduced future replacement rate in the traditional system combined with bond financing ("recognition bonds") to

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[29] Note that this argument against safe debt is quite different from the usual tax smoothing arguments (Lucas and Stokey 1983, Bohn 1988). Here I assume lump-sum taxes, making tax smoothing arguments irrelevant, and I rely heavily on deviations from Ricardian neutrality.

soften the distributional impact (e.g., the PSA plan in Advisory Council 1997; Feldstein, 1996). Such proposals can be interpreted as a reduction in combined with an increase in d. Such reforms would clearly shift productivity risk from the old to the young and to future generations. Based on the analysis above, this would be efficiency-reducing.

A similar risk-shifting occurs if mandatory "individual accounts" are introduced in a way that reduces the social security trust fund. A trust fund holding government bonds effectively reduces the net, publicly-held debt (d) below the gross Treasury debt. If the trust fund is reduced, e.g., when payroll taxes are diverted into individual accounts, this would increase the net debt and thereby the exposure of future workers to productivity risk.[30]

5.2. Other Policy Instruments

In this section, I will examine two additional policy tools that have an impact on the allocation of risk, income-proportional taxes paid by the old and state-contingent government debt.

Income taxes deserve comment in this context because they are widely considered important risk sharing tools. True income taxes are problematic, however, because a tax on the capital income of the old would usually distort savings decisions; the implied second-best considerations are beyond the scope of this paper. It is instructive, however, to consider a setting in which the old pay a lump-sum tax equal to a fraction $_t$ of their factor income. (For example, consider an income tax combined with a tax credit that provides the correct savings incentives on the margin.) For

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[30] These comments should not be interpreted as a complete assessment of any particular policy plan because a specific plan may have many other, potentially offsetting features. For example, a shift from debt to equity holdings within a government-run trust fund will also increase net government debt, which looks efficiency reducing. Such a shift has different implications than an individual accounts plan, however, if the trust fund benefits future generations; see Bohn (1998).

risk-sharing purposes, such a tax is equivalent to an income tax because its revenues vary with productivity in the same way. Since the capital income of the old is $(R^k_t-1) s^k_{t-1}$, the tax on the old collects revenues

(5.7) $\quad \tau^2_t = \tau_t \left[\left(\dfrac{k_{t-1}}{(1+a_t)(1+n)} \right)^{-1} + v-1 \right] k_{t-1} A_{t-1}.$

For the tax rate, three simple cases are worth considering. First, suppose $\tau_t = \tau$ is constant and assume that the young pay whatever residual is needed to maintain debt service. Then the tax payments by the old are an increasing function of the productivity shock a_t. Hence, the residual exposure of the young to productivity shocks is increased. This does not resolve the puzzle of why governments issue safe debt.

Second, suppose that the young pay taxes at a fixed rate, $\tau^1_t/w_t = $ constant. Then the disposable income of the young is wage-proportional and τ_t must vary such that the old pay for the entire debt service. In this case, debt does not alter the allocation of risk, because Ricardian neutrality applies when the debt holders are taxed for the debt service. In this case, debt is uninteresting.

Finally, consider a uniform income tax, i.e., suppose τ_t is equated to the wage tax on the young ($\tau_t = \tau^1_t/w_t$ such that (5.2) and (5.7) are satisfied). Then τ_t must satisfy

$$\tau_t \left[\left(\dfrac{k_t}{1+n} \right)(1+a)^{1-\alpha} - (1-v)\dfrac{k_t}{1+n} \right] = R^b_t d(k_{t-1}) - \dfrac{1+a_t}{1+n} d(k_t).$$

Hence, τ_t is a decreasing function a_t, which means that safe debt still increases the exposure of the young to productivity risk, though less than in the case of $\tau^2_t=0$. The intuition is that debt is neutral to the extent that the old pay part of the debt service.

Overall, one finds that income-proportional taxes imposed on the old do have risk-sharing implications. But safe debt still shifts productivity

risk from old to young--in the wrong direction--except in the case of Ricardian neutrality.[31]

State-contingent debt is perhaps a more promising extension. Such debt is not unrealistic if one interprets nominal debt as state-contingent and treats inflation as a government control variable that determines he state-contingent variations in the real return on nominal bonds. Let η_{Rba} be the elasticity of $R^b_t(a_t)$ with respect to a_t. For any level of debt, an increase in η_{Rba} shifts productivity risk from young to old. Hence, an efficient allocation of risk could be implemented in principle by setting η_{Rba} sufficiently high.

Though promising, the nominal contingency is probably not a plausible rationalization for government debt. This is because an unrealistically high elasticity would be required to shift risk in the "right" direction. For example, one can show that the risk exposure of the old with η_{Rba} 0 is given by

$$\eta_{c2a} = 1- \phi - (1-\phi) s_v/s_2 - (1-\phi - \eta_{Rba}) s_d/s_2$$

(notation as in Table 1; assuming $\sigma^2_t=0$ for simplicity). For $\eta_{Rba}<1-\phi$, government debt is still shifting risk away from the old and onto the young and future generations. Recall from (3.7) that $dR\hat{^k}_t/d\hat{a}_t = \eta_{Rk} 1-\phi <1$. Thus, to be effective, nominal debt would have to have an elasticity coefficient $\eta_{Rba}>1-\phi$ above the corresponding coefficient on capital. Nominal bonds that are more "risky" than claims on capital appears unrealistic. In addition, the argument for nominal debt assumes an inflation rate with deterministic link to productivity growth. If the government cannot control inflation perfectly, "noise" in inflation would add a new stochastic shock to the

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[31] I am not pursuing more general state-contingent taxes on the old because it is complicated to design capital income taxes in a way that avoids distorted savings incentives (Chari et.al, 1991; Zhu, 1992; Bohn 1994). In addition, it is too obvious that general state contingent taxes could achieve efficiency.

economy, a shock that would redistribute resources across generations and therefore be inconsistent with distributional efficiency. Overall, I conclude that nominal debt is less damaging than safe debt, but not much better, unless one allows for extreme parameter values for the state-dependence of returns.

6. Extensions

This section generalizes the main model in several directions to examine the robustness of the above results. Notably, I will discuss other aggregate shocks, elastic labor supply, and CES-production.

6.1. Other macroeconomic risks

In the main model, permanent productivity shocks were the only source of aggregate risk. Here I consider three other sources of risk: temporary productivity shocks, an uncertain salvage value of old capital, and government spending shocks.

Temporary productivity shocks are worth discussing because they raise questions about the role of consumption-smoothing. If productivity shocks are temporary, the young may be able to bear them more easily than the old because they can consumption-smooth over two periods. But since temporary productivity shocks reduce interest rates while permanent productivity shocks increase interest rates, the implications are far from obvious. The differential income effects may be offset by differential substitution effects.

To examine this issue more formally, suppose one adds a temporary productivity disturbance e_t to the model and redefines output to be

$$Y_t = K_t^{} (A_t (1+e_t) N_t)^{1-} .$$

If e_t is i.i.d., both the positive model of Section 3 and social planning problem of Section 4 retain their Markov structures, now with an additional state variable e_t. The z_t-shock of Section 2 becomes a vector $z=(A,e)$. The log-linearized decision rules gain an additional term for the e_t-shocks ($_{xe}\hat{e}_t$ in the positive model, $^*_{xe}\hat{e}_t$ in the normative model). But since the $_{xa}$ and $_{xk}$ coefficients remain unchanged, all previous results about permanent shocks and about initial capital remain unchanged.

The efficient solution for CRRA utility again requires equal coefficients for old and young consumers, $^*_{c1e}= \, ^*_{c2e}$. In the market model, the distinction between permanent and temporary shocks is irrelevant for the old, i.e., $_{c2a}= \, _{c2e}$. For the young, the consumption response to temporary shocks generally differs from the response to permanent shocks because of differential consumption smoothing and intertemporal substitution effects. The difference in the elasticity coefficients is

$$_{c1a} - \, _{c1e} = \frac{(1- \,)}{\, +(1- \, - \, _d)} \, ,$$

which is positive if and only if $\quad$ 1 (see Table 1 for notation). Recall that $=1$ applied in the case of wage-proportional incomes and log-utility, hence $_{c1a}= \, _{c1e}$. In this special case, the differential interest rate movements exactly offset the differential income effects, producing equal consumption coefficients.[32]

For wage-proportional incomes ($v=0$) and a CRRA utility with elasticity of substitution below one (<0), one finds <1 and therefore $_{c1e} < \, _{c1a}$. Then the differential income-effects (consumption-smoothing) dominate and the consumption response of the young $_{c1e}$ is below $1- \, $. If in addition $v \quad 0$, so that $_{c2e} \quad 1- \, $, one may have $_{c1e} < \, _{c2e}$, so that the old

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[32] A positive temporary shock provides a consumption-smoothing motive to save more, but it also raises the prospective capital-labor ratio k_t, and hence depresses interest rates. In contrast, a permanent shock reduces k_t and raises interest rates, providing a substitution motive to save more. In the log-utility case, these two mechanisms are equally strong.

bear too much risk. Safe government debt would have a role as a tool for shifting temporary productivity risk to the young.

Even if $c_{1e} < c_{2e}$, it would be preferable, however, to find a policy instrument that only shifts temporary risk and not permanent risk. In any case, the negative autocorrelation in productivity growth that one should observe with temporary shocks is not apparent in the data.[33] Hence, one should be cautions about drawing policy conclusions from the theoretical case of temporary productivity shocks.

Second, consider <u>adding uncertainty to the return to capital</u>. Productivity shocks (permanent and temporary) imply a deterministic link between wages and the marginal product of capital, which may be considered restrictive. To see the implications of independent movements in the return to capital, suppose the salvage value of old capital is an i.i.d. random variable v_t. In practice, the old hold a variety of long-lived capital goods of uncertain value so that one might think of a stochastic v_t as a general "valuation risk." The return to capital is then still correlated with productivity and wages, but contains additional noise. The model retains its Markov structure, now with v_t as additional state variable and with $\hat{v}_t$-terms in the log-linearized decision rules.

In the market allocation, valuation risk is a generation-specific risk, since the old generation holds all the capital. One finds $c_{2v} > 0$, while $c_{1v} = 0$ and $k_v = 0$, which is inefficient. Bohn (1998) has shown that social security trust fund has an interesting risk-sharing role in this context. With defined benefits, the risk and return of social security equity investments would be carried by future generations, so that equity investments are a means to share valuation risk, to implement an allocation

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[33] With temporary shocks, productivity growth should have a negative autocorrelation: $(1+a_{t+1})(1+e_{t+1})/(1+e_t)-1$ is above $E_t a_{t+1}$ in expectation whenever $e_t < E_t e_{t+1}$.

with $^*_{c1v} = {}^*_{c2v} > 0$. Overall, independent movements in the return to capital are another potential source of inefficiency but they do not overturn previous findings.[34]

Third, <u>shocks to government spending</u> are potentially important source of risk (e.g. war spending). In the normative model, a stochastic, exogenous share of government spending g_t in output can be accommodated easily, because government spending reduces the resources available to consumption and capital investment like a negative productivity shock. To maintain the Markov structure, let the spending share g_t be i.i.d. with mean g. Then an efficient allocation requires a negative response of old and young consumption ($^*_{c1g} < 0$, $^*_{c2g} < 0$, with equal coefficients in case of CRRA) and a burden-sharing with future generations through variations in capital investment, $^*_{kg} < 0$. In a market setting, efficient responses to spending shocks could be implemented in various ways, e.g., by allowing taxes and debt to depend on spending shocks. A more detailed analysis is beyond the scope of this already long paper, but it is worth noting that this source of uncertainty could be included fairly easily and without changing other results.

6.2. Variable labor supply

The assumed inelastic labor supply may be considered restrictive, too. One might argue, for example, that the young can bear more risk, because they can "recover" from bad shocks by increasing their labor supply, whereas the retired old have to live with their given resources.[35]

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[34] A stochastic v_t may modify the results on productivity risk, if v_t and a_t are correlated. If the correlated component is subsumed into a_t, $_{c2a}$ would effectively be increased. This would quantitatively modify the previous results, but qualitatively overturn them.

[35] This concern was raised on several occasions when I presented an earlier draft.

To examine this issue, assume that individuals are endowed with one unit of time and have preferences over consumption and leisure. By assumption, the old are excluded from the labor market and use all their time for leisure. The young consume l_t units of leisure, where $0 \le l_t \le 1$, and provide labor supply $1-l_t$. Efficiency and individual rationality both require that the marginal rate of substitution between young consumption and leisure equals the wage rate. In the normative analysis with time-separable utility, innovations in consumption and leisure would have to satisfy

(6.1) $$\frac{dc^2_t}{c^2_t} \; \sigma_2 = \frac{dc^1_t}{c^1_t} \; \sigma_1 + \frac{U_{cl}}{U_c} \; dl_t,$$

where the subscripts in U_{cl} and U_c denote partial derivatives.

In general, the relative volatility of the old and young generations' consumption depends on the correlation and the substitutability of consumption and leisure. For the special case of $U_{cl}=0$, (6.1) implies $\sigma^*_{c1a}=(\sigma_2/\sigma_1)\sigma^*_{c2a}$, like (4.1b) in Section 4, so that the previous results about relative consumption volatilies remain largely unchanged. If consumption and leisure are substitutes ($U_{cl}<0$) and negatively correlated (as one may suspect in case of productivity shocks), the consumption of the old should actually be <u>more</u> volatile than the consumption of the young.

To say more about the correlation of consumption and leisure, a parametrized model is again needed. As example, consider a time-separable CRRA specification with a Cobb-Douglas aggregator over consumption and leisure, $U_t = u(c^1_t,l_t) + \beta u(c^2_t,1)$ with $u(c,l) = [c^\alpha l^{\gamma}]^{1-\sigma}/(1-\sigma)$, $\sigma>0$. The Cobb-Douglas aggregator implies a unit elasticity of substitution between consumption and leisure, which is necessary for a balanced growth. The efficient log-linearized decision rules can be derived as before.

One finds: (a) In the special case of log-utility (1) and v=0, the labor supply is constant and the wage-proportional allocation is efficient, as in the fixed labor model. (b) In the empirically most relevant case of >1 (low elasticity of substitution), negative productivity shocks induce an increase in labor supply so that consumption and leisure are negatively correlated. Since >1 implies $U_{cl}<0$, a variable labor supply implies that the consumption volatility of the young should indeed be less than the consumption volatility of the old, contrary to the "recovery" argument motivating this section.

Intuitively, the "recovery" argument fails because states of nature with low income are also states of nature in which the marginal product of labor is low. Hence, it would be inefficient to ask the young to work more when aggregate income is low. Overall, the section shows that the labor-leisure option of the young and the exclusion of the old from the labor market do NOT create a presumption that the young are better able to bear risk than the old. As shown above, the labor-leisure choice may be irrelevant (with log-utility) or even call for less risk-bearing by the young. Although other parametrizations may conceivably yield different results (I am not striving for generality here), the main model with fixed labor supply provides a reasonable approximation for the optimal allocation of risk.

6.3. CES Production

The Cobb-Douglas technology in the main model implies a unit elasticity of substitution between capital and labor. This is a significant restriction because a non-unit elasticity changes the relative riskiness of capital and labor incomes. To see this, consider a CES-production function

$$Y_t = [\ K_t + (1-)(A_t N_t)\ ]^{1/}$$

with elasticity of substitution 1/(1-); Cobb-Douglas is the limiting case
 =0. Technological progress is assumed permanent and labor augmenting (to
ensure balanced growth), as in the main model.

Economically most interesting is the case of an elasticity below one,
 <0. A positive productivity shock that raises the effective supply of
labor ($N_t A_t$) relative to the stock of capital will then reduce the labor
share in output relative to the capital share. This magnifies the effect of
productivity shocks on capital income and dampens the effect on labor
income relative to the Cobb-Douglas case. Hence, the old generation faces a
more volatile market income than the young, suggesting that the market
allocation may impose too much risk on the old. This is reinforced by the
fact that the efficient allocation calls for productivity shocks to be
absorbed by variations in the savings rate that reduce the volatility of
old and young consumption relative to the Cobb-Douglas case. The labor
augmenting nature of technical progress is important here, because it makes
the effective labor supply the main source of uncertainty.

Overall, a CES-technology with elasticity parameter below one may
theoretically justify government interventions that shift risk from old to
young, such as safe debt. An elasticity of intertemporal substitution below
one and a non-zero salvage value of old capital would, however, reduce the
relative consumption volatility of the old. An elasticity of factor
substitution below one is therefore by no means sufficient to justify
government intervention.[36]

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[36] A weighting of these factors would require an empirical analysis beyond the scope of
this paper. For the main model, I assume Cobb-Douglas technology because the qualitative
implications of alternative preference and policy parameters are most easily explained in
a Cobb-Douglas setting (yielding wage-proportional incomes), and because Cobb-Douglas is a
standard assumption in the production literature (e.g., Gomme and Greenwood, 1995). The
data are difficult to interpret. In annual U.S. data, the simple correlation between the
log capital share and the log output-capital ratio is actually negative (-0.31 for 1929-
1996, -0.33 for 1954-1996), contrary to what one would need to rationalize safe debt. But

7. Conclusions

The paper has examined the intergenerational sharing of macroeconomic risk in a stochastic OG model. In the market allocation without government, the old and the young share productivity risk through its impact on capital and labor income. A comparison of the market allocation with the set of ex ante Pareto-efficient allocations shows that the market allocation is generally inefficient. The market allocation of risk depends importantly on individuals' intertemporal elasticity of substitution--the willingness to spread risk over time--and not primarily on risk aversion. The optimal allocation requires that old and young bear consumption risk in inverse proportion to their respective relative risk aversion.

For plausible parameters, the market allocation without government impose too much productivity risk on the young and too little risk on the old. Observed government policies are difficult to interpret as efficiency-improving in this context. Notably, safe government debt shifts productivity risk from the old to the young, reinforcing the inefficiency.

A wage-indexed social security system has every different risk sharing properties that government debt. It is essentially neutral with respect to the allocation of risk. Thus, if the government engages in intergenerational redistribution, productivity-contingent transfer schemes such as wage-indexed social security seem preferable to government bonds. This finding has some relevance for the current social security reform debate, suggesting that a reduction in social security accompanied by an increase in net government debt would be welfare-reducing.

Overall, the widespread government practice of issuing essentially safe debt makes one wonder if politicians have been tempted to offer safe

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careful production studies have found evidence for a below-unit elasticity (e.g., Lucas, 1969); a more detailed analysis is best left for future research.

securities to current voters without considering--perhaps without recognizing--the implied risks for future generations. Judging from the public confusion about the merits of social security equity investments, which involves similar risk sharing issues, the hypothesis of imperfect understanding by politicians and/or their voters is perhaps not implausible.

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Table 1: Elasticity Coefficients with Debt and Social Security

Consumption of the old $(c^2{}_t/A_{t-1})$:

$$c2k = \quad + (1- \ ) \ s_v/s_2 - (\ + \ _{Rk} - \ _{dk}) \ s_d/s_2$$

$$c2a = 1- \ - (1- \ ) \ s_v/s_2 - (1- \ ) \ s_d/s_2$$

Consumption of the young $(c^1{}_t/A_{t-1})$:

$$c1k = \frac{+ (\ _{Rk} - \ _{dk}) \cdot s_d}{+ (1- \ - \ _d)}$$

$$c1a = 1- \ + \frac{(1- \ ) \ + (1- \ ) \ _d}{+ (1- \ - \ _d)}$$

Capital-labor ratio (k_t):

$$kk = \frac{+ (\ _{Rk} - \ _{dk}) \cdot s_d}{+ (1- \ - \ _d)}$$

$$ka = - \frac{- \ _d}{+ (1- \ - \ _d)}$$

Notation:

s_1, s_2, s_k = Ratios of worker-consumption to output, retiree-consumption to output, and capital to output, respectively.

$s_v = v/(1+a)/(1+n) \ s_k$ = Ratio of old capital relative to current output

$s_d = R^b/an \cdot d/y$ = Ratio of initial debt incl. interest to current output.

$_{Rk} = (1- \ )(1-v/R^k)$ = Elasticity of interest rates with respect to capital.

$_{dk}$ = Elasticity of debt issue $d(k)$ with respect to k (a policy parameter).

$= s_k/[(1- \ )(1- \ )]$ = Worker savings as share of wage income net of social

 security taxes.

$_d = s_d/[(1- \ )(1- \ )]$ = Initial debt as share of wage income net of social

 security taxes.

$$= \quad + \quad _{Rk}/(1- \ ) + (1- \ ) \ s_v/s_2 - (\ + \ _{Rk} - \ _{dk}) \ s_d/s_2$$

 where d is assumed small enough that >0.