

Social Security Reform and Financial Markets

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1. Introduction

Social security is in trouble. With declining population growth and rising life-expectancy, the cost of social security benefits is rising relative to payroll tax revenues. As result, the social security retirement fund is expected to run out around 2030.¹ Recently, the Advisory Council on Social Security (1997) proposed three different plans to address the problem. Interestingly, all three plans involve an social security investments in the stock market. This paper examines the impact of social security reform on financial markets, commenting specifically on the Advisory Council's proposals and more generally on the question of how to operate social security under uncertainty and under adverse demographic conditions.

The effects of social security on financial markets have long been the subject of debate among economists. The debate has generally focused on issues of intergenerational redistribution, using deterministic or certainty-equivalent economic models and taking for granted that government debt and social security trust funds involve essentially safe securities. The thrust of this literature is that social security reduces individual savings incentives, raises interest rates, and crowds out investment. The debate is about how much and under what conditions.

The Advisory Council proposals about equity investments raise significant new questions about the workings of social security under uncertainty. These questions are fundamentally about the allocation of macroeconomic risks between generations, about intergenerational risk-sharing.² This is an important issue because almost all policies affecting the

¹ The gap between Medicare costs and revenues is even worse. Though this paper focuses on retirement funding, the conceptual points about intergenerational risk sharing and intergenerational redistribution apply analogously to medical and disability funding.

² The literature in this area is much more limited. See Gale (1990), Bohn (1997), and the references therein.

intergenerational distribution of resources also have an impact on the allocation of risk and because risk and insurance are economically valuable. An analysis of redistributive policies is therefore incomplete without an assessment of the risk sharing implications. From this perspective, the Advisory Council has done us a favor in presenting proposals with such blatant risk-shifting implications that the issue cannot be avoided.

Social security reform therefore raises two key macroeconomic questions. First, the distributional question: How does a certain proposal affect the expected cash-flows between different generations and the government? Second, the risk-sharing question: Which generation is responsible for the shortfall (or receives a windfall) if the financing does not work out according to expectations?³ These twin questions cannot be answered separately. On the one hand, the allocation of risk occurs relative to the underlying expected distributional positions, making it difficult to examine risk sharing without also considering redistribution. On the other hand, policy plans rarely work out as expected, making it dangerous to neglect the allocation of risk.

A third question is about the possibility of disguised equivalencies between alternative policies. Policies that look very different at first sight and that involve very different policy instruments may have identical macroeconomic effects. A discussion of neutrality results simplifies the analysis of complex policy plans because it allows one to "strip away" the neutral components and to focus on the items that matter.

The paper is organized as follows. Section 2 reviews the basic principles of pay-as-you-go social security and the Advisory Council proposals. Section 3 examines neutrality results, Section 4 the effects of

³ The paper focuses on macroeconomic issues. Microeconomic questions, e.g., about redistribution across income levels and family structures or about the intra-generational sharing of mortality risk, are undoubtedly important for social security, but they are beyond the scope of this paper.

intergenerational redistribution, and Section 5 the effects of intergenerational risk-shifting. Section 6 compares alternative policies in the context of changing demographics. Section 7 concludes. Many technical results are presented in the appendix.

2. Social Security and Demographic Change

2.1. Pay-as-you-go and the No-Free-Lunch Principle

Some general comments about the nature of social security are appropriate because the social security reform debate is still plagued by misconceptions about what reforms are feasible. The basic principle of social security is to collect a tax from workers against the promise of retirement benefits. If social security taxes were invested by the government and returned to workers with interest, social security would essentially be a system of forced savings. Such a "fully funded" system would likely replace private savings, but--unless the forcing element is binding--have little effect on national savings. But if social security operates as a "pay as you go" (PAYG) system in which worker contributions are immediately transferred to current retirees, the reduction of private savings is not matched by government savings and national savings are likely to decline.⁴

In 1935, U.S. social security was designed as fully funded system but soon converted into a PAYG system. Benefits far in excess of prior contributions were granted to the initial generations of beneficiaries (Boskin et al., 1987). Since 1983, social security has accumulated a growing trust fund (about \$450bill. as of Sept.1995). Despite the impressive dollar value of this fund, it remains small relative to the total obligations of the social security system, which are somewhere between \$3.5 and \$11 trillion (Bohn,

⁴ A well-known caveat is that private savings might not fall if individuals increase their planned bequests (Barro, 1974). I will not reiterate this caveat below because the point should be obvious to readers interested in Ricardian equivalence.

1992; Feldstein, 1996). Hence, the U.S. social security system is still largely a PAYG system.

The welfare effects of a PAYG system depend crucially on the relation between market interest rates, population growth, and wage growth. The cost of social security benefits relative to payroll (the cost rate) is given by the average replacement rate (the ratio of benefits to wages) divided by the ratio of workers to retirees. At a given payroll tax rate, the relation between retiree benefits (a fraction of current wages) to the retiree's past contributions (a fraction of past wages) is therefore determined by the growth rate of wages and the population growth rate (determining the worker-to-retirees ratio). Except under extreme and practically irrelevant conditions, the implied "return" on social security contributions is below comparable market interest rates, making participants worse off than if they had saved for retirement at market rates. This point has sometimes been disputed with reference to "dynamic inefficiency," an extreme scenario in which individuals are so eager to save that market returns fall below the population plus wage growth rate. But the empirical work of Abel et al. (1989) has shown convincingly that dynamic inefficiency does not apply to the United States.

The intuition why social security must offer a bad deal to participants is simply the "no free lunch" principle. The first generation of PAYG beneficiaries receives benefits far in excess of their contributions plus interest. The below-market returns offered by a mature social security system are the necessary counterpart to the initial net transfers. The gap between market returns and the return on social security contributions is in effect a

perpetual tax that is exactly equal to the initial net transfers in present value terms.⁵

The size of this inherited burden is currently obscured by the federal government's misleading accounting methods. In the Social Security Administration's publications, expected receipts from future generations are counted as offsets against the cost of benefit payments to current social security participants, without acknowledging that such receipts would generate new obligations.⁶ Such accounting would be considered fraudulent if used by any private entity. If one treats the net present value of future payments to current participants as a liability, the estimated obligations of the U.S. social security system are staggering. According to my own very conservative estimates, just the obligations to current retirees are \$3.5 trillion as of 1990 (Bohn 1992). Using different assumptions, Feldstein (1996) obtains estimated liabilities as high as \$11 trillion. It would be a real contribution to the reform debate if the federal government were willing to recognize and officially quantify these liabilities.

Both the inevitability of below-market returns and the notion that we are paying for transfers made in the distant past have fundamental implications for social security reform:

1. One should not be surprised that each new generation is upset about the inherited burden of the PAYG system. By construction, a PAYG system offers

⁵ Stiglitz et al. (1997) provide an excellent discussion of this point. Feldstein (1995) makes a convoluted argument against this equality, treating the discount rate as a free parameter instead of applying the usual principle of discounting at market rates.

⁶ The political rhetoric justifying this accounting is simply incoherent. On the one hand, social security is praised as a system worthy of universal political support because it offers secure benefits "in exchange" for contributions. On the other hand, the promised benefits are not recognized as government obligations because payroll taxes do not create liabilities in the legal sense. This is inconsistent. If future receipts do not create obligations, return and "money's worth" calculations (as in Advisory Council, 1997; Gramlich, 1996) are meaningless. Moreover, if claims on social security are not government obligations, the option of abolishing social security overnight without compensation for past contributions (putting the old on general welfare) would be fair game in the policy debate. But if this "option" is considered outrageous--as I think it is--the retirees' entitlement to receive social security is in effect a government obligation and should not be denied.

a below-market returns to current and future participants. Hence, it is fundamentally impossible to raise social security's "money's worth" ratio to 100%; and it is misguided to attempt the impossible.⁷

2. Since those who received the initial transfers have long died, it is impossible to unwind the system without someone paying the price. In an ongoing social security system, each generation pays a fraction of the inherited burden and passes on the remainder to their successors. To end or "privatize" social security, some generation(s) would have to pay off the entire burden, either by suffering huge benefits cuts when old or by paying higher payroll taxes when young without promise of corresponding benefits.
3. In an ongoing social security system, the fraction of the inherited PAYG burden borne by the current generation depends on the gap between market interest rates and the population plus productivity growth rates. Not surprisingly, social security has become more unpopular as U.S. population growth and (since 1973) productivity growth have declined.

Overall, social security reform is about how to cope with the huge unfunded claims created by the existing PAYG system. No reform can realistically promise to make this inherited burden vanish. The real question is how to share the burden at a time of adverse demographic developments.

2.2. The Advisory Council's Proposals

Currently, the combined employer and employee contributions to the Old Age Retirement (OASI) and disability (DI) funds amount to 12.4% of covered payroll. After deducting 1.8% for disability insurance, 10.6% remain for OASI funding, which is the focus of the Advisory Council proposals. Current social security law provides for essentially constant tax rates and a constant

⁷ The Advisory Council's claims of success in this regard are misleading. In short, the Advisory Council assumes that bonds yield an annual real return of 2.3% while stocks yield a real return of 7.0%. Expected future trust fund positions and feasible benefits are computed on this basis. But to calculate present values, all benefits are discounted at a fixed rate of 2.3%--even the benefits funded by risky stock market investments. Not surprisingly, if a dollar is accumulated at 7.0% and then discounted at 2.3%, there is an apparent "free lunch" that allows the Council to claim fictitiously high "money's worth" ratios.

average replacement rate of about 32%.⁸ At the current ratio of 3.2 worker per retiree, OASI operates at a surplus (at a cost rate of about $32\%/3.2=10\%$) and accumulates a growing trust fund. But with rising life expectancy and declining population growth, the ratio of workers to retirees will decline sharply, causing a substantial rise in the cost rate. Over the 75 year horizon used by the social security administration (1996-2070), the average OASI cost rate is about 12.8%, leaving a 2.2% gap to the 10.6% revenues under current law.

The Advisory Council report contains three different proposals. All three claim to cover the gap between estimated cost and revenues. The first proposal, the Maintenance of Benefits (MB) plan, calls for an increase in income taxes on social security benefits, a small reduction in cost of living adjustments, increased coverage of state and local government employees, an increase in payroll taxes by 1.6% in 2045, and "consideration" of investing up to 40% of the social security trust fund in the stock market. Despite the cautious wording, the stock market investment is essential to the plan, because the plan would be unbalanced without a high rate of return on trust fund investments. Importantly, the plan maintains a fixed benefit formula, which means that risk of unexpectedly low or high investment returns is implicitly imposed on future generations of contributors.

The second proposal, the Individual Accounts (IA) plan, calls for an immediate increase in worker contributions by 1.6% of payroll to be invested in "individual accounts" that work like a defined contribution pension plan. The plan includes the same changes in the income taxation of benefits and inflation adjustments as the MB plan, it includes a phased-in changes in benefits that reduce traditional defined benefits by 30%, and calls for an

⁸ To summarize the financial status of social security, I am using rounded numbers and ignore some complicating details that would not change the overall assessment.

accelerated increase in the retirement age. The plan is designed so that the sum of (reduced) defined benefits plus expected returns on individual accounts equals the benefit level under current law.

The third proposal, the Personal Security Accounts (PSA) plan, calls for 5% of contributions to be diverted to individual "personal security" accounts, a new 1.52% "transitional" tax to be imposed from now to the year 2070, and the same changes in the income taxation of benefits and inflation adjustments as the other plans. After a phase-in period, defined benefits are reduced sharply so that retirees will have to rely largely on their individual accounts. The transitional tax is needed because social security will run a deficit after the 5% diversion until the benefit reductions are phased in. The shortfall is supposed to be bond financed and the bonds to be retired by 2070 from the transitional tax.

From a generational perspective, the essence of social security is that the young make contributions and the old receive benefits. Policy changes that reduce the value of social security-related net payments to retirees are therefore best interpreted as benefit reductions (see Proposition 1 below). This includes increases in the retirement age and the taxation of benefits. From this perspective, the IA plan is the most straightforward of the three. Contributions are immediately and permanently increased by 1.6%. Total expected benefits are reduced by about 0.8% of payroll (because of increased taxation, changes in inflation indexation, and changes in retirement age) and made contingent on the individual account returns.

The PSA plan calls for an immediate tax increase of similar size as the IA plan (by 1.52% versus 1.6%) and it calls for larger individual accounts, funded by 5.0% of payroll versus 1.6% under IA. Without itemizing the benefit changes, the comparison to IA makes clear that the PSA plan must be reducing

the cost of defined benefits by an extra 3.5% (5%-1.52%). A key difference between the IA and the PSA plan is the transitional nature of the tax increase under PSA. After 2070, the IA system continues to collect 12.2% of payroll, of which 1.6% go to individual accounts and 10.6% to the "pooled" defined benefits account, while PSA contributions fall back to 10.6% of payroll, of which 5% go to individual accounts and only the remaining 5.6% to the defined benefits pool. The distinction between pooled and individual accounts deserves special emphasis in this context because the size of the pooled fund turns out to be critical for evaluating the plans' effects on interest rates, savings, and intergenerational redistribution.

The MB plan's transitional features go in the opposite direction. Contribution rates are held at the current level until 2045, when they are raised by 1.6% as under the IA plan. The MB and IA plans therefore call for equally high contributions in the long run; but under MB, all contributions go into the defined benefits account. To cover expenses prior to the tax increase in 2045, the plan relies heavily on high returns from stock market investments (penciled in for 0.8% of payroll).⁹

For purposes of intergenerational risk-sharing, a key question is what will happen if investment returns turn out to be above or below expectations. Here the IA and PSA plans differs drastically from MB. Under IA and PSA, the risk of unexpectedly high or low returns is borne by individual participants. The MB plan is silent about this question. But since the thrust of the plan is

⁹ Two provisions deserve separate comment. First, the IA and PSA plans call for an indexation of the retirement age to life-expectancy. This is a quite elegant way to eliminate a major source of cost increases, because without indexation, rising life-expectancy would require ongoing tinkering with the systems' tax and benefit provisions. Second, with regard to the increased coverage of state and local government employees, recall that social security is a bad deal once the initial generation has collected more in benefits than it paid in. By including more workers--this time, state and local employees--the inherited burden is shared among a larger number of participants, cleverly reducing the percentage cost for the existing participants. This financing trick is not new, of course. Social security has become more and more inclusive over time.

to maintain defined benefits, a reasonable interpretation is that all risks are effectively borne by future generations of contributors. This interpretation is also consistent with the fact that a tax increase is scheduled in 2045 when the trust fund is exhausted in expectation. Unexpectedly low or high returns would most likely trigger an advancement or delay in this tax change.¹⁰

The following sections will examine the macroeconomic effects of social security reform in general and the above proposals in particular. Since the focus is on intergenerational issues, I will use an overlapping generations model as the conceptual framework, following Samuelson (1958) and Diamond (1965). The objective is to address three key questions: Which policies matter--versus being neutral? Which policies redistribute across generations? Which provisions affect the intergenerational allocation of risk? I will address these issues in turn.

3. Neutrality Results

Sometimes, alternative government policies are economically equivalent even if they are presented in very different ways. For smart individuals, policy differences are real only to the extent that they affect consumption opportunities. Equivalent policies can be identified by examining the net cash-flows between the government and individuals. Such an accounting for cash flows provides several interesting insights about social security reform and other policy issues:¹¹

¹⁰ There may be an asymmetry, however, in that the old may well demand extra payments in case of unexpectedly high returns, while benefit cuts are less likely in case of low returns. The MB plan is silent about this issue, too. Under the IA and PSA plans, one may similarly wonder if the government would come under political pressure to "bail out" the old if the stock market falls. The distributional implications of such bailouts should be obvious. If one seriously suspects that the young would bear most losses while the old participate in the gains, an options pricing approach would be required to model plans' impact. This might be an important topic for future research.

¹¹ Precise statements of the following propositions are presented in the appendix in the context of an overlapping generations model; see also Stiglitz (1983).

Proposition 1: A higher tax on the old is equivalent to a cut in social security benefits. This equivalence is has been exploited routinely since the 1983 social security reform and it is part of all three Advisory Council proposals. All three plans propose an increased taxation of benefits combined with an transfer of the tax receipts to the trust fund.

This proposition applies not just to the taxation of social security benefits but also to the general tax reform debate. For example, a revenue-neutral shift from income taxes to consumption taxes is bound to increase the taxation of retirees who consume but not work. Consumption taxes reduce the purchasing power of given social security benefits and are therefore economically equivalent to a benefit reduction.

Proposition 2: A trust fund to pay for the contributors' own future retirement has no real effects. This is because such a trust fund is a virtually perfect substitute for private savings. This neutrality proposition is more narrow than Ricardian neutrality (the neutrality of intergenerational redistribution, which requires Barro's (1974) assumptions about bequests), but similar to Stiglitz (1983) result that a shift in the timing of taxes is neutral, if all tax changes involve the same generation.

An important condition for Proposition 2 is the absense of liquidity constraints. With liquidity constraints, a trust fund would increase aggregate savings because constrained consumers would not be able to reduce their private savings in response. But by definition, liquidity-constrained consumers discount future income by more than the market interest rate. A trust fund would then reduce welfare in this case rather than being neutral. Hence, liquidity constraints do not provide an argument in favor of trust funds.

Proposition 2 applies directly to the individual accounts proposed by the IA and PSA plans. Since individuals receive the returns to their own contributions, the present value of benefits from individual accounts must equal the value of contributions. Abstracting from liquidity constraints, such accounts are irrelevant for macroeconomic analysis. At best, they complicate government accounting without doing harm. At worst, they serve as vehicle for accounting gimmicks (e.g., if assumed 7% returns are discounted at 2.3%) and they may impose forced savings upon liquidity-constrained consumers.

The forced savings issue raises some serious questions about the philosophy underlying the current social security reform debate. With rational consumers, forced savings cannot convey welfare benefits, because consumers could save on their own if they wanted to. It seems that some type of paternalistic argument is required to rationalize mandatory individual accounts and other proposals that involve forced savings.

Proposition 3: A trust fund financed by the young to maintain unchanged future retirement benefits is equivalent to a benefit reduction. This is a corollary to Proposition 2. If benefits are paid from a trust fund built up by the generation receiving the benefits, individuals are financing part of their own retirement benefits. The macroeconomic effect is as if the trust fund were never created and the benefits reduced accordingly.

This proposition applies to 1983 social security reform and the IA plan. The 1983 social security reform raised payroll taxes to accumulate a trust fund, but without promising higher future benefits. The IA plan similarly promises to maintain unchanged total benefits (at best) but requires a payroll tax increase.

Proposition 4: Trust fund investments in the stock market are neutral, if and only if the old generation bears the risk of stock price fluctuations. This proposition highlights a key difference between the three Advisory Council proposals. Under the IA and PSA plans, the proposition is satisfied because individuals bear the risk of stock price changes in the accounts set up on their behalf. Under the Maintenance of Benefits (MB) plan, however, promised benefits are independent of the trust fund performance. Unless retirees are subjected to some special tax/transfer scheme contingent on stock returns--which is difficult to imagine--the stock market risk falls on future generations. Hence, the MB plan is not neutral with respect to intergenerational risk sharing.

Proposition 5: The issue of government bonds in exchange for terminating social security can be economically neutral, but only under unrealistic conditions. If the last generation of social security contributors--the generation that pays benefits to the old without itself receiving benefits--is given government bonds with identical payoffs, its consumption opportunities remain unchanged. Similarly, all following generations are unaffected if government debt is perpetually rolled-over from generation to generation at a level equal to the previously scheduled social security benefits. Such a neutral scheme must satisfy two conditions: First, to be distributionally neutral, the debt must grow in expectation at the rate of population plus wage growth. Second, in a stochastic setting with wage-indexed social security benefits, the government must issue wage-indexed bonds to mimic social security. Otherwise, the substitution of bonds for social security benefits has a non-neutral impact on the allocation of risk.

This proposition helps to explain why none of the existing plans to privatize or scale-down social security is neutral. Most plans--including the

PSA proposal and Feldstein's (1996) plan--call for traditional debt rather than wage-indexed claims and they call for the debt to be paid off in finite time. Such plans in effect call for a significant redistribution from the transitional generations that pay off the debt to future generations.¹²

Three caveats about these propositions are in order. First, the above results only apply to the comparison of alternative policies that are implemented with certainty. From a political economy perspective, it is a non-trivial question which policy plans are more or less likely to be implemented. If an "entitlement" to social security benefits is somewhat more or less secure than a claim represented by a Treasury bond, the two would not be equivalent ex ante (see Bohn, 1992). Second, liquidity constraints are relevant for all policy changes involving trust funds, as discussed under Proposition 2. Third, I have ignored the issue of distortionary taxation, on which I will comment below.

4. Intergenerational Redistribution

Two brief comments on intergenerational redistribution should be sufficient, because the effects depend mostly on how individual savings respond to changes in government policy, the topic of another session of this conference.

First, in a standard overlapping generations framework, any permanent policy shift that increases the amount of redistribution from young to old will increase the level of interest rates and put the economy on a growth path with lower per-capita income. Increased redistribution reduces workers disposable income and reduces their need to save for old age. Lower savings

¹² Feldstein (1995) argues that such redistribution is "welfare-improving." This is technically correct if one measures welfare by the present value of all generations' consumption and applies a low enough social discount rate. But the "welfare" label is potentially misleading, because such "welfare" improvements are not Pareto-improvements. The transitional generations are worse off. Feldstein's (1995, 1996) appeals for privatization are therefore best interpreted as expressing a personal value judgment that future generations deserve more resources.

raise the equilibrium interest rate and crowd out capital investment. This reasoning provides an immediate comparison of the three Advisory Council proposals. In the 22nd century--after all transitional provisions have expired--payroll taxes for "pooled" defined-benefit accounts are about 12.2% under the MB plan (1.6% more than now), 10.6% under the IA plan (unchanged), and 5.6% under the PSA plan (5% less). Since the individual accounts are neutral, these percentages indicate the relative scale of the plans' intergenerational redistribution and hence, their relative impact on savings, capital accumulation, and interest rates.

Second, reduced population growth per se has quite positive macroeconomic effects. At a given payroll tax rate, a slowdown in population growth raises the equilibrium capital-labor ratio, which reduces the real return on capital while increasing the wage rate. Reduced population growth is therefore likely to reduce interest rates and to raise per-capita incomes. These positive effects of reduced population growth should not be ignored in the social security reform debate.

5. Intergenerational Risk-Sharing

Among the Advisory Council's proposals, the idea of trust fund equity investments is perhaps the most challenging to evaluate. Fortunately, we have already established that equity investments in individuals accounts are economically neutral. The main issue is therefore how to evaluate proposals such as the MB plan that shift investment risks to future generations.

An assessment of such proposals requires a study of how macroeconomic risks are allocated across generations and of how the government affects the sharing of such risks. Given the focus on intergenerational issues, a stochastic overlapping generations model is the natural tool for analysis. This section describes such a model and its main implications. (For details,

see the appendix and Bohn, 1997.) To anticipate, trust fund equity investments are a surprisingly good idea in principle, though there are many caveats and one has to be careful about the implementation.

5.1. A Framework for Economic Analysis

My results about the macroeconomic implications of alternative trust fund investment policies are based on a standard two-period overlapping generations model. Each generation works, saves, and consumes when young, and consumes all its income when old. Savings are invested in capital or government bonds. Equity securities represent a claim on uncertain future capital income. Bonds offer a safe return. Output is produced with labor and capital and it is used for consumption, capital investment, and government spending. All these are standard assumptions. Going beyond the standard setting, the model includes a social security system with funded and PAYG components, wage-indexed benefits, and a trust fund that can be invested in stocks or bonds. "Regular" government operations include lump-sum taxes on young and old (separate from payroll taxes), real spending, and government debt.

To examine alternative trust fund investments, explicit assumptions about the sources macroeconomic risk are needed. I assume that capital income is risky because of uncertainty about future productivity (productivity risk) and because of uncertainty about the resale value of capital goods (valuation risk). Under standard assumptions about production (Cobb-Douglas technology), productivity risk has a common impact on future output, wages, and capital income. Without another source of risk, all these variables would be perfectly correlated. While a perfect correlation is clearly too extreme (explaining why I assume valuation risk a second source of risk), capital and labor incomes

are indeed highly correlated in the long run.¹³ This has an immediate and perhaps surprising implication: Equities are a much more natural hedging instrument for a wage-indexed social security system than government bonds. (Of course, wage-indexed securities would be even better from this perspective.)

Finally, assumptions about government policy are needed because there are too many policy choices to examine them all. To focus on alternative social security investments, I assume that most policy variables grow at the same rate as the young generation's wage income. This is assumed for government debt, real spending, social security benefits, other taxes and transfers to the old, and the overall trust fund balance. This leaves payroll taxes and regular taxes on the young as the variables that fluctuate as needed to satisfy the social security and the general government budget constraints, respectively.

Given a constant replacement rate and a constant target for the trust fund balance relative to wages, payroll taxes must rise (or can fall) whenever past trust fund investments have yielded particularly low (or high) returns. This is the sense in which the young bear the risk of social security investments. Similarly, regular taxes on the young must rise (or can fall) whenever government debt has increased (or decreased) relative to the debt-income target. Importantly, safe trust fund investments and safe debt will not generally yield stable tax rates, due to the uncertain level of future wages.

¹³ See Baxter and Jermann (1997). Shiller's (1993) finding of a low short term correlation (in 5-year growth rates) is of limited relevance in this context, because correlations at generational frequencies are at issue. Baxter and Jermann's finding of cointegration between capital and labor income implies a high long run correlation.

5.2. Results

The main positive results are about the return on capital, the equity premium and the safe interest rate. Under reasonable simplifying assumptions, one can show that trust fund investments in equities unambiguously reduce the equity premium. The economic argument is that trust fund equity investments reduce the productivity and valuation risks carried by the old generation. On the margin, the old are therefore less willing to accept a lower return on bonds than they are expected to obtain on stocks.

With regard to other macroeconomic effects, it is notable that trust fund equity investments have negligible effects on savings and on the expected return on capital, provided the total trust fund balance remains unchanged. The reduced equity premium therefore implies a higher safe interest rate.

Quantitatively, the effects of alternative trust fund investments are small for realistic parameter values. This is because even a trillion dollar trust fund amounts to only a small share of U.S. households' total assets. For simple benchmark assumptions laid out in the appendix, the equity premium under the MB plan is about 10 basis points lower with bond investments than with equity investments, and the safe interest rate is higher by the same amount. The PSA plan with its transitional debt would reduce the equity premium by about 15-20 basis points. (The safe interest rate would nonetheless fall under the PSA plan because of the reduced scale of intergenerational redistribution; see Section 6.)

On the normative side, the overlapping generations model yields four general insights about the welfare effects of alternative policies:

1. Policy changes may make all generations better off in the Pareto sense, because the intergenerational allocation of risk is generally inefficient in the absence of government intervention. Unborn generation cannot engage in risk-sharing contracts, but the government can do so on their behalf.

2. There is a crucial difference between efficient risk sharing and Pareto-improvement. More efficient risk-sharing has the potential for making all generations better off, but the generations carrying more risk must receive more transfers in expectation.

This point is immediately relevant for the MB plan, which shifts risks to future generations. If higher expected stock returns (versus bond returns) are exploited to justify lower trust fund contributions rather than to reduce future payroll taxes, the current generations are made better off at the expense of future generations.

3. The efficient allocation of risk depends on a number of considerations. One issue is consumption-smoothing, the fact that the young can spread the effect of temporary income shocks over more periods than the old. This enables the young to bear more income uncertainty than the old. A second issue is dynamic hedging, especially in the context of productivity uncertainty. Unexpectedly high productivity growth tends to raise interest rates and future returns on equity by reducing the effective capital-labor ratio. A high exposure to productivity risk enables the young to better exploit such time-varying investment opportunities. This argument applies even if productivity shocks are permanent, making consumption smoothing arguments irrelevant, provided the young have a sufficiently high intertemporal elasticity of substitution. Finally, the young should obviously carry more risk if the old are more risk averse.
4. Policy changes that reduce the variance of both generation's consumption are generally efficiency-increasing. For a given volatility of aggregate consumption, the sharing of generation-specific risks is therefore a straightforward way to improve welfare. Valuation risk is one example.

The practical implications for trust fund investments in equities depend on a comparison of the efficient allocation of risk with the existing allocation. The potential for sharing generation-specific risks provides an immediate argument for social security equity investments. In the market allocation, the old are exposed to the risk of changes in asset prices. This

risk can be shared with subsequent generations through social security equity investments.

With respect to productivity risk, the welfare gains from shifting trust fund balances into equities are more difficult to assess. The efficient allocation depends on all the considerations mentioned above. Existing government debt already provides the old with safe securities that reduces the volatility of their income and thereby reduces the benefits of additional risk-shifting from old to young. It would take a quite ambitious empirical study to determine whether the old are currently too little or too much exposed to productivity risk, nothing less than a comprehensive survey of all relevant sources of risk affecting U.S. households of different ages.

If social security equity investments ARE efficient, one still has to be careful about the distribution of the efficiency gains to ensure that a shift to equities is a Pareto-improvement. The trust fund expects to gain the equity premium times the amount shifted to equities. The old generation (which holds more bonds and fewer equities) gains safety but loses the equity premium. Future young generations bear increased risks. So, who should receive the trust fund's expected gain?

One allocation scheme is to reduce initial trust fund investments so that the balance after expected earnings remains unchanged. An alternative is to leave investments unchanged so that the higher returns reduce expected future payroll taxes. In the first case, the old receive all the trust fund's expected gains plus lower risk, while future generations are stuck with increased risk. This is not a Pareto-improvement. In the second case, future generations receive an expected gain in exchange for taking risk while the old receive a lower but safer income. This allocation is unambiguously Pareto-improving: The equity premium exactly compensates the old for switching from

equities to bonds, and the young are better off because they are better able to carry productivity risk. (Otherwise the shift would not be efficient.) If the young are strictly better off, a slightly higher expected payroll tax and a slightly reduced initial trust fund investment would also yield a Pareto-improvement. Nonetheless, the basic insight is that the bulk of the trust fund's expected gain must be given to the young as compensation for risk. Unfortunately, the MB plan seems to propose the very allocation scheme that is not Pareto-improving.

5.3. Discussion

The notion that the U.S. social security trust fund should buy stocks is clearly a radical idea. Hence, a number of additional issues should be considered before any final judgment is made. Potentially relevant items include distortionary taxation, an imperfect correlation between the returns on publicly traded equities and on capital, the uneven distribution of stock holdings across households, bequests, and the equity premium puzzle. (See also Diamond, 1996; Stiglitz et al., 1997.)

Distortionary taxes are an important complication because they limit the government's ability to enter into risk-sharing contracts on behalf of the unborn. If uncertainty is resolved in a way that the government faces large payments to the old, the government would have to collect from the young by imposing high, and hence highly distortionary taxes. To minimize tax distortions, the government should issue securities that result in "tax-smoothing," i.e., allow a financing of government spending and debt service with minimal variations in tax rates (Bohn 1990). With regard to securities that are positively correlated with the tax base--such as equities--tax-smoothing implies that the government should take a short position. In contrast, social security equity investments would represent a long position.

A reconciliation of the tax-smoothing and the intergenerational risk-sharing perspectives is well beyond the scope of this paper. Even if risk-sharing considerations justify an overall government short position in safe assets, it is not clear that government's current short position in safe assets is inefficiently small--which is the claim one would have to make to establish the optimality of trust fund equity investments.

The case for trust fund equity investments is also weakened if the return on publicly-traded equities is only imperfectly correlated with the return on the nation's overall wealth. This is a significant limitation, because total U.S. wealth far exceeds the capitalization of the stock market. According to the Federal Reserve Flow of Funds accounts, the U.S. net wealth at the end of 1993 amounted to almost \$17 trillion, excluding consumer durables. More than half of this--about \$10 trillion--represents the value of residential real estate and land, while plant, equipment, and inventories amount to less than \$7 trillion. Even if the social security trust fund buys a significant share of the stock market (a claim on plant, equipment, inventories, and some real estate), the old generation would still hold the majority of national wealth.

A third complication is that share holdings are quite unevenly distributed across households (Mankiw and Zeldes, 1991). This unevenness is not well understood. Plausible reasons include liquidity constraints, differences in income, fixed cost of investing in stocks, and/or differences in the degree of risk aversion. Heterogeneous risk aversion is perhaps the most troublesome of these complications. In a market allocation, risky assets will presumably be held by the most risk-tolerant individuals, while more risk-averse individuals will hold safer assets, yielding an efficient cross-sectional allocation of risk. Government holdings of equities would expose all

individuals to equity risk in proportion to their tax liabilities without allowing for any sorting by risk aversion.

A related issue is the equity premium puzzle, the fact that the equity premium is higher than standard models would predict (Mehra-Prescott, 1985). A survey of the policy implications goes well beyond the scope of this paper; see Bohn (1993, 1995). But it seems dangerous to advocate government investments in equities because we do not understand the phenomenon--as opposed to taking the more prudent position of not betting against the market. Moreover, the safe real rate has been much higher since the mid-1980s than in the period studied by Mehra and Prescott (about 3.5% for 1983-95 versus 0.8% in Mehra-Prescott). Hence, the equity premium puzzle does not provide a convincing argument for or against any particular policy.

Bequests and other transfers within families are obvious substitutes for government intervention in the area of intergenerational risk sharing. The results of Altonji et.al. (1996) suggest, however, that such private risk sharing is very incomplete. In any case, bequests would at most neutralize some of the government's risk-shifting policies but they are unlikely to overturn the above results.

Finally, the underlying model could be generalized in many directions, e.g., to reflect additional sources of uncertainty such as government spending shocks and inflation; to model the fact that post-retirement social security benefits are inflation-indexed rather than wage-indexed; or to include the inflation risk inherent in government bonds. But such extensions are unlikely to overturn the basic results about risk sharing. For example, a more elaborate model of inflation risk and of the partially inflation-indexed nature of the social security might soften the distinction between "safe" debt and "risky" (wage-contingent) social security. But it would not change the

more fundamental insight that the government affects the intergenerational allocation of risk by supplying safe assets to the old.

Overall, it is difficult to make a definite case for social security investments in the stock market, but it also surprisingly difficult to make a definite case against such investments.

6. Policy options at a time of demographic change

Since much of the current policy debate is motivated by demographic pressures, social security reform proposals are best compared in a setting with declining population growth. Such a setting is also interesting because it raises questions about the appropriate benchmark for policy reforms: With declining population growth, historical tax and replacement rates are no longer feasible. To compare some basic policy options, I consider a permanent, one-step decline in the population growth rate at some date $t=0$ in the context of a simple calibrated overlapping-generations model.¹⁴ Despite the calibration, the focus should be on the qualitative differences of the principal alternatives, not on the raw numbers. In Tables 1-2 and Figures 1-5, numbers are provided for illustrative purposes and to indicate rough orders of magnitude, but the model is definitely not suitable for precise numerical predictions. With these caveats, here are some policy options.

The two most basic responses to reduced population growth are to cut benefits or to raise taxes. Option 1 freezes the payroll tax rate (at 10%) and cuts the replacement rate to satisfy the PAYG condition that the replacement rate equals the tax rate times the worker-to-retirees ratio. Assuming a drop

¹⁴ In the U.S., the decline in population growth rates did not happen instantaneously and the proposed policy reactions are projected to take several generations. But to understand the conceptually different implications of the alternative proposals, it is sufficient to consider a simpler, one-time shift in population growth.

in annual population growth from 1.5% to 0% and a generational period of 30 years, the replacement rate must fall from 32% to 20.5%.

Option 2 is to raise the payroll tax rate enough to maintain an unchanged replacement rate of 32%; this requires a tax rate of 15.6%.

Option 3 is to maintain constant benefits but with an earlier and smoother path of tax increases. Specifically, let generation $t=1$ be the first non-growing cohort, and assume that generations 0 and 1 are supposed to be taxed at a common rate. Then period-0 taxes must cover current cost plus a "surcharge" that allows the higher cost in period $t=1$ to be financed without a tax increase (see Table 1 for the data).

This option captures the essence of the 1983 social security reform, notably the principle that social security was supposed to show a non-negative trust fund balance with unchanged tax and benefit rules for a finite forecast horizon. Option 3 provides an incomplete solution, however, because as time passes and the forecast horizon extends, it becomes apparent that the period $t=2$ cost rate exceeds the tax rate and that the trust fund is going to be exhausted.¹⁵ Society again faces the choice between further tax increases (Option 3A) and benefit cuts (Option 3B). I interpret Option 3B as a stylized version of the Advisory Council's IA plan. This is because the IA plan calls for various benefit cuts but no higher taxes (disregarding the economically neutral individual accounts). Since the MB plan calls for a tax increase in 2045 (meaning, with a delay of about two generations) I interpret Option 3A as a stylized representation of the MB plan.¹⁶

¹⁵ Alternatively, in case of the 1983 reform, the scale of the demographic shift was perhaps not fully known; in any case, the story serves to motivate an intermediate level of taxation, below the Constant Benefits but above the Constant Taxes level.

¹⁶ Since the MB plan calls for some benefit reductions, too, one might interpret it as being in between Options 3A and 3B; the figures show the basic, stylized alternatives for clarity.

Both Options 3A and 3B treat generations t 2 different than generations 0 and 1. If one wants to give all generations the same "deal" in terms of tax and replacement rates, a natural alternative is Option 4: Keep tax rates constant at the Option 3 level and reduce benefits to the point where the trust fund becomes a permanent endowment. Interestingly, Option 4 is economically equivalent to Option 1, the straight benefit cut. This is because each generation replenishes the trust fund and because self-funded trust funds are neutral. By comparison, Option 4 shows that Options 3A and 3B are "better" for generation 0 and/or generation 1 only because they treat future generations worse.^{17,18}

Finally, consider a stylized version of the PSA plan, Option 5. Option 5 calls for reductions in tax and replacement rates combined with transitional taxes between now and 2070 (roughly, generations 0-2). The need for transitional taxes follows directly from the no-free-lunch principle: The scale of intergenerational redistribution can only be reduced if the inherited burden of PAYG social security is paid off during the transition.

Overall, Figure 1 shows that most policy options involve benefit cuts. Options 3B and 4 imply smaller benefit cuts than Option 1 because they include a "small" tax increase (see Figure 2). Options 3B and 4 differ only in the timing of benefit reductions: Delayed reductions must be larger than earlier ones. Similarly, Options 2 and 3A offer timing versus scale differences on the

¹⁷ For all options with trust funds, the need for higher taxes and lower benefits is reduced if one assumes a high rate of return on the trust fund. This calculation is apparent in the MB plan and it may explain much of the practical appeal of stock market investments in many privatization proposals. But as shown in the previous section, such higher returns cannot be exploited by current contributors without beggaring the future generations that bear the corresponding risks. The numerical illustrations assume that the trust fund holds wage-indexed claims earning the same rate of return as capital (which is an appropriate benchmark in a stochastic environment), without taking the high return as an excuse for reduced trust fund contributions.

¹⁸ To be precise, all three Advisory Council plans provide for a positive trust fund balance at the end of the Council's 75-year planning horizon. (I thank Ned Gramlich for pointing this out.) I interpret these balances as buffer stocks that are too small to significantly affect the interpretation. On account of the positive 75-year ahead balance, one may interpret the actual IA plan as being somewhere in between the stylized Options 3B and 4.

tax side. Only Option 5 calls for reduced tax rates at a time when benefits are already under pressure. Figure 3 shows the trust fund balances and/or transitional debt implied by the alternative options: a one-generation fund under Options 3A and 3B, a permanent fund under Option 4, and transitional debt under the PSA-type plan.

The macroeconomic implications are shown in Figures 4-5. At a given payroll tax rate (Option 1), reduced population growth per se raises the capital-labor ratio and reduces the real return on capital.¹⁹ In comparison, Option 2 reduces the supply of savings and therefore leads to relatively higher (but still falling) interest rates and a lower long-run growth path of per-capita income. The MB- and IA-type plans are in between, while the PSA-type option goes in the opposite direction, yielding much reduced interest rates. Throughout, safe interest rates moves similarly (not shown). Sharply reduced interest rates under the PSA-type option might seem counterintuitive, given the transitional debt. But by design, the debt is much less than the value of social security benefits that it replaces.

Figure 5 shows the corresponding time series of wages, which are essentially a mirror image of Figure 4. The more a policy raises savings and reduces the interest rates, the more it increases the marginal product of labor. In all cases, wages rise relative to the previous trend because the decline in population growth reduces the supply of labor relative to the supply of capital.

Table 1 combines the policy options and their macroeconomic implications and provides a set of generational net cost associated with the alternative

¹⁹ I should note here that the calibration assumes a unit intertemporal elasticity of substitution. This implies that declining population growth per se does not affect the savings rate for given intergenerational transfers, which is a convenient and empirically plausible simplification. A different elasticity value would not significantly affect the relative comparisons but it would affect the absolute changes. The declining interest rates under Option 1 (and the equivalent Option 4) are entirely due to the demographic changes. Under the other options, interest rate movements relative to Option 1 are policy-induced.

policies. For each generation (t), the net cost of social security is defined as the payroll tax minus the present value of retirement benefits in the following period ($t+1$), all expressed as a fraction of period- t wages. The results are perhaps striking: Options 1 and 4 impose equal cost on all generations, despite all the demographic and economic changes. The MB- and IA-style Options 3A and 3B reduce generation 0's net cost at the expense of future generations. The PSA-style Option 5 does the reverse, imposing larger cost on generations 0-2 for the benefit of generations $t \geq 3$.

Why do Options 1 and 4 imply equal cost despite the reduced replacement rates? The economic argument has two parts. First, since social security is wage-indexed and since lower population growth raises the wage rate, the reduced replacement rate overstates the actual cut in benefits. Second, the decline in interest rates associated with lower population growth raises the present value of future benefits. For the case of Cobb-Douglas technology, these two effects exactly cancel out the direct effect of reduced benefits.

While the relative comparisons across options presented in Table 1 are quite robust with respect to changes in the assumptions, the comparisons over time should not be presented without a number of caveats. First, equal net cost does not imply equal utility. *Ceteris paribus*, falling interest rates imply reduced consumption in retirement, a negative income effect. On the other hand, future generations are better and better off because of productivity growth.

Second, the wage and interest rate paths in Figures 4-5 are based on a closed economy model. The closed economy assumption is important because the decline in interest rates and the increase in wages would be less if U.S. savers invested abroad. In the extreme case of an infinitely elastic foreign demand for U.S. savings, all interest rate and wage effects would vanish. This

is the Small Open Economy case in Figures 4 and 5, for which Table 2 shows the implied net cost of social security. (Tax rates, replacement rates, and trust fund levels are also shown because for options involving trust funds or borrowing, the set of feasible tax rates changes somewhat.) The lack of macroeconomic adjustment makes social security look much worse, but the relative comparisons across options remain largely unaffected.

The U.S. economy is certainly not small, and domestic savings and investment are empirically highly correlated (Feldstein and Horioka, 1980). Hence, Figures 4-5 and Table 1 still provide a good benchmark. To the extent that capital is mobile, however, the above arguments for lower interest rates translate directly into arguments for higher U.S. capital outflows. All interest rate and wage movements would then be somewhat smaller than indicated in Figures 4 and 5.

Despite the large size of the U.S. economy, the trends towards global capital movements and increased openness are critical issues for the future of social security--perhaps the most critical and also the most underrated ones. As reduced U.S. population growth reduces the returns to domestic capital investment, U.S. workers have huge incentives to invest their retirement savings abroad, most likely in developing countries where high population growth will generate attractive investment opportunities. Given these demographic trends, globalization and increased investment in emerging markets are natural phenomena. The magnitude of such capital flows is difficult to predict because U.S. investors' willingness to go abroad may depend sensitively on a variety of economic and political developments in the capital-receiving countries. But such capital flows determine whether the future of social security will be more like Table 1 or like Table 2.

Three other modeling issues should be mentioned. First, bequests and the nature of bequest motives are potentially important. To the extent that different generations are altruistically linked through Barro (1974) style bequest motives, the effects of social security may be reduced. But unless all families are altruistically linked, such linkages will only moderate the effects discussed above, but not eliminate or overturn them.

Second, liquidity constraints may matter for the economic effect of trust funds and individual accounts. By definition, liquidity constrained consumers prefer to consume more and save less than their credit limit allows, i.e., they discount future transfers at a higher rate than the market interest rate. Hence, they are unambiguously worse off under a plan with a trust fund or individual accounts than under an otherwise equivalent plan with lower taxes and benefits. (They would, e.g., prefer Option 1 over Option 4). Liquidity constraints also imply that increased government savings in a trust fund are not automatically neutralized by reduced private savings. A trust fund may therefore raise national savings, lower interest rates, and raise long run output as compared to an "equivalent" plan without trust fund. But since the liquidity-constrained consumers are worse off and everyone else is indifferent, the increased savings and higher output do not provide arguments for a trust fund. Instead, the case of liquidity constraints offers a nice example of a welfare-reducing increase in output, i.e., an argument against social security trust funds and against individual accounts.

Finally, note that I treat population growth as deterministic. In principle, one might think of demographic uncertainty as a source of risk that raises similar risk-sharing questions as the uncertainty about productivity

growth.²⁰ If the decline in fertility over recent decades is viewed as an unexpected shock, risk-sharing considerations suggest that the impact should be shared among all generations, perhaps including current retirees. This is another interesting issue for future research.

7. Conclusions

The paper has examined the effects of alternative social security reform proposals on stock and bonds markets, with special emphasis on the recent Presidential Advisory Council plans. The key issues are intergenerational redistribution and intergenerational risk sharing.

The three Advisory Council plans redistribute resources across generations in very different ways. The intermediate IA plan essentially amounts to reducing benefits in response to adverse demographics. Since it calls for an unchanged fraction of wages that is transferred from young to old it is not likely to have significant effects on the savings rate, interest rates, or stock and bond prices. In comparison, the MB plan calls for higher benefits and taxes. Such increased transfers from young to old will lead to relatively higher interest rates and put per-capita incomes on a lower long-run trajectory, but they will make the current generation better off than the alternative plans. The PSA plan, in contrast, outlines a transition to significantly lower long run transfers from young to old. It is likely to reduce interest rates and to put per-capita incomes on a higher long-run trajectory, but at the expense of the generations paying for the transition.

All three plans involve equity investments of retirement funds, but in very different ways. Under the IA and PSA plans, equity investment takes place in individual accounts, which means that the retirees bear the associated

²⁰ One difference is that changes in the work force are predictable (with a caveat about immigration) about one generation in advance, since children do not enter the work force right after their birth. Hence, the demographics are somewhat more foreseeable.

risks and returns. To a first approximation, these accounts are economically neutral and have no effects on interest rates or the equity premium. Under the MB plan, however, the risk of unexpected stock price movements is effectively shifted to future generations. This proposal raises important questions about intergenerational risk sharing. Government policy is potentially important in this context, because the government can enter into insurance contracts on behalf of unborn future generations. Perhaps surprisingly, trust funds investments in equities may in principle be a good idea, because the sharing of equity risk between current and future generations may yield a Pareto-improvement in the intergenerational allocation of risk. The specific MB plan, however, is better described as a disguised increase in risk-adjusted benefits to the old at the expense of future generations.

References

- Advisory Council on Social Security, 1997, Report of the 1994-1996 Advisory Council on Social Security, Washington D.C.
- Abel, Andrew, Gregory Mankiw, Larry Summers, and Richard Zeckhauser, 1989, Assessing Dynamic Efficiency: Theory and Evidence, Review of Economic Studies 56, 1-20.
- Altonji, Joseph, Fumio Hayashi, Laurence Kotlikoff, 1996, Risk Sharing between and within Families, Econometrica 64, 261-294.
- Barro, Robert J., 1974, Are Government Bonds Net Wealth? Journal of Political Economy 82, 1095-1117.
- Baxter, Marianne, and Urban Jermann, 1997, The International Diversification Puzzle is Worse Than You Think, American Economic Review 87, 170-180.
- Bohn. Henning, 1990, Tax-Smoothing with Financial Instruments, American Economic Review 80, 1217-1230.
- Bohn, Henning, 1992, Budget Deficits and Government Accounting, Carnegie-Rochester Conference Series on Public Policy 37, 1-84.
- Bohn, Henning, 1993, Fiscal Policy and the Mehra-Prescott Puzzle: On the Welfare Implications of Budget Deficits when Real Interest Rates are Low, Working paper, UC Santa Barbara.
- Bohn, Henning, 1995, The Sustainability of Budget Deficits in a Stochastic Economy, Journal of Money, Credit, and Banking 27, February 1995, 257-271.
- Bohn, Henning, 1997, Risk Sharing in a Stochastic Overlapping Generations Economy, Working paper, UC Santa Barbara.
- Boskin, M., L. Kotlikoff, D. Puffert, and J. Shoven, 1987, Social Security: A Financial Appraisal across and within Generations, National Tax Journal 40, 19-34.
- Diamond, Peter, 1965, National Debt in a Neoclassical Growth Model, American Economic Review 55, 1126-1150.
- Diamond, Peter, 1996, Proposals to Restructure Social Security, Journal of Economic Perspectives 10, 67-88.
- Feldstein, Martin, and C. Horioka, 1980, Domestic Savings and International Capital Flows, Economic Journal 90, 314-329.
- Feldstein, Martin, 1995, Would Privatizing Social Security Raise Economic Welfare?, NBER Working Paper No.5281.
- Feldstein, Martin, 1996, The Missing Piece in Policy Analysis: Social Security Reform, American Economic Review 86 (May), 1-14.

- Gale, Douglas, 1990, The Efficient Design of Public Debt, in: R. Dornbusch and M. Draghi, Public Debt Management: Theory and History, Cambridge University Press.
- Gramlich, Edward, 1996, Different Approaches for Dealing with Social Security, Journal of Economic Perspectives 10, 55-66.
- Mankiw and Zeldes, 1991, The Consumption of Stockholders and Non-Stockholders, Journal of Financial Economics 29, 97-112.
- Mehra, Rajnish, and Edward Prescott, 1985, The Equity-Premium: A Puzzle, Journal of Monetary Economics 15, 145-162.
- Kotlikoff, Laurence, 1986, Deficit Delusion, The Public Interest, 53-65.
- Samuelson, Paul, 1958, An Exact Consumption-Loan Model of Interest with and without the Social Contrivance of Money, Journal of Political Economy 66, 467-482.
- Shiller, Robert, 1993, Macro markets: creating institutions for managing society's largest economic risks, Oxford University Press.
- Stiglitz, Joseph, 1983, On the Relevance or Irrelevance of Public Financial Policy: Indexation, Price Rigidities, and Optimal Monetary Policies, in: Rudiger Dornbusch and Mario Simonsen (eds.), Inflation, debt, and indexation, Cambridge: MIT Press, 182-222.
- Stiglitz, Joseph, Alicia Munnell, and Jeffrey Frankel, 1997, Economic Report of the President, Washington D.C.

Appendix

A. The Overlapping Generations Model

The results of Sections 3-6 are derived within the following stochastic overlapping generations model. Individuals live for two periods. Generation t consists of N_t individuals who work in period t and retire in period $t+1$. To match empirical worker-to-retiree ratios, I assume that generation t workers receive benefits for a fraction τ_t of period $t+1$. (One may assume that they are "alive" for only this period or with this probability; see below for the ramifications.) Workers earn a wage w_t equal to the marginal product of labor, pay payroll taxes on wages at the rate τ_t , and pay other taxes amounting to λ_t . The disposable income $w_t \cdot (1 - \tau_t) - \lambda_t$ is either consumed (c_t^1) or saved. Savings are either held as equity securities (s_{t+1}^e) or in form of bonds (s_{t+1}^b),

$$(A1) \quad c_t^1 = w_t \cdot (1 - \tau_t) - \lambda_t - s_{t+1}^e - s_{t+1}^b.$$

The rates of return on equities and bonds are denoted by R_{t+1}^e and R_{t+1}^b , respectively. Equity returns are stochastic. Bond returns are assumed known at time t . Retirees receive social security benefits $\tau_{t+1} \cdot \tau_{t+1} \cdot w_{t+1}$ and pay taxes λ_{t+1}^2 , so that their consumption is

$$(A2) \quad c_{t+1}^2 = R_{t+1}^e \cdot s_{t+1}^e + R_{t+1}^b \cdot s_{t+1}^b + \tau_{t+1} \cdot \tau_{t+1} \cdot w_{t+1} - \lambda_{t+1}^2.$$

Savings decisions are determined by the usual first order conditions (see Bohn, 1997, for details).

To study social security, government operations are divided into two parts. In period t , social security collects payroll taxes $\tau_t \cdot N_t \cdot w_t$ from the young and pays benefits $\tau_t \cdot N_{t-1} \cdot \tau_t \cdot w_t$ to the retired generation $t-1$. Relative to the period- t payroll, the cost can be expressed in terms of the cost rate $\tau_t^* = \tau_t \cdot \tau_t \cdot N_{t-1} / N_t$. Under a pure PAYG system, the tax rate has to match the cost rate at all times. In a mixed, partially funded social security system, the difference between payroll tax receipts and benefit cost are invested in a trust fund. The social security budget equation is then

$$(A3) \quad \tau_t \cdot N_t \cdot w_t + TF_t^* + TRS_t = \tau_t \cdot \tau_t \cdot N_{t-1} \cdot w_t + TF_{t+1}^e + TF_{t+1}^b$$

where TF_t^* is the initial trust fund balance, TF_{t+1}^e and TF_{t+1}^b are the new equity and bond investments, respectively, and TRS is a (possible) transfer from the general government to social security. Capital letters are used to denote aggregate quantities. The overall trust fund balance at the start of period $t+1$ depends on market returns,

$$(A4) \quad TF^*_{t+1} = R^e_{t+1} \cdot TF^e_{t+1} + R^b_{t+1} \cdot TF^b_{t+1}.$$

Bond investments are assumed to be in government bonds. Total Treasury debt D_{t+1} minus the social security holdings TF^b_{t+1} can then be interpreted as publicly-held (net) debt D^{net}_{t+1} .

The general government finances its total spending G_t through general taxes and by issuing bonds. In this context, taxes can be interpreted as taxes minus transfers (negative taxes) and spending as total outlays excluding interest payments and excluding transfers to social security, which are tracked separately. The general government budget equation is

$$(A5) \quad N_t \cdot \tau_t + N_{t-1} \cdot \tau^2_t + D_{t+1} = G_t + TRS_t + R^b_t \cdot D_t$$

Tax revenues and new debt issue are used to finance spending, transfers to social security, and interest plus principal on old debt. Combined with the social security budget, one obtains a unified government budget equation

$$(A6) \quad N_t \cdot w_t \cdot \tau_t + N_t \cdot \tau_t + N_{t-1} \cdot \tau^2_t - \tau_t \cdot \tau_t \cdot N_{t-1} \cdot w_t + D_{t+1} + TF^*_{t+1} \\ = G_t + R^b_t \cdot D_t + TF^e_{t+1} + TF^b_{t+1},$$

or equivalently,

$$(A7) \quad N_t \cdot w_t \cdot \tau_t + N_t \cdot \tau_t + N_{t-1} \cdot \tau^2_t - \tau_t \cdot \tau_t \cdot N_{t-1} \cdot w_t + D^{net}_{t+1} \\ = G_t + R^b_t \cdot D^{net}_t + (TF^e_{t+1} - R^e_t \cdot TF^e_t)$$

Net revenues from regular and payroll taxes plus new net debt issues (D^{net}_{t+1}) pay for social security benefits, non-interest spending (G), payment on the initial net debt, and new equity investments (TF^e_{t+1} minus the existing holdings $R^e_t \cdot TF^e_t$). The unified budget equation illustrates the interaction of government debt and the social security trust fund. Bond holdings in the social security trust fund reduce the publicly-held Treasury debt. A shift of trust fund investment from bonds to equity would therefore raise the publicly-held Treasury debt, one-for-one.

Note that τ_t is only applied to social security benefits. I treat it simply as a device to reconcile empirical replacement rates in the 30-35% range with payroll taxes in the 10-15% range. If one seriously interpreted τ_t as a survival rate, it would have to be applied to all old-age incomes. The actuarial fairness of annuity markets would then become an issue; the model should could then be interpreted as a setting with fair annuities paying $R^b/$ or $R^e/$ to survivors (see Bohn, 1997, for alternative interpretations).

An equilibrium on equity and bond markets requires that individual plus social security trust fund holdings of these securities equal the supply. To

simplify the accounting, I assume that firms' capital is equity financed and that the market value of firms is the capital stock. (Adjustment cost that might make firm values deviate from the value of their capital stock are unlikely to be important on the time scale considered here. Leverage could be added in a straightforward way and it would not change any significant results, except that it would improve the model's ability to match the equity premium.) Equity holdings by individuals and the social security trust fund must then add up to the aggregate capital stock K_{t+1} ,

$$(A8) \quad N_t \cdot s_t^e + TR_{t+1}^e = K_{t+1}$$

and their bond holdings must add up to gross government. Or equivalently, individuals must hold the net supply of government bonds,

$$(A9) \quad N_t \cdot s_t^b = D_{t+1}^{net}$$

A key insight for policy analysis is that individual behavior depends on government policy only through the net cash flow to and from the government, regardless of how these payments are labeled. (Similar neutrality results have been derived by Stiglitz (1983) for tax and debt changes and by Kotlikoff (1986) for social security in a deterministic context.) Let

$$(A10) \quad CF_t^1 = w_t \cdot \tau_t + \tau_t + \frac{D_{t+1} - TR_{t+1}^b - TR_{t+1}^e}{N_t}$$

be the net payment from the young to the government and let

$$(A11) \quad CF_{t+1}^2 = \tau_{t+1} \cdot w_{t+1} - \tau_{t+1}^2 + R_{t+1}^b \cdot \frac{D_{t+1} - TR_{t+1}^b}{N_t} - R_{t+1}^e \cdot \frac{TR_{t+1}^e}{N_t}$$

be the net payment from the government to the old. Then the individual budget equations can be written more compactly as

$$\begin{aligned} c_t^1 &= w_t - K_{t+1}/N_t - CF_t^1, \\ c_{t+1}^2 &= R_{t+1}^e \cdot K_{t+1}/N_t + CF_{t+1}^2 \end{aligned}$$

using the above equilibrium conditions. Moreover, the unified government budget equation reduces to

$$(A12) \quad N_t \cdot CF_t^1 = N_{t-1} \cdot CF_{t-1}^2 + G_t.$$

The government collects from the young to pay for real spending and net transfers to the old. Note that CF_t^1 and/or CF_{t+1}^2 can theoretically be negative, though there are defined such that they are likely positive in reality.

The cash flow measures show that a variety of differently labeled policies are equivalent. Propositions 1-5 are examples that can be formalized as follows:

Proposition 1: A change in benefits by τ_{t+1} affects CF_{t+1}^2 in the same way as a change in taxes by $\tau_{t+1}^2 = -\tau_{t+1} \cdot w_{t+1}$.

Proposition 2: A rise in payroll taxes by τ_t increases the trust fund by $TF^b_t + TF^e_t = w_t \cdot N_t \cdot \tau_t$, and leaves CF^1_t unchanged. The higher trust fund balance will be matched by an equal reduction in private savings, leaving national savings, the capital stock and interest rates unchanged. If the trust fund is investing in bonds, individuals will reduce bond holdings by the same amount. If the trust fund is investing in stocks, individuals will reduce stock holdings by the same amount. In either case, the return on the trust fund raises the replacement rate in period $t+1$ by $r_{t+1} = R^b_{t+1} \cdot TF^b_t + R^e_{t+1} \cdot TF^e_t$, leaving CF^2_{t+1} unchanged.

Proposition 3: If a trust fund is used to finance benefits that were supposed to be paid by the next young generation, the generation building up the trust fund is in effect financing part of their own retirement benefits. As in Proposition 2, CF^1_t remains unchanged; but CF^2_{t+1} falls as if τ_{t+1} were reduced.

Proposition 4: Under uncertainty, individual behavior is unaffected by policy changes only if the cash flows of alternative policies are identical across all possible realizations of uncertain future events. If the social security trust fund invests in stocks rather than bonds--raising TR^e but leaving $TR^e + TR^b$ and CF^1_t unchanged, CF^2_{t+1} will change by the stochastic amount $(R^e_{t+1} - R^b_{t+1}) \cdot TR^e$. To keep CF^2_{t+1} unchanged, τ_{t+1} and/or τ^2_{t+1} must vary stochastically by an offsetting amount: If taxes are fixed, benefits must be reduced if $R^e_{t+1} < R^b_{t+1}$ and they can be raised if $R^e_{t+1} > R^b_{t+1}$, always by the amount $\tau^2_{t+1} \cdot w_{t+1} = (R^e_{t+1} - R^b_{t+1}) \cdot TR^e$. Alternatively, if τ_{t+1} is fixed, neutrality would require the government to impose a tax in the amount $\tau^2_{t+1} = -(R^e_{t+1} - R^b_{t+1}) \cdot TR^e$ on the old (or if negative, a transfer).

Proposition 5: Suppose generation t is the "last" young generation that pays benefits to the old without itself receiving benefits. If this generation is given a transfer of government bonds equal to the present value of the previously scheduled benefits, $(D_t/N_t) = (-\tau_t)$ implies equal CF^1_t . But to leave CF^2_{t+1} unchanged, one needs $R^b_{t+1} \cdot (D_t/N_t) = \tau_{t+1} \cdot w_{t+1}$ for all states of nature, i.e., wage-indexed bonds. In future periods $(t+i)$, if the level of debt (D_{t+i}/N_{t+i}) must similarly be maintained at a level equal to the previously scheduled social security benefits.

For the general equilibrium analysis in Sections 4-6, the following assumptions about preferences, technology, and policy are imposed. To

distinguish the elasticity of intertemporal substitution from the degree of risk aversion, I consider a recursive, non-expected utility function

$$(A13) \quad U_t = \frac{1}{1-\alpha} \cdot [(c_t^1)^\alpha + \beta \cdot \{E_t[(c_{t+1}^2)^{1-\alpha}]\} / (1-\alpha)]^{1-\alpha}$$

that even allows the risk aversion of the old (α) to differ from the risk aversion of the young (β). The elasticity of intertemporal substitution is $1/(1-\alpha)$. This specification reduces to the standard CRRA case for $\alpha = \beta = 1/(1-\alpha)$. Regarding production, I assume a Cobb-Douglas technology with stochastic total factor productivity A_t and stochastic depreciation rate $1-\delta_t$,

$$Y_t = F_t(K_t, N_t) = K_t^\alpha \cdot (A_t \cdot N_t)^{1-\alpha} + \delta_t \cdot K_t,$$

where A_t follows an exponential random walk, $A_t = A_{t-1} \cdot (1+a_t)$ with a_t being i.i.d, and δ_t is simply i.i.d. The model is solved by taking a log-linear approximation around the deterministic steady state. Below, the variables d , b , c^1/w , and c^2/w denote the steady state ratios of government debt, capital investment, the trust fund balance, the consumption of the young, and the consumption of the old to the wage rate, respectively; b^b and b^e are the portfolio shares of the trust fund in bonds and equities, respectively; and $\theta^* = (c^1/w) / [\beta \cdot (c^2/w) \cdot (1+a)]$ is a constant. The deterministic steady state is characterized by the constraints

$$\begin{aligned} (c^1/w) &= 1 - g - \delta \\ (c^2/w) &= (1+n) \cdot [d + \frac{\delta}{1-\alpha} + \frac{\delta}{\alpha n}] \end{aligned}$$

and the first order condition

$$(c^2/w) = (c^1/w) / (1+a) \cdot [(\frac{\alpha n}{1-\alpha} \cdot \theta^* + \beta)]^{1/(1-\alpha)}$$

where $\theta^* = \theta^* - c^2/w / (1+n) + (\frac{\delta}{1-\alpha} \cdot \frac{1}{\alpha} + \frac{\delta}{\alpha n}) \cdot (d - \delta)$.

θ^* is a measure summary measure of intergenerational redistribution. Hence, intergenerational redistribution depends on the cost rate and on government debt net of the social security trust fund holdings. It is straightforward to show that θ^* depends negatively on d . One can also show that the log equity premium is

$$\begin{aligned} \ln(E_t R_{t+1}^e) - \ln(R_{t+1}^b) &= 2 \cdot \text{COV}_t(\ln(R_{t+1}^e), \ln(c_{t+1}^2)) \\ &= 2 \cdot \left\{ (\beta / R^e)^2 \cdot (\delta - \delta \cdot e) / (c^1/w) \cdot \theta^* \cdot \text{VAR}(\ln(\delta_t)) \right. \\ &\quad \left. + (1-\beta / R^e) \cdot [1 - \{(\delta - \delta \cdot e) \cdot \beta / R^e + (d - \delta \cdot b)\} / (c^1/w) \cdot \theta^*] \cdot (1-\alpha)^2 \cdot \text{VAR}(\ln(a_t)) \right\}, \end{aligned}$$

a function of the risk aversion of the old and the variance of the two shocks. It is clearly a declining function of e (with $b=1-e$).

B. The Policy Options in Section 6

The assumptions about policy in Section 6 are as follows. The population growth rate falls at time 0 from a high rate $n_0 = n_{-1} = n^H$ (high) to a lower rate $n_i = n^L$ (low), for all periods $i \geq 1$. The shift becomes known in period $t=0$ (when the number of infants who will become workers in period $t=1$ are born) and it is unanticipated (or was considered so unlikely that it did not significantly affect the savings behavior prior to period 0).

Option 1 is to freeze the payroll tax rate (τ) and cut the replacement rate from $\tau_0 = \tau \cdot (1+n^H)$ to $\tau_t = \tau \cdot (1+n^L)$ for $t > 0$. Option 2 is to maintain constant benefits (τ) and to raise the payroll tax rate from $\tau_0 = \tau \cdot (1+n^H)$ to $\tau_t = \tau \cdot (1+n^L)$ to cover the cost increase.

Option 3 is to maintain constant benefits, but with equal taxes on generations t and $t+1$. Taxes in period t are set to $\tau_0 = \tau \cdot (1+n^H) + \tau^*$, where τ^* is the minimum feasible "surcharge" such that if one invests τ^* on financial markets, benefits in period $t+1$ can be financed without a tax increase. That is, set $\tau_0 = \tau_1 = \tau \cdot (1+n^L) - \tau^*$ where $\tau^* \cdot N_1 \cdot w_1 = R^W \cdot \tau^* \cdot N_0 \cdot w_0$ is financed out of the trust fund with earnings and R^W is the rate of return on wage contingent claims, which is the appropriate discount rate in this context. After period $t+1$, either (3A) taxes are raised to the level of Option 2 or (3B) benefits are reduced to the PAYG level.

Option 4 for requires the following: Set $\tau_0 = \tau \cdot (1+n^H) + \tau^* = \tau_t$ equal to the level of Option 3B. Starting period 1, the replacement rate must be reduced to $\tau_t = \tau \cdot (1+n^L)$ for $i \geq 1$ such that

$$\tau^* \cdot N_0 \cdot w_0 = \sum_{t=1}^{\infty} \frac{\tau^* \cdot N_t \cdot w_t - \tau \cdot N_{t-1} \cdot w_t}{R^{W_0,t}}, \text{ where } R^{W_0,t} = \prod_{j=1}^t R^{W_j}$$

which ensures that the trust fund is never exhausted.

For Option 5, I reduce the replacement rate in periods $t+1$ and beyond such that the cost rate is 5.6%, the value discussed in Section 2.2. The payroll tax rate in periods $t+3$ and beyond is set equal to the cost rate as required in a PAYG system. During the transition, taxes $\tau_t = \tau_{t+1} = \tau_{t+2} = \tau^T$ are set such that the social security present value constraint is satisfied, which means: $(\tau_0^* - \tau^T) \cdot w_0 + (\tau_1^* - \tau^T) \cdot w_1 / R^W + (\tau_2^* - \tau^T) \cdot w_2 / (R^W \cdot R^W) = 0$. Since $\tau_0^* < \tau_1^* = \tau_2^*$, this involves a debt-financed deficit followed by surpluses sufficient to retire the debt, and a value τ^T in between τ_0^* and τ_1^* .

The net cost entries in Tables 1-2 for the different generations (t) are defined as the payroll tax minus the present value of retirement benefits in the following period ($t+1$) as a fraction of current wages,

$$\text{Net Cost}(t) = \tau - \tau \cdot \frac{w_{t+1}}{R^w_{t+1} \cdot w_t}.$$

Note that interest rates (R^w) and wage growth are the only required macro data in this context. The small open economy results described in Table 2 can therefore be obtained without any macro modeling, simply by treating R^w and wage growth as constant. In Table 2, the above policy options are calibrated as follows. All "generational" growth rates are compounded from annual growth rates assuming generational period of 30 years. The initial "high" growth rate of the labor force is based on a 1.5% annual growth rate of the labor force, $1+n^H = 1.015^{30} = 1.56$, and future growth is $1+n^L=1$ (close to the Social Security Administration's intermediate forecast for 2020-2070). The current worker-to-retiree ratio of $3.2 = N_0 / (w \cdot N_{-1}) = (1+n^H) / r$ then implies $r = 0.488$. This is set constant, assuming future retirement ages are indexed to life expectancy. For the rate of return, I start from the advisory council's values of 7% for the equity return and 2.3% for the real rate. Since U.S. equity is a leveraged claim on capital at a debt/equity ratio of about 50%, this implies an annual return on unlevered equity of about 5%; I actually use 4.96% for reasons explained below. I use the same rate of return to discount future wages, motivated by the case of Cobb-Douglas technology. Per generation, this yields $R^w = 1.0496^{30} = 4.27$. For wage growth, I use the Social Security Administration's intermediate projection, which calls for an annual wage growth of 1.0%, which implies a generational value of $w_{t+1}/w_t = 1.01^{25} = 1.35$.

For the policy options, I set $r = r^* = 10\%$ for the period- t benchmark, motivated by the current cost rate of about 10%. For plans with trust fund balance and/or debt, the ratio of the trust fund balance to payroll is interpreted as a ratio involving a generation's worth of wages. To convert annual into generational flows, I multiply by an annuity factor of 32.7. This is because the value of annual savings of 1% of payroll for 30 years is worth about 32.7% of the wage at the midpoint of this interval (accumulating and discounting for 15 year forward and backward, respectively). After earning 30-years' worth of interest, the savings are similarly converted back into annual old-age consumption. Or equivalently, an annuity of r -percent of wages is

considered equivalent to a lump-sum of $32.7 \cdot = 15.8$ times the annuity. Since the life-expectancy at age 65 is currently about 17 years (15 for males, 19 for females), these stock/flow conversion values seem reasonable.

C. The Calibrated OG model

The underlying model is the OG model described in Appendix A. In addition to the above assumptions about policy, I assume the following. Technology is Cobb-Douglas with 100% depreciation (over a generational horizon) and with capital share $\alpha = 1/3$. Preferences are as in (A13) with a unit intertemporal elasticity of substitution (the limiting case $\sigma \rightarrow 0$), so that savings behavior is similar to log-utility. The risk aversion parameter γ is chosen below to match the equity premium. The sole source of uncertainty is productivity growth, which is i.i.d. (on a generational basis) with an annual mean of 1%. To model the macroeconomic dynamics along a balanced growth path, most variables are expressed as shares of total payroll, $w_t \cdot N_t$, which grows asymptotically at the rate of productivity plus population growth. The initial social security tax and cost rates are set to $\tau_t^* = 10\%$ and the trust fund relative to payroll is set to zero, $\tau_t = TR_t / (w_t \cdot N_t) = 0$. Regarding government debt, a 53.7% debt-GDP ratio (1993 total public debt according to Federal Reserve Release C.9 divided by GDP) divided by $(1 - \delta)$ yields an annual debt/payroll ratio of 80.5% and a generational ratio of $d = D_t / (w_t \cdot N_t) = 2.49\%$ (using the annuity factor of 32.7 explained above). Government purchases/payroll are 15.7% of GDP divided by $(1 - \delta)$, which is $g = G_t / (w_t \cdot N_t) = 23.5\%$. Lacking precise data on the generational allocation of taxes, I assume the old pay taxes in proportion to their factor share and set $\tau^2/w / (1+n) = 7.85\%$. The 1995 share of gross investment to GDP of 16.4% (including government investment minus foreign borrowing) implies an investment/payroll ratio of $i = 24.65\%$, which is consistent with a capital share of aggregate income of $\alpha = 1/3$ if and only if the annual return on capital is 4.96%. This is how the return on capital is calibrated--without actually looking at return data. But the number is nicely in between the 7% return on levered equity and the 2.3% safe rate. These data imply an initial measure of intergenerational redistribution $\tau = 10\% - 7.85\% + 2.03 \cdot 2.49\% = 7.23\%$ and consumption-wage shares of

$$(A14) \quad (c^1/w)_t = 1 - g - \tau - i = 0.4463 \text{ and}$$

$$(A15) \quad (c^2/w)_t / (1+n_t) = \frac{\tau}{1+n_t} + i = 0.5723.$$

Finally, the discount factor β is set to match the observed consumption growth at the observed interest rates, which implies $\beta = 0.6323$. In the policy examples, τ_t^* and τ_t are varied over time as demanded by the alternative options (see above). For each policy, the implied paths for consumption, savings, and interest rates are computed from (A14) and (A15) and the individual first order condition for optimal savings, using the logarithmic and log-normal approximations described in Bohn (1997). The assumed unit elasticity of intertemporal substitution yields significantly simplified first order condition in this context, namely,

$$(A16) \quad 1 = \frac{E_t[R^e_{t+1} \cdot (c^2_{t+1}/w_t)^{-2}]}{E_t[(c^2_{t+1}/w_t)^{1-2}]} \cdot (c^1_t/w_t).$$

Since wages and capital income are perfectly correlated with Cobb-Douglas production, (c^2_{t+1}/w_t) can be written as sum of a wage-contingent component

$$(1+n) \cdot \left[\frac{1}{1-\tau} + \tau^* - \frac{2}{w/(1+n)} \right] \cdot (w_{t+1}/w_t)$$

and a safe component $R^b_t \cdot (d - \tau)$. Since the safe component is small in all examples, (c^2_{t+1}/w_t) is well-approximated log-linearly by $(c^2_{t+1}/w_t) = (c^2/w) \cdot (w_{t+1}/w_t)^{1-\tau} \cdot (1+a)^{\tau}$, where $\tau = R^b_t \cdot (d - \tau) / (c^2/w)$ is the safe component of old-age income. Since τ is generally small, one may further approximate $E_t(w_{t+1}/w_t)^{-\tau} \cdot (1+a)^{\tau} \approx 1$. As result, the first order condition (A16) reduces to

$$(A16') \quad 1 = \left(\frac{1}{1-\tau} \cdot \frac{1}{\tau} \right) \cdot \frac{1 - g - \tau - \tau}{/(1-\tau) + \tau_{t+1}}.$$

$$\text{Since } \tau_{t+1} = \tau^*_{t+1} - \frac{2}{w/(1+n)} + \left(\frac{1}{1-\tau} \cdot \frac{1}{\tau} \right) \cdot (d - \tau)$$

is known at time t , the optimal investment share is a deterministic function of alternative social security policies. This explains why the figures are not contingent on the realizations of future productivity growth.

Equation (A16) combined with the analogous condition for the equilibrium bond rate R^b_{t+1} imply an equation for the log-equity premium,

$$\begin{aligned} \ln(E_t R^e_{t+1}) - \ln(R^b_{t+1}) &= 2 \cdot \text{COV}_t[\ln(c^2_{t+1}), \ln(R^e_{t+1})] \\ &= 2 \cdot \text{VAR}_t[\ln(w_{t+1}/w_t)] \cdot (1 - \tau), \end{aligned}$$

using again the log-linear approximation for c^2_{t+1} and assuming log-normal wage growth. In levels, this implies

$$PR_t = E_t[R^e_{t+1}] - R^b_{t+1} = R^b_{t+1} \cdot (\exp\{2 \cdot \text{VAR}_t[\ln(w_{t+1}/w_t)] \cdot (1 - \tau)\} - 1).$$

To match the initial equity premium of $PR = 4.96\% - 2.3\% = 2.68\%$ at $\beta_0 = 0.088$, one needs $2 \cdot \text{VAR}_t[\ln(w_{t+1}/w_t)] = 0.8446$. If the variance of wage growth is proxied by aggregate consumption growth with an annual standard deviation of 3.6%, $\text{VAR}_t[\ln(w_{t+1}/w_t)] = 0.039$ implies a rather high risk aversion parameter of

$\sigma^2=21.7$, in line with Mehra-Prescott (1985). But if the variance of wage growth is proxied by the standard deviation of stock price (18% annual for the S&P500 scaled down to 13.5% to adjust for leverage), $\text{VAR}_t[\ln(w_{t+1}/w_t)]=0.55$ implies a risk aversion parameter of only $\sigma^2=1.54$. For the relative comparisons in Sections 5-6, these alternative parametrizations are equivalent because they imply the same negative dependence of the equity premium on r_t .

The statements in Section 5 about the likely impact of the MB and PSA plans on the equity premium are motivated as follows. The MB plan is interpreted Option 3A and the PSA plan as Option 5 described in Section 6. If the trust fund accumulated under Option 3A is invested entirely in equity (unlevered, implying an actual stock market investment of slightly above 50%), r_t will remain virtually constant, implying an unchanged equity premium. If the trust fund is invested in bonds, however, r_t will fall by more than half to $r_1=0.041$, raising the equity premium to $PR_1=2.77\%$ from 2.66%, by 11 basis points. Under the PSA plan, if the borrowing were done with wage-indexed bonds (which would leave the intergenerational allocation of risk essentially unchanged), the equity premium would decline slightly to 2.62%, in line with generally falling interest rates. With bond financing, however, the safe component of old age consumption would rise to $r_1=0.15$ and $r_2=0.136$ before returning to $r_3=0.089$; this reduces the equity premiums to $PR_1=2.46\%$ and $PR_2=2.49\%$ for two generations, by 20 and 17 basis points, respectively. The fact that these changes in the equity premium are so small explains why I simply assumed wage-indexed financing in constructing Figures 1-5.

Table 2 provides the same summary data as Table 1, but for the case of exogenous interest rates and wages. The comparison shows that the feasible tax and replacement rates for different generations are fairly robust with respect to alternative assumptions. The relative comparison of net cost across options is also similar, but the absolute level of net cost is much higher because the failure for interest rates to fall and wages to rise implies a sharply rising net cost. A non-unit intertemporal elasticity of substitution would have similar implications: A high elasticity would yield results similar to Table 2, while an elasticity below 1 would go in the opposite direction.

Table 1: Policy Options as Population Growth Declines

t	Period/Generation:						Memo: Trust Fund	
	-1	0	1	2	3	4	0	1
<i>Option 1: Constant Taxes/Lower Benefits</i>								
Tax rate	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%		
Repl.rate	32.0%	32.0%	20.5%	20.5%	20.5%	20.5%		
Net Cost	5.1%	5.1%	5.1%	5.1%	5.1%	5.1%		
<i>Option 2: Constant Benefits/Higher Taxes</i>								
Tax rate	10.0%	10.0%	15.6%	15.6%	15.6%	15.6%		
Repl.rate	32.0%	32.0%	32.0%	32.0%	32.0%	32.0%		
Net Cost	5.1%	2.8%	9.1%	9.1%	9.1%	9.1%		
<i>Option 3A: A stylized MB-type plan</i>								
Tax rate	10.0%	11.3%	11.3%	15.6%	15.6%	15.6%	42.9%	
Repl.rate	32.0%	32.0%	32.0%	32.0%	32.0%	32.0%		
Net Cost	5.1%	3.9%	4.4%	9.1%	9.1%	9.1%		
<i>Option 3B: A stylized IA-type plan</i>								
Tax rate	10.0%	11.3%	11.3%	11.3%	11.3%	11.3%	42.9%	
Repl.rate	32.0%	32.0%	32.0%	23.2%	23.2%	23.2%		
Net Cost	5.1%	3.9%	6.1%	6.0%	6.0%	6.0%		
<i>Option 4: Making the trust fund permanent</i>								
Tax rate	10.0%	11.3%	11.3%	11.3%	11.3%	11.3%	42.9%	42.9%
Repl.rate	32.0%	32.0%	26.0%	26.0%	26.0%	26.0%		
Net Cost	5.1%	5.1%	5.1%	5.1%	5.1%	5.1%		
<i>Option 5: A stylized PSA-type plan</i>								
Tax rate	10.0%	8.3%	8.3%	8.3%	5.6%	5.6%	-55.2%	-42.0%
Repl.rate	32.0%	32.0%	11.5%	11.5%	11.5%	11.5%		
Net Cost	5.1%	5.5%	5.4%	5.3%	2.5%	2.5%		

Notes: Repl.rate = replacement rate in period t (benefits/wages). Net Cost = net cost of participating in social security for generation t = Tax rate in period t - present value of benefits in period t+1. Trust fund = End of generation balance as share of annual payroll. See the appendix for the underlying calculations.

Table 2: Policy Options with Exogenous Interest Rates

t=	Period/Generation:						Memo: Trust Fund	
	-1	0	1	2	3	4	0	1
<i>Option 1: Constant Taxes/Lower Benefits</i>								
Tax rate	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%		
Repl.rate	32.0%	32.0%	20.5%	20.5%	20.5%	20.5%		
Net Cost	5.1%	6.8%	6.8%	6.8%	6.8%	6.8%		
<i>Option 2: Constant Benefits/Higher Taxes</i>								
Tax rate	10.0%	10.0%	15.6%	15.6%	15.6%	15.6%		
Repl.rate	32.0%	32.0%	32.0%	32.0%	32.0%	32.0%		
Net Cost	5.1%	<u>5.1%</u>	10.7%	10.7%	10.7%	10.7%		
<i>Option 3A: A stylized MB-type plan</i>								
Tax rate	10.0%	11.4%	11.4%	15.6%	15.6%	15.6%	43.6%	
Repl.rate	32.0%	32.0%	32.0%	32.0%	32.0%	32.0%		
Net Cost	5.1%	<u>6.4%</u>	<u>6.4%</u>	10.7%	10.7%	10.7%		
<i>Option 3B: A stylized IA-type plan</i>								
Tax rate	10.0%	11.4%	11.4%	11.4%	11.4%	11.4%	43.6%	
Repl.rate	32.0%	32.0%	32.0%	23.2%	23.2%	23.2%		
Net Cost	5.1%	<u>6.4%</u>	7.8%	7.8%	7.8%	7.8%		
<i>Option 4: Making the trust fund permanent</i>								
Tax rate	10.0%	11.4%	11.4%	11.4%	11.4%	11.4%	43.6%	43.6%
Repl.rate	32.0%	32.0%	29.2%	29.2%	29.2%	29.2%		
Net Cost	5.1%	6.8%	6.8%	6.8%	6.8%	6.8%		
<i>Option 5: A stylized PSA-type plan</i>								
Tax rate	10.0%	8.7%	8.7%	8.7%	5.6%	5.6%	-41.7%	-31.7%
Repl.rate	32.0%	32.0%	11.5%	11.5%	11.5%	11.5%		
Net Cost	5.1%	6.9%	6.9%	6.9%	<u>3.8%</u>	<u>3.8%</u>		

Notes: Repl.rate = replacement rate in period t (benefits/wages). Net Cost = net cost of participating in social security for generation t = Tax rate in period t - present value of benefits in period t+1. Trust fund = End of generation balance as share of annual payroll. See the appendix for the underlying calculations.

Figure 1: Average Replacement Rates

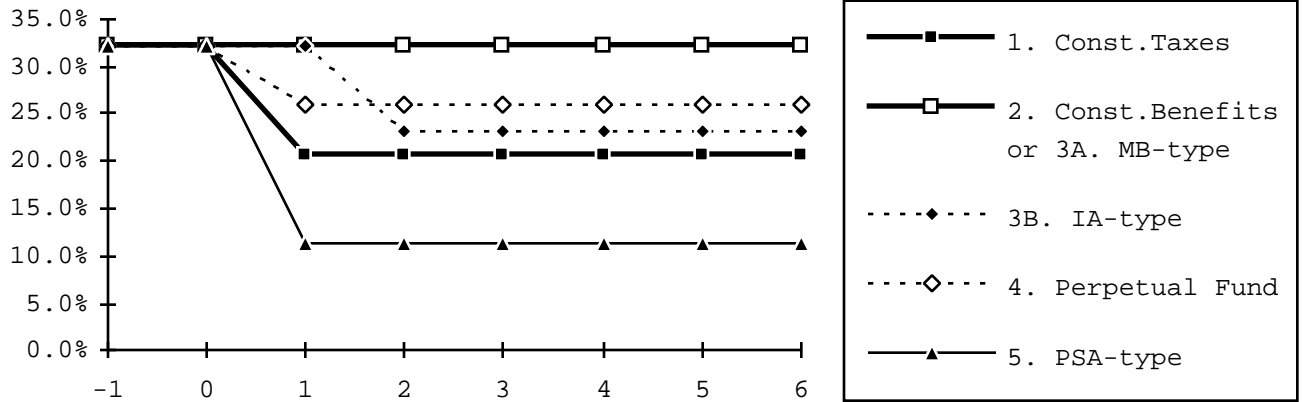


Figure 2: Average Payroll Tax Rates

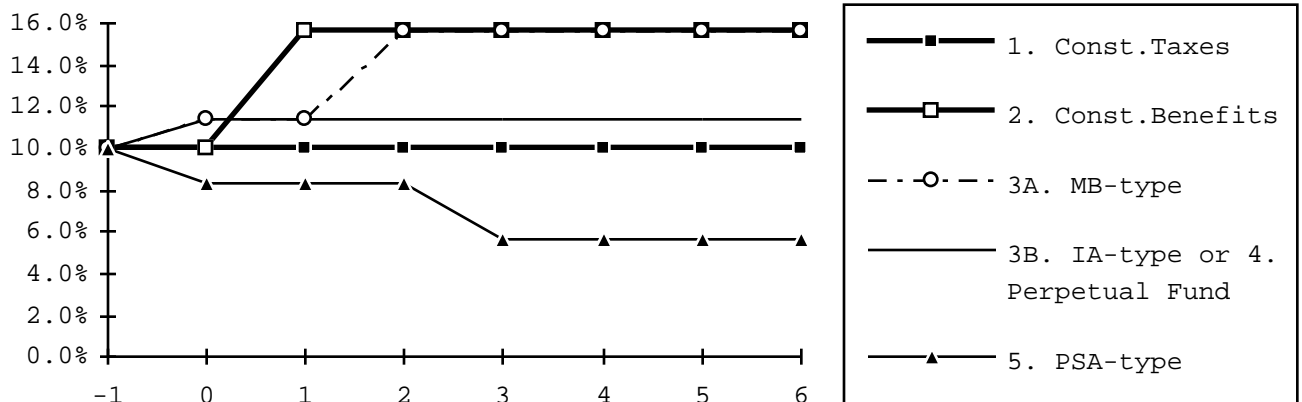


Figure 3: Trust Fund/Annual Payroll

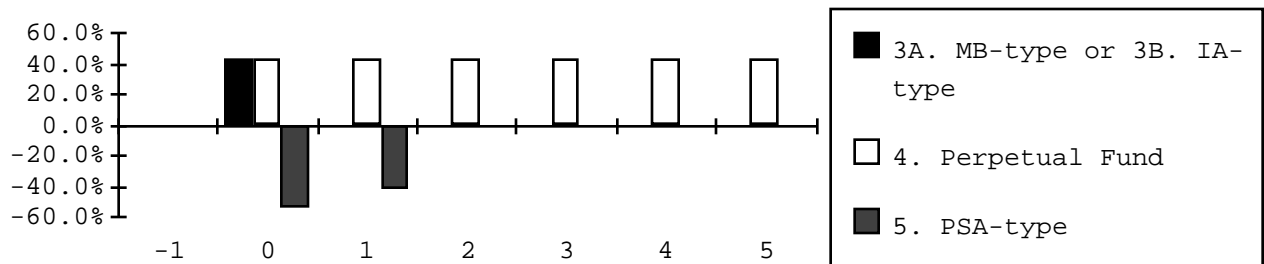


Figure 4: Expected Return on Capital

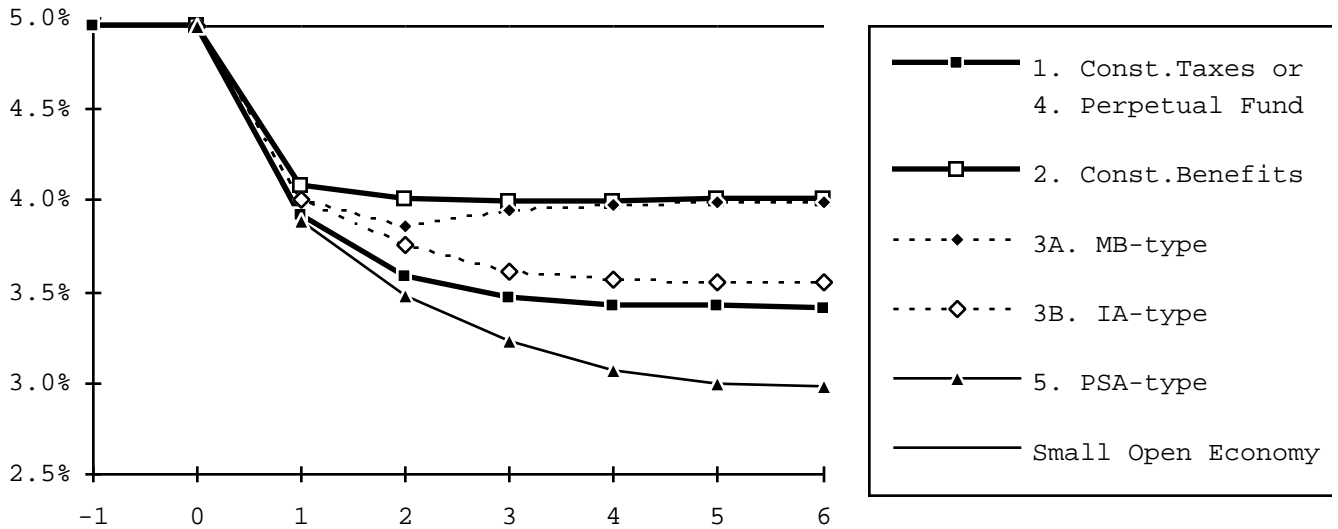


Figure 5: Wage Incomes/Growth trend

