# Fiscal Policy and the Mehra-Prescott Puzzle: On the Welfare Implications of Budget Deficits when Real Interest Rates are Low 

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#### Abstract

Historically, average real returns on U.S. government debt have been far below the rate of economic growth, allowing the U.S. government to roll over its debt at a rather low cost. At the same time, the rate of return on capital has generally been above the growth rate, suggesting that the U.S. economy is dynamically efficient. The paper shows that the welfare implications of budget deficits in this scenario depend critically on why interest rates have been so low. If the government can offer low returns on its debt because of some unique ability to create default-free claims, persistent primary budget deficits may be unproblematic. But if low interest rates are due to high risk aversion, policies that exploit the low cost of government debt to run frequent budget deficits will impose significant risks on future taxpayers. In essence, safe government debt is safe for the debt holders, but it is very risky for the taxpayers who are implicitly taking a short position in the safe security.


## 1. Introduction

The paper examines the welfare implications of government budget deficits. The real return on U.S. government debt has historically been far below the average return on equities and substantially below the average rate of economic growth. If the interest rate is below the average growth rate, a government may roll-over its debt with interest-i.e., run a primary deficit and not provide debt service—and still expect a declining ratio of debt to GDP. Are persistent budget deficits therefore unproblematic?

An analysis of deficit financing strategies that rely on low interest rates faces the problem that we do not have a fully convincing explanation why the real return on safe assets has been so low. As Mehra and Prescott (1985) have pointed out, standard asset pricing models have considerable difficulties in explaining the equity premium. Complicating the puzzle, the relatively high return on equity capital suggests that the low safe interest rate is not a sign of dynamic inefficiency (see Abel et al. 1989, Zilcha 1992).

This paper examines the fiscal policy implications of different "explanations" of the equity premium puzzle. The various explanations are best organized in two groups. One group of authors maintains either that a high risk aversion parameter should not be viewed as puzzling or that the puzzle can be resolved by more general models within the framework of complete, frictionless markets. A second group of authors argues that a solution to the Mehra-Prescott puzzle can only be found in models with transactions cost that deviate significantly from the complete markets paradigm. (Specific references are in Section 2; see also the survey by Kocherlakota, 1996).

Using two simple models, I show that a welfare assessment of persistent budget deficits depends crucially on which of these two lines of argument is correct. If low safe interest rates are due to risk aversion, the government can sustain frequent primary budget deficits by rolling over safe debt, but such policies generally involve an inefficient allocation of risk. This is because safe debt implies high future tax rates in states of nature with low consumption, i.e., it creates systematic risks for the taxpayers who implicitly back the debt. A welfare assessment of government debt policy in a stochastic setting must therefore include an analysis of how government debt affects the
allocation of risk, and not just focus on expected values. In practice, government policy plans-e.g., administration or CBO budget projections—are unfortunately almost always expressed in terms of expected values. Hence, the paper raises serious questions about the interpretation of such projections. One may even wonder to what extent the widespread use of expected value projections provides political incentives to issue debt securities that carry a low interest rate.

Very different welfare implications can be obtained in models that explain low interest rates on government debt by market imperfections. I demonstrate this point in a model with intermediation costs in which the government can issue debt at an interest rate below the interest rate on private loans. Then permanent primary deficits are possible and they do not have adverse implications for future taxes.

Overall, the contrast between the two types of explanations for the Mehra-Prescott puzzle suggests that we do not have definitive answers on how to assess budget deficits in a situation with low real interest rates. If low interest rates on government debt are due to risk aversion, a policy that exploits low real rates to sustain budget deficits will impose risks on future taxpayers. But if low interest rates on government debt are due to special advantages of government borrowing over private borrowing, persistent budget deficits may be sustainable without creating risks and they may be part of a welfare-maximizing policy. Since we do not have a definitive answer which of these lines of argument is empirically more relevant, we have to conclude that the Mehra-Prescott puzzle has created a puzzle for assessing fiscal policy.

Note that safe government debt raises troubling questions about risk taking even if bond returns exceed the average growth rate. ${ }^{1}$ At an interest rate above the average growth rate, primary deficits would trigger an exponential growth in the debt-GDP ratio. Even if the government then runs primary surpluses to stabilize the debt-GDP ratio, safe debt allows the government to run lower average primary surpluses than it could run without relying on the bondholders' risk aversion. Irrespective of the interest rate level, safe debt imposes risk on future taxpayers.

The paper is organized as follows. Section 2 reviews the relationship between risk aversion, interest rates, and budget deficits. Section 3 examines low-interest debt in a setting where low safe
interest rates are due to risk aversion. Section 4 considers a model in which low interest rates in government debt are due to intermediation costs. Section 5 concludes.

## 2. Interest Rates, Risk Aversion, and Budget Deficits

To start, consider a Lucas (1978) exchange economy with frictionless markets. This model has served as benchmark for much of the asset pricing literature. Identical, infinitely lived agents are each endowed with an exogenous, stochastic income stream $Y_{t}$ and they have preferences over statecontingentconsumptionct. The price of any financial asset is determined so that its distribution of returns, $R_{t+1}$, satisfies the Euler equation

$$
\begin{equation*}
\mathrm{E}_{\mathrm{t}}\left[\left(1+\mathrm{R}_{\mathrm{t}+1}\right) \cdot \mathrm{MRS}_{\mathrm{t}, \mathrm{t}+1}\right]=1 \tag{1}
\end{equation*}
$$

where $\mathrm{MRS}_{t, t+1}$ is the marginal rate of substitution evaluated at the equilibrium allocation $\left\{c_{t}=Y_{t}\right\}$. In this setting, all differences in expected returns across securities are due to risk-aversion. A low interest rate of government securities must therefore be due to the fact that governments tend to issue safe (or in practice, nearly safe) debt securities.

To be specific, assume that utility is CRRA with risk-aversion parameter $\alpha$ and discount factor $\beta, U=E\left[\sum_{t \geq 0} \beta^{t} \cdot c_{t}^{1-\alpha} /(1-\alpha)\right]$, so that $\operatorname{MRS}_{t, t+1}=\beta \cdot\left(c_{t+1} / c_{t}\right)^{-\alpha}$. Let the growth rate of income $x_{t}$ be an i.i.d. process and let $\alpha$ and $\beta$ be such that utility is finite, $E\left[\beta \cdot\left(1+x_{t}\right)^{1-\alpha}\right]<1$. For sufficiently high risk aversion, the safe real interest rate r defined by $1 /(1+r)=\mathrm{E}\left[\beta \cdot\left(1+\mathrm{x}_{\mathrm{t}}\right)^{-\alpha}\right]$ can be less than the mean growth rate $x$ and it may be negative. In contrast, finite utility restricts the return on incomeindexed claims, denoted by $1+R_{t}=\left(1+x_{t}\right) \cdot(1+v)$, to lie above $x_{t}$, because eq. (1) and finite utility imply $1=\mathrm{E}\left[\beta \cdot\left(1+\mathrm{R}_{\mathrm{t}}\right) \cdot\left(1+\mathrm{x}_{\mathrm{t}}\right)^{-\alpha}\right]=\mathrm{E}\left[\beta \cdot\left(1+\mathrm{x}_{\mathrm{t}}\right)^{1-\alpha}\right] \cdot(1+\mathrm{v})<1+\mathrm{v}$. (That is, income indexed-claims have a positive dividend yield $v>0$.)

To illustrate the policy implications, suppose one adds a government that levies lump-sum taxes to finance an outstanding debt $\mathrm{D}_{\mathrm{t}}$. The government policy is to issue safe debt and to maintain a constant end-of-period debt-income ratio $D_{t} / Y_{t}$. This policy implies primary (excluding interest) surpluse $Z_{t}=\left(r-x_{t}\right) \cdot D_{t-1}$. If $r<E\left[x_{t}\right]$, the primary surplus will be negative on average, $E\left[Z_{t}\right]<0$. Depending on the density of $x_{t}$, the ex ante probability of $Z_{t}<0$ may be near one so that surpluses will
be observed quite rarely and long strings of realized budget deficits will be quitecommon. ${ }^{2}$ Although the government is subject to an intertemporal budget constraint, namely

$$
\begin{equation*}
\left(1+\mathrm{R}_{\mathrm{t}}\right) \cdot \mathrm{D}_{\mathrm{t}-1}=\mathrm{E}\left[\sum_{\mathrm{n} \geq 0} \mathrm{MRS}_{\mathrm{t}, \mathrm{t}+\mathrm{n}} \cdot \mathrm{Z}_{\mathrm{t}+\mathrm{n}}\right] \tag{2}
\end{equation*}
$$

(see Bohn 1995), so that a positive initial debt must be backed by future primary surpluses, high risk-aversion allows the government to satisfy this constraint by running surpluses in "a few" states of nature with a high MRS-value (low consumption).

For the United States, Mehra and Prescott (1985) document that the long-run average return on equity has been close to $7 \%$, that the safe real interest rate has been about $0.8 \%$, and that consumption growth had a mean of about $1.8 \%$ with a standard deviation of about $3.6 \%$. If one interprets equity as a claim on the present value of income and assumes log-normal income growth, these data would imply a risk-aversion parameter of about $\alpha \cong 28$. If, as Mehra and Prescott argue, a reasonable upper bound on risk aversion is $\alpha \leq 10$, the observed high equity premium is a puzzle. For a given equity return, the premium puzzle may also be interpreted as a puzzle of why the safe interest rate has been so low. ${ }^{3}$

These puzzles have generated a huge literature. (See Kocherlakota, 1996, for an excellent recent survey.) One set of responses involves generalizations of the CRRA model that maintain an asset pricing equation of the form (1). This includes models with non-time-separable or non-expected utility (e.g., Abel 1990; Constantinides 1990; Weil 1989), models with stochastic processes that allow for "crashes" (Rietz 1988), models with leverage effects (Benninga and Protopapadakis, 1990), and papers arguing that CRRA utility functions with parameters $\beta>1$ (Kocherlakota, 1990) or $\alpha>10$ (Kandel and Stambaugh 1990, 1991) are not unreasonable. In contrast, a second set of papers argues that significant transactions cost or other deviations from the frictionless markets paradigm are needed to explain the high equity premium (e.g., Aiyagari and Gertler 1991, Heaton and Lucas 1996).

This paper does not attempt to resolve the puzzle. Instead, I will show that the two lines of argument above yield very different policy implications. The welfare implications are important,
because regardless of why the safe interest rate has been so low-how the asset pricing puzzles are eventually resolved-interest rates below the growth rate imply that the government can run primary deficits quite easily. Historically, the U.S. primary budget balance has been negative on average over long periods, suggesting that the U.S. government has in fact exploited it's ability to run frequent deficits. ${ }^{4}$

To do a meaningful policy analysis, the above model must be modified in a way that makes debt policy non-neutral. First, I consider a frictionless markets setting in which an Euler equation of the form (1) applies and deficits are non-neutral for distributional reasons. Policies that sustain primary deficits by low-interest debt turn out to be suboptimal because they generally imply an inefficient allocation of risk. Second, I consider a model with intermediation cost and show that low-interest government debt can be Pareto-improving if such debt reduces transactions cost.

## 3. A Model with Risk Aversion

This section examines budget deficits in a model in which low safe interest rates are due to high risk aversion. For simplicity, I assume CRRA utility and I interpret a low safe interest rate as result of a high $\alpha$ value (above 10 if necessary; similar arguments would apply in more general asset pricing models, provided they explain low interest rates on government debt by risk aversion, as noted below).

To obtain interesting welfare effects of government debt and deficits, some deviation from Ricardian neutrality is required. Consider therefore a model with two types of agents. Type I (investor) is infinitely lived and has a CRRA utility function with parameters $\alpha=\alpha^{\mathrm{I}}$ and $\beta$, as in Section 2. Type T (taxpayer) agents live for one period-a new generation being born every periodand have a CRRA utility function with risk aversion $\alpha^{T}$. The government imposes equal taxes $T_{t}$ (transfers, if negative) on both types to finance expenditures and an initial government debt. (Tax collectors cannot distinguish the types.) This model is a tractable extension of the representative agent asset pricing framework, because asset returns depend only on the preferences of type-I agents, and it has interesting normative implications because type-T agents are directly affected by changes in the intertemporal and the across-states distribution of the tax burden. ${ }^{5}$

To prevent asset pricing complications, I take another step away from the Lucas model by adding a production sector. ${ }^{6}$ For each state of nature $s$ in period $t+1$, there is a production technology that produces one consumption good in that state at a cost of $p_{t}(s)$ units of consumption in period $t$. Provided all technologies are used, this assumption fixes all asset prices and rates of return. An asset with return distribution $\mathrm{R}_{\mathrm{t}+1}(\mathrm{~s})$ has to satisfy the arbitrage condition

$$
1=\sum_{s}\left(1+R_{t+1}(s)\right) \cdot p_{t}(s)=E_{t}\left[\left(1+R_{t+1}(s)\right) \cdot p_{t}(s) / \pi_{t}(s)\right]
$$

where $\pi_{\mathrm{t}}(\mathrm{s})$ is the conditional probability of state s . Given the state-contingent claims prices $\mathrm{p}_{\mathrm{t}}(\mathrm{s})$, the optimal consumption plan $c^{\mathrm{I}}$ t of type I agents will satisfy the Euler equations $\beta \cdot\left(c^{\mathrm{I}}{ }_{t+1}(\mathrm{~s}) / c_{t}{ }^{\mathrm{I}}\right)^{-\alpha \mathrm{I}}=$ $p_{t}(\mathrm{~s}) / \pi_{\mathrm{t}}(\mathrm{s})$ for all $(\mathrm{t}, \mathrm{s})$. Hence, eq. $\left(1^{\prime}\right)$ is the same as the standard asset pricing eq. (1) except that c is replaced by $\mathrm{c}^{\mathrm{I}}$.

To obtain similar consumption processes as in Section 2, let type-T agents receive a stochastic endowment income $Y_{t}$ with i.i.d. growth rate $x_{t}$, let productivities be such that $\left[\beta \cdot \pi_{\mathrm{t}}(\mathrm{s}) / \mathrm{p}_{\mathrm{t}}(\mathrm{s})\right]^{1 / \alpha^{\mathrm{I}}}=$ $1+x_{t+1}(\mathrm{~s})$ for all times and states, and let type I agents have an endowed wealth in period zero equal to the present value of the sequence $\left\{Y_{t}\right\}$. Without government activity, type-I would then run all production technologies at a positive level and choose to consume the same amount $c^{\mathrm{I}_{\mathrm{t}}}=\mathrm{Y}_{\mathrm{t}}$ as type-T. Thus, the exchange economy of Section 2 is essentially embedded in this production model.

Turning to the government, let $D_{t}$ be the end-of-period debt. Debt may consist of safe securities or of state-contingent liabilities. Let $R_{t}$ be the (generally stochastic) return on period $t-1$ debt, let $\theta \in(0,1)$ be the fraction of type I agents in the population, let the population size be normalized to one (to avoid separate notation for aggregate and per capita quantities), and assume that government spending is proportional to income, $\mathrm{G}_{\mathrm{t}}=\mathrm{g} \cdot \mathrm{Y}_{\mathrm{t}}$ with $\mathrm{g} \geq 0 .{ }^{7}$ Then government debt and taxes must satisfy the budget equation

$$
\begin{equation*}
D_{t}=\left(1+R_{t}\right) \cdot D_{t-1}-Z_{t}=\left(1+R_{t}\right) \cdot D_{t-1}-T_{t}+G_{t} \tag{3}
\end{equation*}
$$

and the intertemporal budget constraint (2).
Given an initial debt $\left(1+\mathrm{R}_{0}\right) \cdot \mathrm{D}_{-1}$ and given the present value of government spending, alternative tax and debt policies do not affect type-I agents; their consumption is always proportional to $Y_{t} .{ }^{8}$ The consumption of type-T, $c^{T}=Y_{t}-T_{t}$, does depend on tax policy and therefore
on the underlying debt-management policies. Overall, the model describes an economy with nonneutral debt policy and with asset returns that satisfy an Euler equation of the form (1). ${ }^{9}$

As in Section 2, the government can finance its debt at a low interest rate and run primary deficits frequently and/or in expectation by issuing securities with payoffs that have a low or negative correlation with consumption growth. ${ }^{10}$ Prop. 1 shows, however, that such policies are unlikely to be optimal:

Proposition 1: If taxpayers are equally or more risk averse than bondholders ( $\alpha^{\mathrm{T}} \geq \alpha^{\mathrm{I}}$ ), all Paretooptimal tax and debt policies imply an expected return on government debt $E\left[R_{t}\right] \geq R^{x}>x$ above the return on income-indexed debt and strictly above the growth rate x (provided optimal policies exist and debt is positive). If $\alpha^{\mathrm{T}}=\alpha^{\mathrm{I}}$, optimal debt must be income-indexed.

Proof: An intuitive argument is below; see Bohn (1997) for details.

Intuitively, the Pareto-optimal allocation of income risk depends on the relative risk aversion of the type-T taxpayers versus the type-I debt holders, regardless of welfare weights. In the benchmark case of equal risk aversion $\left(\alpha^{\mathrm{T}}=\alpha^{\mathrm{I}}\right.$ ), the Pareto-optimal levels of type-T consumption and taxes are proportional to income. To support such a tax policy, the initial debt across states of nature must also be proportional to income, i.e., it must consist of income-indexed securities. Such securities have an expected return $\mathrm{R}^{\mathrm{X}}$ above the growth rate x , proving the $\alpha^{\mathrm{T}}=\alpha^{\mathrm{I}}$ case. If taxpayers are more risk averse than bondholders $\left(\alpha^{\mathrm{T}}>\alpha^{\mathrm{I}}\right.$ ), the Pareto-optimal type-T consumption is less volatile than income (namely, proportional to $\mathrm{Y}_{\mathrm{t}}{ }^{\alpha \mathrm{I}} / \alpha^{\mathrm{T}}$ ). Pareto-optimal government debt must then consist of income-indexed securities plus a hedge fund that insures type-T agents against income risk. Since type-I agents demand positive expected payoffs for taking income risk, the expected return on the overall debt portfolio is above $R^{x}$ and even further above $x$ than in the $\alpha^{I}=\alpha^{T}$ case. Since $r<R^{x}$, safe debt is clearly suboptimal. ${ }^{11}$

The only case in which Pareto-optimal government debt may carry an interest rate below x is the case of $\alpha^{\mathrm{T}}<\alpha^{\mathrm{I}}$, when taxpayers are strictly less risk averse than debt holders-and so much less risk averse that the premium for insuring bondholders against income risk exceeds the gap $R^{\mathrm{x}}$ - x .

This case seems far fetched, however, going against the conventional wisdom that financial markets tend to be populated by relatively risk-tolerant agents and that a major function of financial markets is to absorb risk. Except in this rather pathological case, policies that sustain primary deficits through low interest debt are suboptimal.

Overall, the model yields negative conclusions about government policies that exploit safe or other low interest debt. Such polices may be feasible, but they are not optimal unless taxpayers are less risk averse than debt holders. Safe debt allows the government to keep expected taxes loweven below government spending if $\mathrm{r}<\mathrm{x}$ —but at the price of promising a fixed, non-contingent return that must be backed by relatively high taxes in states of nature with low consumption. ${ }^{12}$ For riskaverse taxpayers, the utility cost of such a tax policy exceeds the benefits of having a low average tax burden.

Note that the same intuition applies in more elaborate models of risk aversion in the frictionless-markets tradition, such as models with habit formation, fear of a crash, leverage effects, or Epstein-Zin utility functions. If taxpayers and debt holders have similar preferences, the same source of risk aversion that explains why investors are willing to hold safe debt at an interest rate far below the equity return makes taxpayers very averse to being responsible for the debt service.

## 4. A Model with Restrictions on Private Intermediation

This section examines budget deficits in a model with intermediation cost. The model is meant to be a simple representation of models such as Aiyagari-Gertler (1991) and Heaton-Lucas (1996) that try to resolve the Mehra-Prescott puzzle by assuming intermediation cost and heterogeneous agents. Since I focus on the fiscal implications of low safe interest rates rather than their explanation, I do not attempt to develop an elaborate, "realistic" model of market imperfections. Instead, I simply assume that there is an intermediation cost that implies a lending rate below the growth rate and allows the government to run primary deficits. ${ }^{13}$ To eliminate the risk aversion issue, the model is deterministic.

The model has an equal number of two types of infinitely lived agents, A and B. Type A has income $Y^{A}{ }_{t}=Y_{t}$ in even periods (zero otherwise) and type $B$ has income $Y^{B}{ }_{t}=Y_{t}$ in odd periods (zero otherwise). Income grows at a constant rate $x, Y_{t}=Y_{0} \cdot(1+x)^{t}$. To simplify the notation, let there be one representative agent of each type so that the individual income $Y_{t}$ also denotes the aggregate income. Both types have identical CRRA utility functions $U=\sum_{t \geq 0} \beta^{t} .\left(c^{i}{ }_{t}\right)^{1-\alpha} /(1-\alpha)$. Government spending is $G_{t}=g \cdot Y_{t}$. The government finances its spending by imposing lump-sum taxes $T_{t}$ on the type that receives income, and by issuing debt. ${ }^{14}$

If there were a frictionless credit market, the market clearing condition $c^{A}{ }_{t}+c^{B}{ }_{t}=(1-g) \cdot Y_{t}$ and the standard Euler equations would imply

$$
c^{A}{ }_{t+1} / c^{B}{ }_{t}=c^{B}{ }_{t+1} / c^{B}{ }_{t}=[\beta \cdot(1+r)]^{1 / \alpha}=Y_{t+1} / Y_{t}=1+x,
$$

where $r$ denotes the frictionless interest rate. The consumption of both types would be proportional to income and grow at the rate $x$. Finite utility again requires $\beta \cdot(1+x)^{1-\alpha}<1$, which implies a frictionless interest rate above the growth rate, $r>x$.

Turning to the market imperfections, suppose private lenders have to bear an proportional intermediation cost $\kappa>0$. One may interpret $\kappa$ as a cost of monitoring, of identifying a borrower, or of verifying credit. The cost is a real resource cost (say, wasted time) that does not benefit anyone. If a loan is made at the borrowing rate $R^{*}$, the after-cost return to the lender is $1+R_{t}=\left(1+R_{t}^{*}\right) /(1+\kappa)$. The key assumption of the model is that loans to the government can be monitored without cost, so that the government can borrow at the lending rate $\mathrm{R}_{\mathrm{t}} .{ }^{15}$

To show that budget deficits have very different welfare implications in this setting than in Section 3, it is sufficient to focus on symmetric steady states with a constant debt-income ratio. Start-of-period government debt is a fixed fraction $d \geq 0$ of income, agents earning income in the period consume a fixed fraction $\gamma$ of income, and agents without income consume a fraction $\gamma^{*}$ of income. Such symmetric steady states have the convenient property that the borrowing and lending rates are constant over time $\left(\mathrm{R}_{\mathrm{t}}^{*}=\mathrm{R}^{*}, \mathrm{R}_{\mathrm{t}}=\mathrm{R}\right)$. The equilibrium allocation depends on the debt-income ratio d as follows:

## Proposition 2:

(a) For government debt $0 \leq \mathrm{d}^{2} \mathrm{~d}_{1}$ below a critical level $\mathrm{d}_{1}$, agents that do not earn income in the period borrow a non-zero amount from the other type. Income-earning agents are lenders and holders of government debt. The borrowing and lending rates are above and below the frictionless rate, respectively: $1+\mathrm{R}^{*}=(1+r) \cdot \sqrt{1+\kappa}>1+r>1+R=(1+r) / \sqrt{1+\kappa}$.
(b) For an intermediate range of government debt, $\mathrm{d}_{1} \leq \mathrm{d} \leq \mathrm{d}_{2}$, agents smooth consumption by buying government debt in the period with income and liquidating it in the period without income. There is no private borrowing. The bond rate $R=R(d)$ is a strictly increasing function of $d$, rising from $1+R\left(d_{1}\right)=(1+r) / \sqrt{1+\kappa}$, the value in $(a)$, to $R\left(d_{2}\right)=r$.
(c) For government debt $\mathrm{d}>\mathrm{d}_{2}=(1-\mathrm{g}) / 2$, the bond rate is the frictionless interest rate $\mathrm{R}=\mathrm{r}$. Both types agents hold government bonds in all periods and they have equal consumption $\gamma=\gamma^{*}=(1-\mathrm{g}) / 2$.

Proof: An intuitive argument is below; see Bohn (1997) for details.

Corollary: If $\kappa$ is large enough that $R_{\kappa} \equiv(1+r) / \sqrt{1+\kappa}-1<x$, there is a range of debt levels $d<d_{\kappa}$ (where $\mathrm{d}_{1}<\mathrm{d}_{\kappa}<\mathrm{d}_{2}$ ) at which the government runs permanent primary deficits.

Proof: If $R_{K}<x$, Prop. $2(b)$ implies a unique $d_{\kappa} \in\left(d_{1}, d_{2}\right)$ such that $R\left(d_{\kappa}\right)=x$. For $d<d_{\kappa}, R=R(d)<x$ implies $\mathrm{Z}_{\mathrm{t}}=(\mathrm{R}-\mathrm{x}) \cdot \mathrm{d} \cdot \mathrm{Y}_{\mathrm{t}}<0$.

To understand Proposition 2, note that agents not earning income must either carry assets into the period or borrow. Without government debt, private loans are the only store of value. Hence, non-zero borrowing and lending must occur and agents must alternate between borrowing and lending. Over any two periods, the steady state consumption growth rate must match the growth rate of income. Hence, the two-period return to savings (more lending then reduced borrowing, or reduced borrowing then lending) must equal the two-period return in the frictionless markets model, $(1+\mathrm{R}) \cdot\left(1+\mathrm{R}^{*}\right)=(1+\mathrm{R})^{2} \cdot(1+\kappa)=(1+r)^{2}$, which implies (a). Because of the intermediation cost, individuals consume more in the period with income than in the other period, $\gamma>\gamma^{*}$.

Government bonds are an alternative store of value that crowds out private lending. For debt levels low enough that private lending occurs, debt does not affect interest rates. If $\kappa$ is large enough,
$R=R_{K}$ is below the growth rate $x$, so that the government runs primary deficits all the time, as noted in the Corollary. Intuitively, agents know that if they do not buy debt or make loans in a period with income, they will have to borrow at a much higher rate in the next period. Hence, they are willing to buy low interest government debt.

For a range of higher debt levels, government debt provides a large enough store of value to eliminate private lending, but agents without income still consume less than agents with income $\left(\gamma^{*}=\mathrm{d}<\gamma\right.$ for $\left.\mathrm{d}_{1} \leq \mathrm{d}^{2} \mathrm{~d}_{2}\right)$. Consumption smoothing is imperfect, but the shadow value of private borrowing is not high enough to compensate for the intermediation cost. The bond return $1+\mathrm{R}$ is above the value $(1+\mathrm{r}) / \sqrt{1+\kappa}$ that would trigger private lending but below the frictionless value $1+\mathrm{r}$. Finally, if debt is high enough that agent can smooth consumption perfectly by holding government bonds ( $\gamma=\gamma^{*}$ for $\mathrm{d} \geq \mathrm{d}_{2}$ ), bond returns must equal the frictionless interest rate, $\mathrm{R}=\mathrm{r}$.

In this model, primary deficits do not signal risk-taking or any problems for the future. Instead, the ability to run primary deficits if $R<x$ is perhaps best interpreted as a monopoly rent on the government's unique ability to create securities that agents can hold without incurring monitoring cost. Not surprisingly, the government can improve welfare by supplying such securities to the point of satiation:

Proposition 3: For all $\mathrm{d}^{2} \mathrm{~d}_{2}$, the government can increase the debt-income ratio d in a way that increases all agents' utility. For $\mathrm{d}>\mathrm{d}_{2}$, changes in d are neutral.

Proof: An intuitive argument is below; see Bohn (1997) for details.

For $\mathrm{d} \geq \mathrm{d}_{2}$, enough safe assets exist for agents to smooth consumption perfectly and to attain the frictionless markets allocation; hence, Ricardian neutrality applies. Below $\mathrm{d}_{2}$, more debt is efficient because it alleviates the wasteful monitoring cost associated with private borrowing (for $\mathrm{d}<\mathrm{d}_{1}$ ) and the distortions associated with imperfect consumption smoothing (for $\mathrm{d}<\mathrm{d}_{2}$ ). (Despite the clear intuition, the precise proof is rather lengthy, because once has to construct a Paretoimproving transition path between steady states; here, this requires at least two steps, to give both types a share of the tax rebate associated with additional government debt.)

Note that since the interest is above the growth rate at efficiently high levels of debt, $R\left(d_{2}\right)=r>x$, permanent primary deficits are a signal that the debt-income ratio is too low $\left(d_{<}<d_{2}\right)$ and government should flood the market with more debt. This somewhat radical policy result can be avoided, however, if one assumes (realistically) that regular taxes are distortionary, so that debt financing at a low interest rate has a value in terms of reduced excess burden. This can be demonstrated most easily by adding proportional tax collection cost to the model:

Proposition 4: If $\mathrm{R}_{\boldsymbol{K}}<x$ and if taxes require a sufficiently high proportional collection cost $k$, then an efficient symmetric allocation must have a debt-income ratio $d \in\left[d_{1}, d_{\kappa}\right)$ such that $R(d)<x$, and it will display permanent primary deficits.

Sketch of the Proof (see Bohn (1997) for details): For $\mathrm{d}^{2} \mathrm{~d}_{1}$, debt is too low because a higher debt would reduce taxes by $\left(x-R_{\kappa}\right) \cdot d$. For $\mathrm{d} \geq \mathrm{d}_{2}$, more debt is welfare-reducing because of $\mathrm{k}>0$. Within $\mathrm{d}_{1}<\mathrm{d}<\mathrm{d}_{2}$, debt improves consumption smoothing, generates revenues $\left(-\mathrm{Z}_{\mathrm{t}} / Y_{\mathrm{t}}\right)=(\mathrm{x}-\mathrm{R}(\mathrm{d})) \cdot \mathrm{d}>0$ for $\mathrm{d}<\mathrm{d}_{\kappa}$, and requires debt service $(\mathrm{R}(\mathrm{d})-\mathrm{x}) \cdot \mathrm{d}>0$ for $\mathrm{d}>\mathrm{d}_{\mathrm{\kappa}}$. At $\mathrm{d}=\mathrm{d}_{\mathrm{\kappa}}$, the marginal revenue $\partial(-$ $\left.Z_{t} / Y_{t}\right) / \partial d=x-R(d)-d \cdot R^{\prime}(d)$ is negative because $x=R\left(d_{k}\right)$ and $R^{\prime}>0$. If $k$ is sufficiently high, the negative revenue effect outweighs consumption smoothing, so that the optimal debt must be below $\mathrm{d}_{\mathrm{K}}$. QED.

Overall, government debt and primary deficits are rather benign in this model. An observed low interest rate on government debt provides a signal that the government should perhaps go ahead and borrow more. But low interest rates do not provide an unlimited borrowing opportunity. At a sufficiently high level of debt, the interest rate rise above the growth rate so that the government would have to run primary surpluses.

At this point, one may wonder about the welfare implications of deficits in economies with both uncertainty and intermediation cost. While a general answer is beyond the scope of this paper, a simple thought experiment provides some insights. Suppose income growth $x_{t}$ is stochastic rather than deterministic and the government issues income-indexed debt (with all other assumptions as above). Since all agents have equal risk aversion, income-indexed loans are the optimal type of
private loan contract, too. For sufficiently low debt-income ratios and high $\kappa$, income-indexed debt has a return below the growth rate, as in Prop.2, and primary deficits sustained by such debt have the same benign welfare implications as in Prop.3-4. But at higher debt-income ratios, the return on income-indexed debt is unambiguously above the growth rate. Expected primary deficits can then only be sustained by low-interest debt that imposes excessive income risks on taxpayers, as in Section 3. ${ }^{16}$ Thus, when one combines uncertainty and intermediation cost, the question if the low interest rate on a given government bond is primarily explained by risk aversion or by intermediation cost is still the critical question for welfare analysis, as in the simpler models above.

## 5. Conclusions

Different "solutions" to the Mehra-Prescott puzzle produce very different policy implications for government debt and deficits. If low real interest rates on government debt are due to high risk aversion, debt financing strategies that exploit low interest rates impose significant risks on future generations of taxpayers. Unless taxpayers are far more risk tolerant than debt holders, such policies are not Pareto-optimal. Primary deficits can then be interpreted as a signal that current taxpayers are trying to shift the payment for current government services to future taxpayers. This is true even if the government promises that in expectation future taxpayers will be allowed to run budget deficits, too. On the other hand, if the government can offer low returns on its debt because of some unique ability to create safe claims, persistent primary deficits may be unproblematic, perhaps even desirable.

An assessment of actual U.S. government budget deficits would require information to what extent the historically low interest rates on U.S. government debt are the result of risk aversion or the result of unobserved "rents" on the government's ability to create safe claims. The literature on the Mehra-Prescott puzzle has unfortunately not provided a definitive answer to this question. Hence, the Mehra-Prescott low real rate puzzle has in effect created a fiscal policy puzzle.

The paper nonetheless provides a warning against the common practice of using expected values to assess government policy plans, e.g., in the CBO and Administration deficit projections. Unless one is convinced that the equity premium is due to some special advantage of government
borrowing over private borrowing rather than due to risk aversion, the use of safe, low-interest debt securities creates systematic risks for future taxpayers that are not properly reflected in the standard expected value projections.

## Footnotes

1 Since about 1980, U.S. real bond returns have been higher than in the Mehra-Prescott sample. It remains to be seen if this is a permanent structural break or a disinflationary phenomenon. But equity returns have been very high, too, so that the equity premium puzzle remains.

2 One could construct more extreme examples where primary surpluses are negative with probability one for many periods (see Bohn, 1991a, 1994, 1995; and Ball et al. 1995). But since this paper is about the welfare implications of alternative "reasonable" deficit policies and not primarily about the limits of feasibility, a simple scenario with constant debt-income ratio is sufficient. The point is that examples with frequent and/or expected primary deficits are easy to construct.
${ }^{3}$ Different sample periods may yield somewhat different numbers, but without resolving the equity premium puzzle (e.g., see Siegel 1992). The log-normal i.i.d. scenario is sufficient to illustrate the puzzle and its policy implications, and it is not far fetched because consumption growth rates are indeed almost uncorrelated.

4 Bohn (1991b) documents an average primary deficit of 0.4 percent of GDP over the period 18001988. The debt-GDP ratio is not much higher now than after the War of Independence. More recent periods, such as 1960-95, show average primary deficits, too.
5 Mankiw and Zeldes (1991) report that only a relatively small fraction of the U.S. population holds equities, providing empirical support for the notion that asset prices are determined by some fraction of the population, while others are, for whatever reasons, not active on financial markets.

6 Otherwise, budget deficits would have large interest rate effects, especially for high $\alpha$ values (see below). This should be avoided, because empirical studies find at most a small effect of budget deficits on interest rates (Plosser 1982, Evans 1987).

7 Government spending is introduced only to distinguish results about primary surpluses from results about taxes; otherwise, one could set $\mathrm{g}=0$ without loss of generality.
8 Namely, $\mathrm{c}_{\mathrm{t}}=\mathrm{Y}_{\mathrm{t}}-\mathrm{Y}_{\mathrm{t}} \cdot\left[(\right.$ Present value of $\left.G)+\left(1+\mathrm{R}_{0}\right) \cdot \mathrm{D}_{-1}\right] /($ Present value of Y$)$. Production is important at this point. Type I agents hold $D_{t} / \theta$ units of government debt. In an exchange economy, their budget equation $c^{I_{t}}+D_{t} / \theta=Y_{t}-T_{t}+\left(1+R_{t}\right) \cdot D_{t-1} / \theta$ would make consumption $c_{t}=Y_{t}-G_{t}+(1-\theta) / \theta \cdot\left[\left(1+R_{t}\right) \cdot D_{t-1}-\right.$ $D_{t}$ ] a function of the budget deficit. Through (1), deficits would have interest rate effects, huge ones if $\alpha^{\mathrm{I}}$ is as high as the equity premium requires. The production setting avoids these implausible interest rate effects; it may also be interpreted as model of a small open economy with exogenous intertemporal terms of trade. The government still operates under constraint (2) because rational type-I agents would otherwise refuse to lend (as in Bohn, 1995). The additional restriction $\mathrm{T}_{\mathrm{t}} \leq \mathrm{Y}_{\mathrm{t}}$ applies if $\mathrm{c}^{\mathrm{T}} \mathrm{t} \geq 0$ is bounded.
9 The labeling of the two types is motivated by the fact that type I (investor) determines asset prices and type T (taxpayers) determines the welfare effects of tax and debt policy. Of course, the
roles overlap, in reality as well as in the model (type I pays taxes, too). But conceptually, the model is about what happens when not all taxpayers hold exactly the amount of debt that would render debt and tax policy neutral.

10 For example, if $r<x$, a policy of issuing safe debt and keeping $D_{t} / Y_{t}$ constant yields negative expected primary surpluses $\mathrm{E}_{\mathrm{t}-1}\left[\mathrm{Z}_{\mathrm{t}}\right]=\mathrm{E}_{\mathrm{t}-1}\left[\mathrm{~T}_{\mathrm{t}}-\mathrm{G}_{\mathrm{t}}\right]=(\mathrm{r}-\mathrm{x}) \cdot \mathrm{D}_{\mathrm{t}-1}<0$ for all t .
${ }^{11}$ Note that this applies for any degree of risk aversion $\alpha^{\mathrm{I}}$ and whether or not the safe interest rate is below the growth rate.
12 Note a caveat: This argument implies a negative correlation over time horizons associated with major debt accumulations-several decades-but not necessarily at cyclical frequencies. Given a fixed debt and a fixed $g$-value, tax rates must eventually rise if growth is unexpectedly low for extended periods, but not necessarily soon.

13 Government policy in economies with intermediation cost has also been examined in the Ricardian equivalence literature (Yotsuzuka 1987). As noted there, it is important whether the government is subject to the same information problems as the private sector. Here, a key assumption is that the government has a cost advantage.
14 The timing of taxes would be irrelevant in a frictionless markets setting. With frictions, the assumption that taxes can only be levied in periods with income rules out atemporal redistributions from one type to the other, which would make the model uninteresting by removing the source of heterogeneity.
${ }^{15}$ If monitoring cost in the private sector are due to the cost of tracking "small" borrowers, such an asymmetric treatment of government debt may be reasonable because the government is by nature large and easily identifiable. If private monitoring costs are credit verification cost, it may also be reasonable to assume that such cost do not apply to the government whose credit is backed by tax revenues.
16 Results analogous to Prop.2-4 with uncertainty and income-indexed debt are derived in Bohn (1997). Results analogous to Prop. 1 that imply negative welfare effects of safe debt can be shown in a model that combines uncertainty and intermediation cost (essentially combining types A, B, and T, $\kappa>0$, and the production setting of Section 3; see Bohn 1997). Such a combined model is not presented here because it would be more much complicated than the models above without providing new conceptual insights.

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