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Should the Social Security Trust Fund hold Equities?
An Intergenerational Welfare Analysis

Henning Bohn

Department of Economics
University of California at Santa Barbara
Santa Barbara, CA 93106
Phone: (805)-893-4532
E-mail: bohn@econ.ucsb.edu

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ABSTRACT

In a stochastic economy with overlapping generations, fiscal policy affects the allocation of aggregate risks. The paper shows how to compute the welfare effects of marginal policy changes that shift risk across cohorts, in general and for an application to social security equity investments. I estimate the relevant correlations between macroeconomic shocks and equity returns from 1874-1996 U.S. data, calibrate the model, and find positive welfare effects for equity investments. Since stock returns are positively correlated with social security's wage-indexed benefit obligations, equity investments would also help to stabilize the payroll tax rate.

1. Introduction

The recent proposals by the Social Security Advisory Council (1997) to invest social security reserves in the stock market have triggered a lively debate about the merits of such investments; see Bohn (1997), Dotsey (1997), Smetters (1997). This paper examines the investment policy of the social security trust fund in the context of a simple stochastic growth model with overlapping generations.

The theoretical framework is a stochastic Diamond (1965) style economy with two-period lived agents.[1] The old receive capital income and social security transfers, consume, and pay taxes. The young receive wage income, pay regular and social security taxes, consume, make capital investments, and buy government bonds. The government sector includes real spending, regular taxes, and safe debt as well as a social security system with trust fund. Social security promises a fixed replacement rate, i.e., benefits indexed to wages at a fixed ratio.

Without government intervention, both generations share the risk of uncertain productivity growth, but only the old bear the risk of fluctuations in the value of old capital. The latter, which I call the valuation risk, provides the risk-sharing argument for trust fund equity investments. Since risks should generally be shared across generations (Bohn 1998), an allocation in which only the old bear valuation risk is inefficient. The trust fund is a device to share this risk.[2]

[1] In terms of economic theory, the paper draws on the OG literature studying intergenerational risk sharing, e.g., Enders and Lapan (1982), Gordon and Varian (1988), Gale (1990), and Bohn (1998).

[2] There are of course other devices that could be used to share such risk, e.g., state-contingent taxes (see Bohn 1998). Trust fund equity investments stand out, however, as a practically feasible policy tool that directly addresses valuation risk.

For given defined benefits to retirees, the risks and returns of alternative social security investments are borne by future generations of tax payers. In an OG setting, unborn future generations are naturally excluded from financial markets. They cannot insure themselves against fluctuations in future taxes. Because of this incomplete access to financial markets, government policy influences the allocation of macroeconomic risks across generations.

Specifically, if the trust fund shifts from debt to equity (claims to capital), the composition of private savings will shift from capital investment to fixed income investments. Individuals release equity to the trust fund and instead hold government debt. Net government debt rises as less of the gross Treasury debt is held by the social security system. Since future generations are implicitly responsible for keeping social security solvent, future payroll taxes will vary inversely with the equity returns of the trust fund portfolio. Hence, future generations bear part of the valuation risk. This is unambiguously welfare-improving.

Trust fund equity investments have two additional risk sharing implications, however, that must be addressed because they are likely negative. First, in practice, the trust fund will have to purchase specific securities, presumably a portfolio of corporate stocks. If the portfolio return is imperfectly correlated with the return on the aggregate capital stock, the idiosyncratic component of the portfolio return raises the income of the young but reduces the income of the old, i.e., it creates new generation-specific risk. I will call this the relative return risk. It is an empirical question if the better sharing of valuation risk outweighs the welfare-loss from creating relative return risk.

Second, the increase in net government debt implied by trust fund equity investments alters the allocation of productivity risk, the uncertainty about future productivity growth. If government debt is safe in real terms (here, a worst case scenario), the additional debt forces future young cohorts to pay a fixed debt service (through taxes) out of an uncertain wage income, which increases their exposure to productivity risk. The old, on the other hand, will hold more safe debt and bear less productivity risk. Bohn (1998) has argued theoretically that the government already supplies too many safe claims to the old, suggesting that an increased public debt has a negative welfare effect. Note that the type of debt matters for the magnitude of this effect. Increased debt would imply a smaller shift in productivity risk, for example, if the debt were nominal and if inflation covaried negatively with wage income. To prevent a bias in favor of trust fund equity investments (and for simplicity), I assume safe real debt and leave a discussion of alternatives to the sensitivity analysis.

Overall, the case for trust fund equity investments depends on the trade-offs between an improved sharing of valuation risk against the creation of new idiosyncratic risk (when trust fund returns and aggregate capital returns diverge) and against the potentially negative risk-sharing impact of more government debt.

The empirical part of the paper examines these trade-offs quantitatively. I use VAR and error-corrections techniques to estimate the relevant long-run correlations between U.S. wages, capital income, GDP, and stock returns. The empirical correlations are then combined with calibrated macroeconomic and policy data to estimate the net welfare effect of trust fund equity investments. While the relative return risk turns out to be small, the shifting of productivity risk through safe debt has a substantial negative welfare effect--large enough to cancel out much of the benefits from an improved sharing of valuation risk. Nonetheless, for a number of different specifications, I find that a shift to equity investments would be welfare-improving. For most specifications, the optimal portfolio consists of 100% equity.

The calibration can also be used to compute efficiency gains in terms of consumption equivalents. But the values depend significantly on the assumed relative risk aversion, a controversial parameter. If the entire trust fund was invested in claims on corporate capital (unlevered) and if one assumes a relative risk aversion of about 25 to match the historical equity premium, the estimated welfare gain is about 0.2% of lifetime consumption. With a lower risk aversion, the values would be much smaller, however; e.g., only 0.012% for log-utility.

Efficiency does not imply that government equity holdings are politically desirable, of course. The paper takes a strict welfare approach and does not address, e.g., issues of corporate control or time-consistency. Such issues are undoubtedly important for policy makers, but beyond the scope of this paper. In addition, the numerical results should be interpreted cautiously because they are based on a quite simple macroeconomic model and on covariance estimates that are subject to substantial specification uncertainty. In the policy experiment above, for example, alternative estimates of the long-run covariance matrix yield welfare gains that range from 0.1% to more than 2.5% of consumption (versus 0.2% in the benchmark case).

Separately, the paper provides a simple finance argument for trust fund equity investments. Namely, since social security benefits are linked to aggregate wages and since aggregate wages, capital income, and equity returns are correlated, equity investments can help to stabilize future payroll tax rates. My time-series estimates imply that a trust fund equity share of 50-70% would minimize the variance of payroll tax rates. From a risk-sharing perspective, however, variations in payroll tax rates are desirable if they are correlated with valuation risk. Hence, the welfare-maximizing equity share generally differs from the tax-stabilizing share.

The paper is organized as follows. Section 2 lays out the model. Section 3 derives the equilibrium allocation and examines theoretically under what conditions a debt-for-equity swap in the trust fund is welfare-improving. In Section 4, I estimate the relevant components of macroeconomic risk and I calibrate the OG model. Section 5 combines the theoretical model with the estimated risk structure to compute welfare effects. Section 6 concludes.

2. The Model

The model is a standard two-period OG economy. Generation t consists of N_t individuals who work in period t (one unit of labor, supplied inelasticly) and are retired in period t+1. Workers earn a wage w_t, pay payroll taxes at the rate θ_t, and pay other taxes τ^1_t.[3] The disposable income $w_t(1-\theta_t)-\tau^1_t$ is either consumed (c^1_t) or saved,

$$c^1_t = w_t(1-\theta_t) - \tau^1_t - s_t. \qquad (1)$$

Savings s_t are invested in a portfolio of financial assets consisting of capital assets and government bonds.

[3] The distortionary effect of taxation are ignored for simplicity. Retirement savings are assumed untaxed, implicitly assuming that such savings takes place (at least on the margin) through tax-sheltered instruments like pension plans, variable annuities, or IRA accounts.

In practice, claims on capital are represented by shares of corporate stocks, corporate bonds, and various other capital assets, e.g., claims to smaller, privately-held companies, or real estate. All proposals to invest social security trust fund in equities assume passively-managed equity investments in well-established, exchange-traded corporations, i.e., in a subset of the capital stock. Hence, the return on trust fund holdings will almost inevitably differ from the return on aggregate capital. To distinguish these returns, let $R^k{}_{t+1}$ be the return on the total capital stock K_t (between periods t and t+1), let $R^e{}_{t+1}$ be the total return on the equity portfolio suitable for trust fund investments, and let $R^b{}_{t+1}$ be the return on bonds. For simplicity, corporate bonds are considered equivalent to government bonds. Generically, returns are denoted by $R^i{}_{t+1}$ for $i\ I$, where I = {e,b,k} is a list of relevant investments. Note that the return on the assets <u>not</u> held by social security (a long position in R^k combined with a short position in R^e and/or R^b) is spanned by (R^k,R^b,R^e).

Individual savings are allocated over different investments $s^i{}_t$, $s_t = \sum_{i\ I} s^i{}_t$. In period t+1, the old receive a return $\sum_{i\ I} R^i{}_{t+1} s^i{}_t$ on savings, they receive wage-indexed social security benefits at fixed replacement rate β (assuming a defined-benefits system), and they pay taxes $\tau^2{}_{t+1}$. Their consumption is

$$c^2{}_{t+1} = \sum_{i\ I} R^i{}_{t+1} s^i{}_t + \beta w_{t+1} - \tau^2{}_{t+1}. \tag{2}$$

Preferences are CRRA with risk aversion η,

$$U_t = \frac{1}{1-\eta}\left\{(c^1{}_t)^{1-\eta} + \rho E_t[(c^2{}_{t+1})^{1-\eta}]\right\}, \tag{3}$$

where ρ is the rate of time preference.[4] Individual savings and investment choices are then characterized by the optimality conditions

[4] CRRA implies a tight link between the risk-aversion η and the elasticity of intertemporal substitution $1/\eta$. If η is calibrated to match the equity premium, this may impose undue restrictions on savings behavior. To avoid this linkage, an earlier version of this paper considered Epstein-Zin (1989) preferences. Since the empirical findings are

$$E_t[\rho\,(c^1{}_t/c^2{}_{t+1})^{-\eta}\,R^i{}_{t+1}] = 1 \quad i \;\; I. \tag{4}$$

For production, I assume that aggregate output is produced from aggregate capital K_t and labor N_t according to a Cobb-Douglas technology with capital coefficient α,

$$Y_t = K_t^{\alpha}\,(A_t\,N_t)^{1-\alpha}. \tag{5}$$

Total factor productivity A_t is stochastic with i.i.d. growth rate a_t.[5] The total resources available for consumption and new capital investment are $Y_t + v_t\,K_t$, where v_t is the value of old capital.

The marginal products of labor and aggregate capital are then

$$w_t = (1-\alpha)\,A_t^{1-\alpha}\,K_t^{\alpha}\,N_t^{-\alpha} = (1-\alpha)\,A_t\,\Big(\frac{k_t}{(1+a_t)\,(1+n)}\Big)^{\alpha} \tag{6}$$

$$R^k{}_t = \alpha\,K_t^{\alpha-1}\,(A_t\,N_t)^{1-\alpha} + v_t = \alpha\,\Big(\frac{k_t}{(1+a_t)\,(1+n)}\Big)^{\alpha-1} + v_t \tag{7}$$

where $k_t = K_t/(A_{t-1}\,N_{t-1})$ is the component of the effective capital-labor ratio known at time t-1, $1+a_t=A_t/A_{t-1}$ is the stochastic productivity growth rate, and $1+n=N_t/N_{t-1}$ is the population growth rate, assumed constant.

Thus, the return on capital depends in part on capital income and in part on the value of capital, v_t. For Cobb-Douglas production, the capital income part is perfectly correlated with output and wages.[6] The value v_t is assumed to be stochastic, i.i.d., to allow the total return on capital to vary somewhat independently from the wage rate. One may interpret $(1-v_t)$ as a stochastic depreciation rate, but I will interpret the randomness in v_t

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similar for both specifications (see Table VIII below), the exposition here focuses on the simpler CRRA case.

[5] I assume i.i.d. noise throughout the paper because even highly autocorrelated annual time series are close to i.i.d. at generational frequencies. The issue of unit root growth versus deterministic trend is discussed in Bohn (1998). Briefly, trend breaks cannot be ignored at generational frequencies, and if trend breaks are possible, uncertainty about future productivity rises with the forecast horizon; this is best captured by a unit root process. In the data (see below), a unit root for real GDP cannot be rejected.

[6] Empirically, the capital and labor shares in GDP show some short-run variability (Shiller, 1993; Gomme and Greenwood, 1995) suggesting slight deviations from the Cobb-Douglas assumption. But capital and labor shares are so highly correlated at generational frequencies (see below) that a time-varying capital share would be more distracting than insightful.

more broadly as representing all shocks that make the market return on capital assets uncertain.[7]

The government sector is modeled in a way that focuses on social security. A simple representation of other government activities is needed, however, to capture the risk sharing implications of other policy instruments--notably, government debt. Government debt D_t is assumed stationary relative to trend productivity growth, $D_t = d\,(A_t N_t)$, and government spending is a constant fraction of GDP, $G_t = g\,Y_t$. The taxes on old and young must satisfy the budget equation

$$G_t + R^b_t D_{t-1} = N_t\,\tau^1_t + N_{t-1}\,\tau^2_t + D_t. \tag{8}$$

For the calibrations, I assume that $\tau^2_t = \xi^2\,Y_t/N_t$ is a constant share of per-capita income, which leaves τ^1_t to be determined by (8).

Social security provides wage-indexed benefits $\beta\,w_t$ to the old that are financed by payroll taxes θ_t and by a trust fund. To obtain balanced growth, I assume that new investments in the trust fund TR_t are proportional to the growth trend with proportionality factor σ, $TR_t = \sigma\,N_t\,A_t$. A share ι^e is invested in an equity portfolio with return R^e_{t+1}, the remainder in bonds, $\iota^b = 1-\iota^e$. To maintain the trust fund balance at the specified level, payroll taxes must be

$$\theta_t\,w_t = \beta\,w_t + \sigma\,A_t - (R^e_t\,\iota^e + R^b_t\,\iota^b)\;\sigma A_{t-1}/(1+n). \tag{9}$$

That is, the level of payroll taxes depends on productivity growth and on the investment performance of the trust fund.

[7] The assumption that R^k depends only on “fundamentals” is conservative in this context. If there is excess volatility in stock prices--meaning existing capital trades at prices different from fundamental--the return on capital would co-vary with the “gap” between price and fundamental that the young must pay to buy up the capital stock. Since such a gap would ceteris paribus reduce the consumption opportunities of the young, it would create an element of negative correlation between the effective resources of old and young. Here I will show that variations returns that do NOT affect the young are sufficient to provide a rationale for trust fund equity. A negative correlation would strengthen the rationale for sharing valuation risk across generations.

The distinction between R^e_{t+1} and R^k_{t+1} is important because any idiosyncratic risk in the trust fund would weaken the case for equity investment. To model R^e_{t+1} parsimoniously, I assume that the firms in the trust fund portfolio have a Cobb-Douglas technology with the same capital coefficient α as the aggregate production function. Hence, they operate at the same capital-labor ratio. I will, however, allow their productivity level to diverge from the aggregate and I will allow for leverage. (The specific assumptions are motivated by the empirical work below.) Let

$$R^f_t = \alpha A_t \left(\frac{k_t}{(1+a_t)(1+n)}\right)^{\alpha-1} (1+\mu_t) + v_t (1+\mu^*_t) \qquad (10)$$

be the total return on the firms' capital (equity and debt), where μ_t is an i.i.d. shock to the firm's capital income relative to aggregate capital income and μ_t is a shock to the firm's relative value. Assuming values are driven by earnings, I let μ^*_t be a deterministic function of μ_t, $\ln(1+\mu^*_t) = \pi_{v\mu} \ln(1+\mu_t)$, where $\pi_{v\mu} \geq 0$ is a constant elasticity coefficient. Finally, the return on equity is

$$R^e_{t+1} = \lambda R^f_{t+1} - (\lambda-1) R^b_{t+1}. \qquad (11)$$

where $\lambda>1$ is the ratio of firm capital to equity (= 1 + debt-equity ratio). Note that the trust fund could hold an unlevered claim on firm capital by setting $\iota^e=1/\lambda$ (≈74% for the S&P500); but the portfolio return would still depend on the relative return shock μ_t.

Overall, the accounting for income is as follows. The young earn a wage income that depends on the productivity shock a_t. The old receive the aggregate capital income that depends on a_t and on the valuation shock v_t, plus a wage-indexed social security income that depends on a_t, plus safe debt. Without trust fund equity holdings, the return on equity relative to aggregate capital is irrelevant, because the old hold the entire capital stock.

With trust fund equity holdings, the defined-benefit nature of social security implies that future generations of taxpayers bear the risk of fluctuations in the value of the trust fund.[8] They effectively own the trust fund because their payroll taxes must rise whenever the trust fund earns a low return, and vice versa. Since R^e_{t+1} depends positively on a_t, v_t, and μ_t, the next young cohort obtains a positive exposure to v_t and μ_t shocks and an increased exposure to a_t shocks. In equilibrium, the old hold the aggregate capital stock except for the trust fund portfolio. Hence, for $\iota^e>0$, their income depends negatively on the relative return shock μ_t, and their pre-existing exposure to a_t and v_t shocks is reduced.

Intuitively, the sharing of valuation risk v_t should be welfare-improving. The μ_t-shocks are generally welfare-reducing because they affect old and young in opposite directions. And the re-allocation of productivity risk may have a positive or negative welfare-effect, depending on how efficiently this risk is allocated initially. To determine if the positive or the negative welfare effects dominate, one has to examine the equilibrium allocation of risk and its dependence on policy.

3. General Equilibrium and Welfare Analysis

This section examines the equilibrium allocation of risk and the welfare implications of alternative policies.

3.1. General Equilibrium

For any given set of policy rules, the equilibrium allocation is determined by successive generation’s savings decisions. Each period, the initial

[8] Note the key role of defined benefits in this argument. If retiree benefits were made a function of trust fund returns, the old would bear investment risk. Less risk would be shifted across generations. The defined benefit nature of social security explains why the trust fund has fundamentally different risk sharing implications than, say, private pension funds or “privatized” social security accounts. The latter would be irrelevant here because they do not shift risk across generations (see Bohn 1997).

capital-labor ratio k_t and the three shocks (a_t, v_t, μ_t) determine the resources available to the old and to the young. The old consume their income and assets. The young divide their disposable income between consumption and savings. Aggregate savings then determine the next period's initial capital labor ratio. Since total factor productivity is growing, per-capita incomes and consumption levels are non-stationary. However, the ratios of capital to labor, wage to productivity, and consumption to productivity converge to a stochastic steady state. In terms of productivity ratios, the economy is a Markov process with state variables (k_t, a_t, v_t, μ_t).[9]

For realistic policies and preferences, the dynamics are sufficiently non-linear that the individual decision problems have no closed form solution. Hence, I follow the business cycle and finance literature and log-linearize constraints and the first order conditions. However, in contrast to much of the literature, I derive analytical formulas for the log-linearized solutions. The resulting elasticity coefficients describe the movements of consumption and capital investment as functions of the shocks for any set of policy parameters, and they can be used to determine the approximate welfare effects of arbitrary policy changes.[10]

I consider two versions of the log-linearization. The most straightforward approach is to linearize around the deterministic steady state, as is common in the business cycle literature (e.g., King-Plosser-

[9] As usual, an equilibrium is defined as sequence of savings choices such that (i) individuals satisfy the Euler equations and budget constraints for given wages, return distributions, and policy rules (as explained above); (ii) firms maximize profits; (iii) individual and firm choices are consistent with the aggregate constraints. Throughout, I assume that the scale of intergenerational redistribution is such that the economy is dynamically efficient.

[10] In contrast, calibration usually provides numerical solutions for only a few discrete parameter settings. Though I will also present discrete policy comparisons later, my analytical approach yields derivatives with respect to policy variables, i.e., it allows an analysis of marginal policy changes and the resulting marginal cost/benefit tradeoffs.

Rebelo, 1988). For asset pricing issues, it is more instructive, however, to log-linearize only the budget equations and to assume log-normality. The stochastic Euler equations can then be evaluated exactly, without further approximation. This approach is motivated by recent work in finance (Campbell and Viceira, 1996). Both approximations yield the same slope coefficients for the decision rules, but the stochastic Euler equations yield additional intercept terms that capture the "displacement" of the stochastic from the deterministic steady state. Since the intercept terms are inessential for many results (and complicated), I compute the intercept terms only when they are conceptually important (e.g., for the equity premium) and otherwise use the King-Plosser-Rebelo approach.

For either method, let $\hat{x}_t = \ln(x_t)-\ln(x)$ denote the log-deviation of a variable x_t from its steady state x (without subscript). The log-linearized laws of motion can then be written as

$$\hat{x}_t = \pi_{x0} + \pi_{xk}\,\hat{k}_t + \pi_{xv}\,\hat{v}_t + \pi_{x\mu}\,\hat{\mu}_t + \pi_{xa}\,\hat{a}_t, \tag{12}$$

for all relevant variables (e.g., $x=c^1,c^2,k_{t+1}$).[11] The coefficients π_{xs} can be interpreted as the elasticities of the endogenous variable x_t with respect to the state variables ($s=k,v,\mu,a$). The elasticity coefficients are fixed for given policy parameters (d,σ,ι^e), but they change when policy is altered.

Applied to the model of Section 2, the formulas for the elasticity coefficients essentially confirm the intuition presented above (a list of formulas is therefore omitted, but available from the author): Without trust fund equity investments (at $\iota^e=0$), the young carry substantial exposure to productivity risk through their wage income, which is magnified

[11] The intercept terms π_{x0t} are always formally included, but set zero in the King-Plosser-Rebelo approximation. For consumption, the coefficients π_{c1s} and π_{c2s} refer to the stationarity-inducing transformations $x_t=c^1{}_t/A_{t-1}$ and $x_t=c^2{}_t/A_{t-1}$, but this does not change the economic interpretation.

by government debt (π_{c1a}>0), but they do not bear valuation risk (π_{c1v}=0). The old bear productivity and valuation risk, π_{c2v}>0, π_{c2a}>0. For ι^e=0, the relative return risk is irrelevant, $\pi_{c1\mu}=\pi_{c2\mu}=0$. But if the trust fund invests in equity (ι^e>0), valuation, productivity, and relative return risk is shifted from the old to the young:

$$d\pi_{c1v}/d\iota^e>0,\ d\pi_{c1a}/d\iota^e>0,\ d\pi_{c1\mu}/d\iota^e>0$$

whereas $$d\pi_{c2v}/d\iota^e<0,\ d\pi_{c2a}/d\iota^e<0,\ d\pi_{c1\mu}/d\iota^e<0.$$

For ι^e>0, the young are therefore positively exposed to relative return risk whereas the old are negatively exposed ($\pi_{c1\mu}>0>\pi_{c2\mu}$).

3.2. Welfare Analysis

For any generation, the effect of any policy change on expected utility can be determined by taking the derivative of the utility function (3) at the consumption path implied by the equilibrium allocation. Often, the effects will be positive for some generations and negative for others, due to transition effects. Hence, to focus on efficiency without getting distracted by distributional complications, I will use a social planning approach.

The social planner’s problem is to maximize a weighted average of all generation’s utilities,

$$W_0 = E_0[\sum_{t=0}^{\infty} \omega(t)\, U_t], \tag{13}$$

where $\omega(t)>0$. The question if a policy change is welfare-improving can be answered by differentiating (13) subject to the log-linearized macroeconomic dynamics. The derivative-taking involves some technical subtleties that are discussed in a technical appendix available from the author. Briefly, I focus on marginal, one-time variations in a single policy parameter ξ (e.g., $\xi=\iota^{e}_{0}$ in the trust fund application);[12] I assume that the welfare weights are consistent with balanced growth, which implies weights of the form $\omega(t)=[\omega^{*}/(1+a)^{1-\eta}]^{t}$ for a fixed ω^{*} (0,1); and I take the welfare-derivative at a point where the level of intergenerational redistribution matches the planners welfare weights.

The latter assumption implies that small deterministic transfers across generations have a zero welfare effect on the margin. Therefore, whenever a policy change reallocates risk in a way that the welfare function strictly increases on the margin, there exist compensating transfers such that the overall change is Pareto-improving. Moreover, if a policy change is efficiency-improving in the sense of increasing the welfare function, the derivatives of individual utilities with respect to the change will reveal which generations (if any) would have to receive the compensating transfers. Overall, this approach separates efficiency and redistributional considerations and prevents the analysis of risk-sharing from being contaminated by distributional side-effects.

Using log-linear approximations, the derivative of W_0 with respect to any policy parameter ξ can be written as a linear combination of two quadratic forms involving the stochastic shocks and the elasticity coefficients, namely

$$\frac{dW_0}{d\xi} = \eta\,\bar{u} \left\{ \text{QFORM1} - \Omega\,(\pi_{c1k}-\pi_{c2k})\ \ \text{QFORM2} \right\} \tag{14}$$

where

$$\text{QFORM1} = (\pi_{c1s}-\pi_{c2s})_s{}'\ \text{COV}_s\ \left(\frac{d\pi_{c2,s}}{d\xi}\right)_s; \text{ and}$$

$$\text{QFORM2} = (\pi_{ks})_s{}'\ \text{COV}_s\ \left(\frac{d\pi_{k,s}}{d\xi}\right)_s.$$

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[12] In principle, welfare could be maximized over a variety of policy instruments, either chosen period-by-period or fixed for all times. I focus on one-dimensional, one-period changes because the question of social security equity investments is one-dimensional and because multi-period or permanent changes could always be interpreted as a succession of one-period changes. The one-period case also helps to emphasize that even one-time changes have long-lasting effects.

Here, COV_S is the 3×3 covariance matrix of the shocks s (s=a,ν,μ), the $(\frac{d\pi_{x,s}}{d\xi})_s$ are 3×1 vectors of derivatives with elements $d\pi_{x,s}/d\xi$; $(\pi_{c1,s}-\pi_{c2,s})_s'$ and $(\pi_{k,s})_s'$ are transposed 3×1 vectors of elasticity coefficients; $\bar{u}=(c^2{}_1)^{1-\eta}/A_0>0$ and $\Omega = \frac{y^1}{(c^2/A)/(1+n)} \frac{\pi_{y1k} \quad \omega}{1-\omega \quad \pi_k^2} >0$ are constants.

The intuition is as follows. For each of the shocks, $\pi_{c1s}-\pi_{c2s}$ is positive whenever the exposure of the young exceeds the exposure of the old. Holding capital investment constant, a policy change that reduces the risk-exposure of the old ($d\pi_{c2s}/d\xi<0$) will equally increase the exposure of the young ($d\pi_{c1s}/d\xi>0$). Hence, a policy change makes a positive contribution to QFORM1 if it shifts risk to the generation that is initially less exposed to it, i.e., if it leads to a more equal risk-sharing. If the shocks are correlated or several shocks are involved, their impact is weighted by the covariance matrix.

The term proportional to $(\pi_{c1k}-\pi_{c2k})$ captures the impact of a time-zero policy change on future generations through variations in the capital stock. (Intuitively, this is the part omitted by “holding capital constant” above.) If one were solving for the first-best optimal policy, this term could be ignored, because $\pi_{c1k}=\pi_{c2k}$ is a necessary condition for a first-best allocation of risk (Bohn 1998). But in a generic market allocation, even if one optimizes over one policy variable, π_{c1k} and π_{c2k} generally differ. Hence, this “capital term” cannot be omitted in an analysis of marginal policy changes (a second-best setting). It can be positive or negative depending on the signs of $\pi_{c1k}-\pi_{c2k}$ and of QFORM2. (A more detailed interpretation is omitted because this term turns out to be small empirically; see Table VIII below.)

In general, to determine the sign and magnitude of $dW_0/d\xi$ for a particular policy experiment, one needs an estimate of the covariance matrix COV_S and information about the elasticity coefficients and constants in (14).

In the application to trust fund equity investments, some properties of the variables in (14) are already implied by the theoretical model. Notably, since $d\pi_{c2,s}/d\iota^e<0$ s, $\pi_{c1\mu}=\pi_{c2\mu}=0$ at $\iota^e=0$, and $\pi_{c2v}>\pi_{c1v}=0$ at $\iota^e=0$, we know that at $\iota^e=0$, the v_t-component of QFORM1 makes a positive contribution and the μ_t-component is zero (and second-order for small ι^e). If there were no productivity risk, QFORM1 would be unambiguously positive at $\iota^e=0$ and declining with ι^e. Hence, the theoretical analysis leaves three open questions that call for an empirical examination. The question are (a) about the impact of non-zero productivity risk, (b) about the magnitude of the $(\pi_{c1k}-\pi_{c2k})$-term, and (c) how fast the marginal welfare gain declines as ι^e rises above zero.

The data to answer these questions are assembled in the next section. Once the data are obtained, one may also address a fourth question, namely about the welfare effects of discrete shifts in the portfolio share ι^e.[13]

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[13] Before moving on, note that even if a policy change raises W_0, it does not necessarily raise the expected utility of every generation. One can show that variations in ξ have an approximate utility impact of $E_0[\frac{dU_0}{d\xi}] = \eta\ \bar{u}\ (\pi_{c2,s})_s\ COV_S\ (-\frac{d\pi_{c2,s}}{d\xi})_s$ on generation zero. If the covariance matrix COV_S is dominated by the diagonal, this is positive for the social security equity experiment: At $\iota^e=0$, μ_t is irrelevant ($\pi_{c2\mu}=0$) while $d\pi_{c2S}/d\iota^e<0$ and $\pi_{c2S}>0$ for s=a,v, making a positive contribution to the quadratic form. Thus, generation t=0 benefits. For generations $t\geq1$, $dU_t/d\xi$ may have either sign. But for large t, one can show that the utility change is proportional to the quadratic form (1,0,0) COV_S $(-d\pi_{kS}/d\xi)_S$. If COV_S is dominated by the diagonal, the sign is given by $(-d\pi_{ka}/d\xi)$. For $\xi=\iota^e_0$, $d\pi_{ka}/d\iota^e>0$ implies a negative welfare impact on future generations. Hence, compensating transfers from generation 0 to later generations are likely required to implement a Pareto-improving policy change. (Intuitively, the future generations taking equity risk must receive most of the equity premium resulting from the social security debt-equity swap.) Such distributional issues are discussed in more detail in Bohn (1997); here I focus on efficiency questions.

4. Estimation and Calibration

As discussed above, the welfare effects of alternative policies depend importantly on the covariance matrix of macroeconomic shocks (COV_S) and the elasticity coefficients π_{XS} in (14). This section explains how these components of (14) are obtained. In passing, I also estimate the trust fund portfolio that would stabilize payroll tax rates.

4.1. Estimation of Long-Run Risks

This section examines the time series of U.S. aggregate income, wages, equity prices, and corporate earnings to draw inferences about the relevant variances and correlations at generational frequencies.

Since long-run variances and correlations are at issue, I focus on long-run data, a 1871-1996 sample and a 1929-96 sample (as opposed to simply using post-war data). The data sources are the National Income Accounts (NIPA) for post-1929 GDP and its components, Romer’s (1989) data for pre-1929 output, and Shiller’s (1989) data on equity prices, dividends, and earnings as proxied by the S&P500, updated to 1996. The GDP components necessary to compute capital and labor shares are, to my knowledge, only available for 1929-96; this motivates the shorter sample. Standard time series tests show that one cannot reject a unit root in real GDP and in equity prices, while the capital and labor shares, the price-earnings ratio, and dividend-earnings ratio are stationary in all samples. In addition, the ratio of aggregate capital income to S&P500 earnings is trend-stationary, which will be important below.

A preliminary issue is to verify the reasonableness of the Cobb-Douglas specification with its constant capital and labor shares. Empirically, capital and labor shares are not strictly constant, but their stationarity combined with the non-stationarity of GDP and capital and labor income implies cointegration, i.e., an asymptotic unit correlation.

The relevant time horizon for the OG model is long but finite. To estimate the relevant correlations of capital and labor, I use two alternative statistical models. First, I estimate a VAR with wage growth and the log-capital share and infer the long run correlations from the estimated VAR companion matrix. The VAR specification imposes the cointegration restriction. Second, as a robustness check, I have run a VAR with wage growth and capital income growth and include the lagged capital-labor ratio as regressor, making it an error corrections model (ECM). This specification allows the data to determine if the error-corrections term has empirical relevance. The VAR-based correlations of capital and labor income (VAR with two lags for 1932-1996) are shown in Figure 1. The correlations are clearly increasing with the time horizon and are close to one for 20-30 years, the time scale relevant for the OG model. This confirms similar findings in Baxter and Jermann (1997). The ECM correlations are similar, too, and therefore not shown separately.

Since wage-growth and GDP-growth are virtually identical over generational horizons, I will use GDP-growth as proxy for wage-growth in the following analysis. This allows me to use the longer 1871-1996 sample for which explicit wage data are not available. The high correlation also serves as justification not to include a time-varying capital share in the theoretical model above.

For the main task of estimating the correlation matrix of productivity, valuation, and relative valuation shocks (COV_S), the lack of market values for the aggregate capital stock is a significant obstacle. Productivity shocks are identified in the data by innovations in wage income (or as proxy, GDP). But variations in equity returns might reflect either aggregate valuation shocks (v_t) or changes in the relative value (μ_t) of the selected companies.

My approach is to exploit time series data on S&P500 earnings for the identification.[14] If firm earnings are interpreted as capital income minus accounting depreciation, the relationship between firm earnings and aggregate capital income allows inferences about the relative performance of the firms in the equity portfolio, i.e., about the μ_t-shock. Namely, one can express the log-variations in firm earnings ($E^f{}_t$) relative to GDP as an approximately linear function of a_t and μ_t, $(\hat{E^f/Y})_t = (\lambda^E-1)(1-\alpha)\hat{a}_t + \lambda^E\hat{\mu}_t + E_{t-1}[(\hat{E^f/Y})_t]$, where $\lambda^E>1$ is the steady state ratio of firm earnings to aggregate capital income. (The $E_{t-1}[]$-term is uninteresting here. A derivation is available from the author.) The key identifying assumption is that ordinary accounting earnings are unaffected by unexpected changes in market prices (the v_t-shock). Since general productivity shocks are identified by the innovations in GDP-growth, $Y_t-E_{t-1}[Y_t] = (1-\alpha)\hat{a}_t$, the earnings-income ratio identifies the relative shock. Given $\hat{a}_t$ and $\hat{\mu}_t$, the valuation shock $\hat{v}_t$ is identified by the equity return $\hat{R}^e{}_t$, the log-linearized version of (11). Since $\pi_{v\mu}$ in (11) already parametrizes the interaction between relative earnings and aggregate values, a correlation estimate for $\hat{\mu}_t$ and $\hat{v}_t$ would be redundant. Hence, I assume that $\hat{v}_t$ and $\hat{\mu}_t$ are conditionally uncorrelated, conditional on $\hat{a}_t$. Then $\hat{v}_t = \pi_{va}\hat{a}_t+\hat{v}^0{}_t$ and $\hat{\mu}_t = \pi_{\mu a}\hat{a}_t+\hat{\mu}^0{}_t$ can be decomposed into orthogonal components so that

[14] The alternative would be to ignore the problem and to interpret a broad basket of equities such as the S&P500 as an accurate measure of aggregate asset values. But this would not be adequate here, because it would assume away the μ_t-shock and bias the analysis in favor of social security equity investments. Accounting data may include measurement error in the sense that accounting and economic concepts do not match; but such measurement error is likely to inflate the estimated variance of μ_t, i.e., to bias the analysis against social security equity investments.

$(\hat{v}\theta_t, \hat{\mu}\theta_t, \hat{a}_t)$ has a diagonal covariance matrix. Overall, the covariance matrix of innovations in $(Y_t$, $(E^f/Y)_t$, $R^e{}_t)$ exactly identifies the six parameters Var$(\hat{a}_t)$, Var$(\hat{v}\theta_t)$, Var$(\hat{\mu}\theta_t)$, π_{va}, $\pi_{\mu a}$, and $\pi_{v\mu}$.

Thus, we are interested in the long-run covariance matrix of $(Y_t$, $(E^f/Y)_t$, $R^e{}_t)$. To be specific, I will use T=30 years as generational time unit and focus on the 30-year ahead covariance matrix. Long run stock returns are conveniently obtained as the sum of a dividend and a capital gains component, using log-linear approximations as in Campbell et al. (1997). Hence, the times series analysis involves stock prices (P), dividends (DIV), earnings (E^f), and output (Y). The covariance matrix is computed from either (i) a VAR that imposes the appropriate unit root and cointegration restrictions or (ii) an error-corrections model that lets the data determine the relevance of the cointegrating relationships.

For the VAR specification, I exploit the trend-stationarity of the earnings-output ratio and the stationarity of the price-earnings and dividend-earnings ratios to estimate the system $[\Delta\ln(Y_i)$, $\ln(E^f/Y)_i$, $\ln(P^e/E^f)_i$, $\ln(DIV/E^f)_i]$ with two lags, constant, and time trend. Table I displays the estimates for the main 1874-1996 sample (1871-1996 data minus lags). Unit root statistics are also provided to show that the unit root properties suggested by the theoretical model are consistent with the data. Table V, Column 1, shows the implied 30-year covariances and correlations of output, earnings, and returns. As specification check, I also estimate the model for a shorter 1932-1996 sample (1929-96 data minus lags) and with wage income instead of GDP. The results are similar and shown in Tables II-III and Columns 2-3 of Table V.

For the error corrections specification, I estimate the system $\Delta\ln(Y_i)$, $\Delta\ln(E^f{}_i)$, $\Delta\ln(P^e{}_i)$, $\Delta\ln(DIV_i)$ in first differences, also for 1874-

1996. As regressors, I include two-lags, a constant, and a time trend as well as the lagged values of the stationary variables $\ln(E^f/Y)_i$, $\ln(P/E^f)_i$, and $\ln(DIV/E^f)_i$. Table IV shows that one or more of the error corrections terms are significant in all but the $\Delta\ln(Y)$ equation. Most importantly, the error-corrections effect linking aggregate output to corporate earnings (and therefore indirectly to prices and dividends) is highly significant, showing that the performance of equities is linked to the performance of the macroeconomy.[15] Table V, Column 4, shows that the error corrections specification implies similar structural parameters as the VARs, except that the estimated 30-year variance of equity returns is higher than in the VAR estimates (to be discussed below).

Separately from the welfare analysis, the above results have some direct implications for social security investment policy if the stability of payroll tax rates is a policy objective. As an approximation, the log-variance of the payroll tax rate θ_{t+1} depends on the “misalignment” between wage growth and the return on equity,

$$VAR_t(\ln(\theta_{t+1})) \approx (\sigma/\theta\, w/A)^2\, VAR_t[\hat{Y}_{t+1} - \iota^e \hat{R}^e_{t+1}].$$

The equity share in the minimum variance portfolio is therefore

$$\iota^{e*} \approx \frac{COV_{t-1}[\hat{Y}_{t+1}, \hat{R}^e_{t+1}]}{VAR_{t-1}[\hat{R}^e_{t+1}]}. \tag{15}$$

This is positive if productivity and equity returns are positively correlated, as they are in the data. Estimates for the variance-minimizing equity shares are displayed in Table V. They range from 49-72%, depending on the specification.[16] In terms of financial management, the intuition is

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[15] The error corrections terms are important for obtaining the high 30-year correlations between output and stock returns shown in Table V. These regressors--which are theoretically motivated and empirically significant--explain why I obtain much higher correlations between macroeconomic and stock market data than Shiller (1993).

[16] Note that actual U.S. social security benefits are only wage indexed until retirement and inflation indexed thereafter, so that not all benefit obligations are wage indexed. The numerical values should therefore be interpreted cautiously.

that the wage-indexed liabilities of the social security system are better matched by equities than debt because equity returns and wage growth have a similar exposure to productivity risk. Note, however, that stabilizing the payroll tax rate is not the same as maximizing welfare.

4.2. Calibration

Returning to the welfare analysis, this section derives calibrated values for the policy coefficients π_{xz} and other items in the welfare condition (14).

As a first step, a conversion of annual into generational quantities is required to interpret standard macro data in the context of a two-period OG model. To calibrate generational quantities, I assume that individuals follow a stylized life-cycle pattern of a work/savings/asset-accumulation phase of T years (within generational period t) followed by a retirement/asset-decumulation phase (period t+1) of the same length. In every year i of period t, working individuals have wage income $w_i = (1-\alpha)Y_i/N_i$. (Years are indexed by i; symbols are as in the OG model.) They pay a cash flow amounting to $CF_{i,t}/(1+n)+G_i/N_i$ <u>to</u> the government, where $CF_{i,t}$ (defined below) is the per-capita cash flow that the old receive <u>from</u> the government; $1+n=(1+n^*)^T$ is the T-th power of the annual population growth rate n^*. Of the disposable income $y^1_{i,t}=w_i-(CF_i/(1+n)+G_i/N_i)$, a fraction $(1-s)=c^1_{i,t}/y^1_{i,t}$ is consumed. The remainder is invested in claims on capital. Claims on capital have an annual return r^k. (Since the OG model is linearized around a steady state, deterministic calculations are sufficient here; time indices are omitted for simplicity.) Let ACC_t be the individual wealth accumulation over period t, discounted forwards and backwards to the midpoint of period t. If per-capita incomes grow at the annual rate a^*,

$$ACC_t = \sum_{i=1}^{T} (1+r^k)^{T/2-i}\, s\, y^1_{i,t} = s\, y^1_{T/2,t}\ \Psi$$

where $\Psi = \sum_{i=1}^{T} [(1+r^k)/(1+a^*)]^{T/2-i}$ is a conversion factor that translates annual savings into generational quantities. Moving forward one generational period, the value of ACC_t in the middle of period t+1 is $ACC_t\, R^k_{t+1}$, where $R^k_{t+1} = (1+r^k)^T$ is the T-th power of the annual return. The amount $ACC_t\, R^k_{t+1}$ can be converted back into an annual flow of retirement income that enables the old to consume $c^2_{i,t+1} = ACC_t\, R^k_{t+1}/\Psi\, (y^1_{i,t+1}/y^1_{T/2,t+1}) + CF_{i,t+1} = s\, y^1_{i,t+1}\, R^k_{t+1}/(1+a) + CF_{i,t+1}$.

This stylized individual model can be embedded in a production economy by assuming that individual net accumulations are pooled into a fund making capital investments I_i. With annual depreciation d_i, the capital stock is $K_{i+1} = (1-d_i)\, K_i + I_i$. Returns are $r^k_i = \alpha\, Y_i/K_i - d_i$. The fraction $\alpha\, Y_i/K_i/(1+r^k_i)$ of the return is productivity-dependent, while the remainder, $(1-d_i)/(1+r^k_i)$, depends on the value of old capital. I therefore equate v/R^k with annual data on $(1-d_i)/(1+r^k_i)$ to calibrate the elasticity of R^k_t with respect to v_t.

In steady state, capital income plus the savings of the young must finance gross investment plus the withdrawals of the old,

$$\alpha\, Y_i + s\, y^1_i\, N_i = I_i + s\, y^1_{i,t}\, R^k_t/(1+a)\, N_i/(1+n).$$

The savings rate of the young can therefore be calibrated as

$$s = \frac{\alpha\, Y_i - I_i}{y^1_i\, N_i\, [R^k/(1+a)/(1+n)-1]}, \tag{16}$$

a function of observable annual variables. (If these calculations look heroic, keep in mind that this is just to calibrate the steady state.)

The cash flow from the government to the old includes social security benefits, other net transfers (deducting taxes), the interest on the government debt, and principal payments on the government debt such that the debt is turned over to the next generation after T years. Assuming $CF_{i,t}$

is proportional to Y_i/N_i within a period, and the debt-GDP ratio is constant in steady state, this implies

$$CF_{i,t} = \beta w_i - \tau^2{}_i + \frac{1}{\psi} D^*{}_i/N_i.$$

Overall, this year-by-year interpretation of the life cycle makes explicit how exactly two-period OG model abstracts from infra-period variations in economic activity. One can think of economic activity as taking place continuously along a balanced growth path (hence the assumed proportionality to Y_i/N_i) and then being time-aggregated into broad periods for analytical purposes. For individuals, uncertainty at generational frequencies is effectively injected at the end of period t, when $R^k{}_{t+1}$ and $CF_{i,t+1}$ may jump relative to the expected values. Note that the annual steady state capital-output ratio K_i/Y_i (the appropriate proxy for K_t/Y_t in the OG model) is not directly related to the individual wealth accumulation ACC_t. If capital mostly depreciates in less than T years, retirement savings require repeated reinvestment along the way.

For the calibration, I use average 1929-96 values to estimate the technological and behavioral parameters, such as the capital share and the depreciation and investment rates. But to assess current policy alternatives, I use more recent values for policy parameters and for interest and growth rates. The main parameters and their sources are listed in Table VI, and some features of the implied steady state are shown in Table VII. The most tenuous choice is probably the division of regular taxes between old and young, the choice of $\tau^2{}_t$. Lacking better data, I allocate net taxes (from NIPA 1995, excluding OASDI and Medicare) to old and young in proportion to their factor shares. Note that the assumed size of the trust fund (σ) is 7.2% of GDP (the 1997 value). A fractional shift in the trust fund's equity <u>share</u> ι^e should be interpreted relative to this asset base; but the results could easily be re-scaled if one were interested in welfare effects for other σ values (say, for 2010 when σ is likely higher).

Finally, the welfare assessment requires preference parameters. The risk aversion η is most naturally identified by the equity premium. A fairly high risk aversion parameter is needed, however, to rationalize the historical data (here, η=24.6). This is the well-known equity premium puzzle; see Mehra-Prescott (1985), Kocherlakota (1996). For any given η-value, the time preference parameter ρ follows from the steady state Euler equations, and the social planner's time preference can be inferred from the steady state relationship $\omega^*=(1+a)(1+n)/R^k$.

To understand which results are sensitive to the equity premium puzzle, note that the parameter η matters for the welfare analysis in two ways. Most obviously, η enters as proportionality factor in (14). A high risk aversion means that better risk-sharing is very valuable in terms of average consumption. Uncertainty about the true η-value implies that the quantitative value of risk sharing in terms of consumption equivalents will necessarily be uncertain. Such uncertainty does not, however, affect the sign of $dW_0/d\xi$.

Secondly, η matters because it influences the elasticity coefficients π_{C1s} and π_{Ks} that appear in (14). This is because with CRRA utility, $1/\eta$ is the elasticity of substitution, which governs savings behavior. This linkage between risk-aversion and substitution is not necessarily appropriate in the asset pricing context (see Epstein-Zin, 1989; Weil 1989). To explore alternatives, I have also derived $dW_0/d\xi$ for Epstein-Zin (1989) preferences, which sever the linkage between η and intertemporal substitution. (This is a non-trivial extension because the derivative of the welfare function is more complicated than in the CRRA case when utility is not time-separable. Details are in a technical appendix available from the author.) Because the specification of η is controversial, calibration results will be presented for a range of risk-aversion and substitution parameters.

5. Results

This section combines the covariance estimates from Section 4.1 with the calibrated elasticities from Section 4.2 to evaluate the welfare effects of trust fund equity investments.

Table VIII shows the main results. As the benchmark, I use the macro parameters of Tables VI-VII, the covariances from Table V, Column 1 (based on the 1874-1996 VAR in Table I), and CRRA preferences with η calibrated to the equity premium. In all cases, the policy change (ξ) is a shift of trust fund investments from debt to equity for one generational period (30 years). Col.1 shows the marginal welfare effect $dW_0/d\xi$ evaluated at $\iota^e=0$, when the trust fund holds debt; Col. 2 shows $dW_0/d\xi$ evaluated at $\iota^e=1/\lambda=74\%$, when the trust fund holds a balanced portfolio of stocks and bonds that represents and unlevered claim on corporate capital; and Col. 3 shows $dW_0/d\xi$ evaluated at $\iota^e=100\%$, if the trust fund is fully invested in S&P500 stocks.

For each specification, Table VIII first shows the differences $\pi_{c1s}-\pi_{c2s}$ (for s=v,μ,a) that reveal to what extent the young are more exposed to risk than the old. Next, the table shows how the three shocks combine in the quadratic form QFORM1. Using the vector of orthogonalized innovations $(\hat{v}^{\theta}_t,\hat{\mu}^{\theta}_t,\hat{a}_t)$, one can rewrite QFORM1 as a sum of three components,

$$\begin{aligned} \text{QFORM1} = {} & (\pi_{c1v}-\pi_{c2v})\,\text{VAR}(\hat{v}^{\theta}_t)\,(d\pi_{c2v}/d\xi) \\ & + (\pi_{c1\mu}-\pi_{c2\mu})\,\text{VAR}(\hat{\mu}^{\theta}_t)\,(d\pi_{c2\mu}/d\xi) \\ & + (\pi^*_{c1a}-\pi^*_{c2a})\,\text{VAR}(\hat{a}_t)\,(d\pi^*_{c2a}/d\xi), \end{aligned} \tag{17}$$

where the coefficients $\pi^*_{xa}=\pi_{va}\,\bar{\pi}_{xv}+\pi_{\mu a}\,\bar{\pi}_{x\mu}+\pi_{xa}$ (x=c1,c2) absorb the off-diagonal components of COV_S. To avoid scale factors, the derivatives are taken with respect to the normalized policy variable $\xi=\iota^e_\theta\,\sigma_\theta/k$, which can be interpreted as the share of aggregate capital held by the trust fund. In the table, the v^θ-, μ^θ- and a-parts of QFORM1 refer to the corresponding components in (17). Next, I show the QFORM1 total, the "capital term" $\Omega\cdot(\pi_{c1v}-\pi_{c2v})$ QFORM2 in (14), and their sum, which equals $(dW_0/d\xi)/(\eta\bar{u})$ in (14). These $(dW_0/d\xi)/(\eta\bar{u})$ values are central to the welfare analysis of this paper: They reveal the sign of $dW_0/d\xi$, i.e., they show if the overall welfare effect of a marginal change in ξ is positive or negative.

Following the literature, I also display the approximate welfare effects of a discrete policy change expressed in terms of consumption equivalents. Specifically, the "discrete shift" row shows the effect of moving from all debt to a portfolio representing unlevered claims on corporate capital (to $\iota^e=1/\lambda\approx74\%$). As usual, the consumption equivalent is the percentage increase in lifetime consumption (here, of generation 0) that would raise expected utility by the same amount as the policy change.

The numbers in Table VIII reflect certain properties of the theoretical model that one should keep in mind: First, since $\pi_{c1\mu}=\pi_{c2\mu}=\pi_{c1v}=0$ and $\pi_{c2v}>0$ holds for $\iota^e=0$, the table shows $\pi_{c1v}-\pi_{c2v}<0$ and $\pi_{c1\mu}-\pi_{c2\mu}=0$ for any calibration with $\iota^e=0$. Second, recall that $d\pi_{c2,s}/d\xi<0$ s, since all forms of risks are shifted from old to young. Hence, the v^θ-component of QFORM1 in (17) is necessarily positive at $\iota^e=0$ and declining as ι^e rises, while the μ^θ-component is zero at $\iota^e=0$ and negative for $\iota^e>0$.

Now we can answer the four empirical questions posed at the end of Section 3. First, Table VIII shows that the impact of shifting productivity risk is substantial and negative, but not enough to outweigh the positive effect of the improved sharing of valuation risk. In the benchmark case (Col.1), the productivity component (a-term) of QFORM1 is negative, but smaller than the positive v^0-term. The a-term is negative because the young bear more productivity risk than the old ($\pi_{c2a}-\pi_{c1a}<0$), so that a further shift of such risk from old to young is welfare reducing. Given the estimated covariance matrix of shocks, the combined impact of valuation and productivity risk is nonetheless positive, QFORM1>0.

Second, the “capital term” in (14)--the welfare impact on future generations through capital accumulation--is also negative, but small relative to the effects of valuation and productivity risk. Hence, if one deducts the capital-term from QFORM1, the net welfare effect remains positive: $dW_0/d\xi>0$. Thus, <u>in the benchmark scenario, a marginal increase in trust fund equity investments has a positive welfare effect</u>.

Third, a comparison across Columns 1-3 shows how fast the marginal welfare benefits from additional equity investments decline with ι^e. One finds that the decline is slow enough that the marginal benefit remains positive even at a 100% equity share. Interestingly, $dW_0/d\xi$ declines with ι^e mostly because $|\pi_{c1v}-\pi_{c2v}|$ falls, while $VAR(\hat{\mu}^0{}_t)$ small enough that the negative μ-term is negligible even at ι^e=100%. (The a-part of (17) becomes more negative, too, but this not an independent change: it occurs largely because the decline in $|\pi_{c1v}-\pi_{c2v}|$ reduces the a-term through the covariance component $\pi_{va}\ \pi_{xv}$ in $\pi^*{}_{xa}$.) The last line of Table VIII shows ι^e values at which $dW_0/d\xi=0$, i.e., the theoretically optimal portfolio share. Values above 100% may well be practically unrealistic, but they provide another perspective on how slowly $dW_0/d\xi$ declines with ι^e.

Fourth, by integrating over the marginal effects, one can obtain the welfare impact of discrete portfolio shifts. As an illustration, Table VIII displays the welfare effects of shifting the social security trust fund from bonds to claims on unlevered capital ($\iota^e=1/\lambda$). In the benchmark case, such a shift has a consumption-equivalent value of 0.23% of a generation's steady state consumption. The consumption-equivalents are, however, sensitive to alternative assumptions about the risk aversion parameter η (as discussed above). This is illustrated in Columns 4-5, which show welfare results for CRRA preferences with $\eta=5$ and for log-utility ($\eta=1$). As η is reduced, the consumption-equivalents decline about linearly, down to 0.012% for log-utility.[17] Not surprisingly, the value of better risk-sharing depends on the price of risk. Note, however, that the marginal welfare effects $dW_0/d\xi$ and the optimal portfolios are quite robust to changes in risk aversion and intertemporal substitution. To confirm that η serves mainly as a proportionality factor, Col.6 shows welfare effects for Epstein-Zin preferences with $\eta=24.6$ (as in Col.1) and unit elasticity of substitution (as in Col.5). The resulting consumption equivalent of 0.25% is similar to Col.1.

Returning to the benchmark setting, the analysis of QFORM1 suggests that the main issue in assessing the optimal trust fund portfolio is the trade-off between valuation and productivity risk. The sensitivity analysis below therefore focuses on two items that influence this tradeoff, safe debt and the estimates of long-run uncertainty.

Safe government debt contributes in two ways to the negative welfare effect of shifting productivity risk from old to young. First, productivity risk enters negatively because a debt-to-equity swap increases the amount

[17] These small percentage values should not be viewed as disappointing, because lifetime consumption (the denominator) is large relative to the trust fund principal. The main purpose of the paper (and of the calibration) is to answer the qualitative question if trust fund equity investments are desirable (if $dW_0/d\xi>0$). The consumption equivalents are provided because such measures are standard in the calibration literature, but they should be interpreted cautiously. In Col.4-5, no attempt is made to match the equity premium.

of safe debt. The derivative $d\pi_{c2,a}/d\xi<0$ would be smaller, if one assumed instead that the return on government debt were contingent on economic growth. This applies, e.g., if debt is nominal and if inflation and growth are negatively correlated at long horizons. (See Bohn 1990 for empirical support.) Though monetary policy and inflation are beyond the scope of this paper, the effect of removing the negative a-term can be illustrated easily. Column 7 shows the welfare effects that one would obtain if the new government debt created by the trust fund's debt-to-equity swap were as productivity-contingent as equity, i.e., if $d\pi_{c2,a}/d\xi=0$, so that productivity risk is not re-allocated. Then the welfare effects are overwhelmingly positive and several times larger than in Col.1-6. One may even argue that Col.7 should be considered the benchmark for evaluating trust fund investments: Since the negative $d\pi_{c2,a}/d\xi$-term in the other columns is due to safe debt, one may interpret this term as capturing the cost of an inappropriate debt management policy, i.e., as a problem for the Treasury that should not be attributed to social security. Col.1 remains the appropriate benchmark, however, if one takes debt management (safe debt) as given.

Secondly, safe debt is important because it explains in part why the old bear less productivity risk than the young in the initial allocation, why $\pi_{c1a}-\pi_{c2a}>0$ at $\iota^e=0$.[18] To highlight this role of safe debt, Col.7 sets d-σ=0, i.e., assumes away the initial debt. Compared to Col.1, the welfare benefit of trust fund equity investments is clearly increased. However, $\pi_{c1a}-\pi_{c2a}$ remains positive, so that a debt-equity swap still re-allocates productivity risk in the wrong direction.

[18] Safe debt held by the old reduces their exposure to productivity risk, but increases the effective exposure of future young generations, which have to fund the debt service out of a productivity-contingent wage income; see Bohn (1998).

Table IX summarize the results with alternative estimates of aggregate uncertainty taken from Tables II-IV. Throughout, I use the benchmark calibration, but with modified η-values to match the equity premium. While the short-sample VAR-estimates (Col.1-2) produce slightly smaller welfare gains than the benchmark case, the ECM estimate (Col.3) implies drastically larger welfare benefits. Intuitively, the ECM estimate implies a much higher long-run variance of equity returns (recall Table V) and therefore gives a larger weight to the improved allocation of valuation risk relative to the negative effect from shifting productivity risk. The VARs in Col.1-2, on the other hand, yield much smaller welfare benefits. (Their sample period, 1932-96, excludes most of the Great Depression.) They are only scenarios for which $dW_0/d\xi$ falls to zero for $\iota^e<1$. The optimal portfolio nonetheless includes about 68% equity.

Overall, the wide range of estimates in Table IX suggests that our knowledge about the relevant long-run variances is highly imperfect. This is perhaps not surprising, because if a generation is 30 years, even the long, 123-year sample of 1874-1996 covers just four data points. For this reason, I have not even attempted to provide standard errors: All numbers are best interpreted as point estimates subject to potentially large errors. All estimates in Tables VIII and IX indicate, however, that the marginal benefits of trust fund equity investments are positive.

6. Conclusions

The paper examines the effects of alternative government policies on the allocation of aggregate risks across generations. The main application is to the question of social security trust fund investments in the stock market. I show that the welfare effects of such investments depend significantly on the correlation structure of macroeconomic shocks, the risk-characteristics of equities, and on individual preferences.

Overall, my estimates suggest that trust fund equity investments have positive net benefits on the margin. These findings should be interpreted cautiously, however: Our knowledge of the long-run sources of aggregate risk is highly imperfect, the analysis is based on a very stylized macroeconomic model, the quantitative benefits in terms of consumption are sensitive to the risk aversion parameter, and the paper does not address the political economy implications of social security equity investments. While political economy issues are beyond the scope of this paper, the finding that such investments appear to have efficiency benefits suggests that the issue deserves further study.

Separately, the empirical data imply that equity investments would help to reduce the variance of payroll taxes rates in a system with wage-indexed benefits. Since equity returns are correlated with GDP and wages, a trust fund portfolio with a mix of debt and equity securities provides a better match for wage-indexed obligations than a pure debt portfolio.

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Table I: VAR Estimates of Macroeconomic Uncertainty

Full Sample: 1874-1996

Regressor	Equation for: $\Delta\ln(Y)_t$	$\ln(E^f/Y)_t$	$\ln(PE)_t$	$\ln(DIV/E^f)_t$
$\Delta\ln(Y)_{t-1}$	0.30 (3.04)	-0.42 (-1.13)	-0.05 (-0.10)	0.18 (0.42)
$\Delta\ln(Y)_{t-2}$	0.13 (1.31)	-0.33 (-0.91)	0.23 (0.48)	-0.02 (-0.05)
$\ln(E^f/Y)_{t-1}$	0.15 (3.31)	1.26 (7.12)	-0.75 (-3.22)	-0.27 (-1.34)
$\ln(E^f/Y)_{t-2}$	-0.13 (-3.10)	-0.50 (-3.06)	0.80 (3.73)	0.34 (1.84)
$\ln(PE)_{t-1}$	0.12 (4.85)	0.54 (5.88)	0.35 (2.88)	-0.30 (-2.88)
$\ln(PE)_{t-2}$	-0.15 (-5.21)	-0.39 (-3.63)	0.46 (3.26)	0.32 (2.60)
$\ln(DIV/E^f)_{t-1}$	0.03 (0.68)	-0.06 (-0.36)	-0.23 (-1.08)	0.65 (3.59)
$\ln(DIV/E^f)_{t-2}$	0.03 (0.94)	-0.02 (-0.17)	0.20 (1.10)	-0.07 (-0.42)
Time	0.0006 (1.01)	-0.0050 (-2.40)	0.0013 (0.47)	-0.0002 (-0.06)
R^2	0.269	0.951	0.530	0.548
F-Tests to exclude:				
$\Delta\ln(Y)$	0.1%	23.8%	89.3%	91.4%
$\ln(E^f/Y)$	0.5%	0.0%	0.1%	16.1%
$\ln(PE)$	0.0%	0.0%	0.0%	1.4%
$\ln(DIV/E^f)$	19.6%	17.8%	47.9%	0.0%
Memo: Unit root test [a]	-3.096 (>10%)	-4.859 (<1%)	-4.916 (<1%)	-5.870 (<1%)

Notes: T-statistics are in brackets. The F-test values are the significance levels of the respective exclusion restrictions.

[a] T-values in a Phillips-Perron unit root test with constant and time trend. Rejection probabilities are in brackets. The critical values are 10%=3.15, 1%=3.73.

Table II: VAR Estimates of Macroeconomic Uncertainty

Sample with NIPA data: 1932-1996

Regressor	Equation for: $\Delta\ln(Y)_t$	$\ln(E^f/Y)_t$	$\ln(PE)_t$	$\ln(DIV/E^f)_t$
$\Delta\ln(Y)_{t-1}$	0.59 (4.51)	-0.77 (-1.90)	-0.50 (-0.72)	-0.07 (-0.16)
$\Delta\ln(Y)_{t-2}$	-0.10 (-0.74)	-0.31 (-0.74)	1.40 (1.92)	0.48 (0.98)
$\ln(E^f/Y)_{t-1}$	0.10 (1.50)	1.04 (5.26)	-0.24 (-0.71)	-0.01 (-0.05)
$\ln(E^f/Y)_{t-2}$	-0.04 (-0.64)	-0.36 (-2.03)	0.59 (1.91)	0.18 (0.85)
$\ln(PE)_{t-1}$	0.10 (3.25)	0.30 (3.28)	0.43 (2.78)	-0.02 (-0.24)
$\ln(PE)_{t-2}$	-0.12 (-3.46)	-0.10 (-0.98)	0.11 (0.60)	-0.01 (-0.11)
$\ln(DIV/E^f)_{t-1}$	-0.02 (-0.46)	-0.23 (-1.37)	0.14 (0.49)	0.66 (3.38)
$\ln(DIV/E^f)_{t-2}$	0.07 (1.49)	-0.03 (-0.17)	0.58 (2.25)	0.12 (0.69)
Time	0.0009 (1.11)	-0.0073 (-2.92)	0.0125 (2.94)	0.0014 (0.49)
R^2	0.443	0.874	0.658	0.689
F-Tests to exclude:				
$\Delta\ln(Y)$	0.0%	2.6%	16.0%	57.1%
$\ln(E^f/Y)$	14.9%	0.0%	3.6%	28.8%
$\ln(PE)$	0.2%	0.3%	0.0%	91.4%
$\ln(DIV/E^f)$	31.6%	24.3%	1.8%	0.0%

Notes: T-statistics are in brackets. The F-test values are the significance levels of the respective exclusion restrictions.

Table III: VAR Estimates of Macroeconomic Uncertainty

Using labor income, Sample 1932-1996

Regressor	Equation for: $\Delta\ln(w)_t$	$\ln(E^f/w)_t$	$\ln(PE)_t$	$\ln(DIV/E^f)_t$
$\Delta\ln(w)_{t-1}$	0.58 (4.62)	-1.09 (-2.75)	-0.35 (-0.52)	0.25 (0.55)
$\Delta\ln(w)_{t-2}$	-0.09 (-0.64)	-0.10 (-0.24)	1.42 (1.94)	0.22 (0.45)
$\ln(E^f/w)_{t-1}$	0.09 (1.52)	1.09 (5.73)	-0.22 (-0.67)	-0.07 (-0.31)
$\ln(E^f/w)_{t-2}$	-0.02 (-0.41)	-0.42 (-2.41)	0.60 (2.01)	0.23 (1.16)
$\ln(PE)_{t-1}$	0.09 (3.26)	0.30 (3.40)	0.44 (2.86)	-0.02 (-0.23)
$\ln(PE)_{t-2}$	-0.12 (-3.63)	-0.11 (-1.11)	0.10 (0.55)	0.00 (-0.02)
$\ln(DIV/E^f)_{t-1}$	-0.04 (-0.69)	-0.19 (-1.13)	0.11 (0.40)	0.63 (3.28)
$\ln(DIV/E^f)_{t-2}$	0.09 (1.95)	-0.04 (-0.26)	0.59 (2.35)	0.11 (0.67)
Time	0.0011 (1.44)	-0.0072 (-2.99)	0.0132 (3.20)	0.0013 (0.46)
R^2	0.482	0.881	0.669	0.695
F-Tests to exclude:				
$\Delta\ln(w)$	0.0%	0.5%	13.8%	59.7%
$\ln(E^f/w)$	7.1%	0.0%	1.8%	21.0%
$\ln(PE)$	0.2%	0.2%	0.0%	95.9%
$\ln(DIV/E^f)$	14.7%	32.5%	1.3%	0.0%

Notes: T-statistics are in brackets. The F-test values are the significance levels of the respective exclusion restrictions.

Table IV: Error Correction Estimates

Full Sample: 1874-1996

	Equation for:			
Regressor	$\Delta\ln(Y)_t$	$\Delta\ln(E^f)_t$	$\Delta\ln(P)_t$	$\Delta\ln(DIV)_t$
$\Delta\ln(Y)_{t-1}$	0.22 (2.12)	-0.23 (-0.50)	0.40 (0.92)	-0.09 (-0.37)
$\Delta\ln(Y)_{t-2}$	0.13 (1.24)	-0.07 (-0.17)	0.01 (0.02)	-0.26 (-1.11)
$\Delta\ln(E^f)_{t-1}$	0.02 (0.73)	-0.20 (-1.78)	-0.09 (-0.86)	0.14 (2.28)
$\Delta\ln(E^f)_{t-2}$	0.00 (-0.07)	-0.29 (-2.52)	-0.03 (-0.27)	0.21 (3.43)
$\Delta\ln(P)_{t-1}$	0.11 (4.41)	0.65 (6.27)	0.03 (0.29)	0.37 (6.81)
$\Delta\ln(P)_{t-2}$	-0.10 (-3.36)	-0.13 (-0.98)	-0.23 (-1.85)	0.16 (2.24)
$\Delta\ln(DIV)_{t-1}$	0.06 (1.48)	0.05 (0.29)	-0.23 (-1.33)	-0.41 (-4.21)
$\Delta\ln(DIV)_{t-2}$	0.01 (0.19)	-0.01 (-0.07)	0.11 (0.64)	-0.39 (-3.91)
$\ln(E^f/Y)_{t-3}$	0.02 (0.93)	-0.28 (-2.72)	-0.25 (-2.58)	-0.16 (-3.02)
$\ln(PE)_{t-3}$	-0.01 (-0.69)	0.19 (2.15)	-0.02 (-0.20)	0.12 (2.70)
$\ln(DIV/E^f)_{t-3}$	0.06 (1.68)	-0.06 (-0.38)	-0.11 (-0.70)	-0.39 (-4.64)
Time	0.0006 (1.06)	-0.0056 (-2.29)	-0.0050 (-2.15)	-0.0047 (-3.62)
R^2	0.341	0.433	0.151	0.492

Notes: T-statistics are in brackets. In this table, F-tests for excluding variables would not meaningful because of the error corrections terms.

Table V: Long Run Variances and Correlations

	Estimates based on: Table I	Table II	Table III	Table IV
	VAR 1874-1996	VAR 1932-1996	VAR 1932-96 with wages	ECM 1874-1996
Generational Variances:				
Output Y_t	0.124	0.116	0.107	0.105
Returns R^e_t	0.141	0.161	0.163	0.199
Earnings E^f_t	0.192	0.163	0.159	0.188
Correlations:				
Y_t & R^e_t	0.77	0.80	0.79	0.68
Y_t & E^f_t	0.76	0.85	0.84	0.76
R^e_t & E^f_t	0.65	0.69	0.67	0.76
Variances of				
Productivity a_t	0.261	0.245	0.226	0.221
Valuation v_t	0.134	0.117	0.105	0.529
Rel. Risk μ_t	0.076	0.067	0.062	0.062
Coefficients:				
π_{va}	0.60	0.55	0.52	1.46
$\pi_{\mu a}$	-0.52	-0.51	-0.51	-0.51
$\pi_{v\mu}$	0.27	0.05	-0.06	1.85
Var. of v^0_t	0.0410	0.0431	0.0443	0.0576
Var. of μ^0_t	0.0053	0.0030	0.0031	0.0052
Min.Variance Portfolio ι^{e*}	0.72	0.68	0.64	0.49

Notes: No standard errors are provided. The numbers should be interpreted cautiously, because for variances at a horizon of T=30 years, even the long 1874-1996 sample amounts to only about four observations. Equity returns are based on Campbell et al.'s (1997) log-linear approximation, using values for 1/(1+exp{log dividend yield}) of 0.9572 for 1874-1996 and 0.9614 for 1929-96.

Table VI: Parameters for the Calibration

Variable	Symbol	Value	Source/Method
Return on equity	r^e	7.0%	Advisory Council (1997)[a]; $R^e=(1+r^e)^N$
Return on safe bonds	r^b	2.3%	Advisory Council (1997)[a]; $R^b=(1+r^b)^N$
Population growth	n^*	1.0%	Social security projections[a]
Wage growth	a	1.0%	Social security projections[a]
Capital Share	α	0.311	Average from NIPA,[b] using Cooley-Prescott (1995) method
Leverage	λ	1.351	Hall&Hall (1993): Debt/assets=0.26
Depreciation	d_i	4.84%	Average for private capital from NIPA,[b] using Cooley-Prescott (1995) method
Old capital/Return	v/R^k	0.867	Average of $(1-d_i)/(\alpha\, Y_i+1-d_i)$ in NIPA[b]
Investment rate	I_i/Y_i	0.137	Average Gross private investment/GDP[b]
Soc.Sec. Benefits	β	10.4%	Cost rate for OASDI+HI for 1997
Net Debt/GDP Ratio	D^*_i/Y_i	0.441	Publicly-held debt/GDP (CBO 1998)
Trust Fund/GDP Ratio	TR_i/Y_i	0.072	1997 Actuarial Report; Dec.1996 assets divided by 1997 GDP
Gov.Spending/GDP	G_i/Y_i	17.1%	Government consumption/GDP, 1995 NIPA
Taxes on the old/GDP	$\tau^2{}_t$	5.3%	NIPA 1995; taxes-transfers, excl. social security, pro-rated by factor shares

[a] I use recent values since safe interest rates have been well above their historical means since about 1980. For equity, the Advisory council’s value matches the historical average reported by Mehra-Prescott (1985). For population and wage growth, the numbers are close to the social security 10-year ahead projections.

[b] Unless otherwise stated, all averages refer to annual 1929-1996 averages.

Table VII: Characteristics of the calibrated economy

Output Shares		Values	Parameters		Values
Income of the Young	$\frac{y^1_t}{(Y_t/N_t)}$	0.501	Savings Rate of the Young	s	0.176
Consumption of the Young	$\frac{c^1_t}{(Y_t/N_t)}$	0.413	Conversion Factor	Ψ	33.2
Consumption of the Old	$\frac{c^2_t}{(Y_t/N_t)}$	0.396	Risk aversion	η	24.6
Wealth Accumulation	$\frac{ACC_t}{(Y_{N/2,t}/N_t)}$	2.926	Planner's Time Discount (p.a.)	ω^*	3.7%

Table VIII: Welfare Effects

Column:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Specification:	Benchmark CRRA	Benchmark $\iota^e=1/\lambda$	Benchmark $\iota^e=1$	CRRA $\eta=5$	log-utility	Epstein-Zin	$\frac{d\pi_{c2a}}{d\xi}=0$	$d\text{-}\sigma=0$
Risk Aversion η	24.6	24.6	24.6	5.0	1.0	24.6	24.6	24.6
Equity share ι^e for marginal analysis[a]	0	74%	100%					
$\pi_{c1,v}-\pi_{c2,v}$	-0.772	-0.732	-0.718					
$\pi_{c1,\mu}-\pi_{c2,\mu}$	0.0	0.014	0.019					
$\pi_{c1,a}-\pi_{c2,a}$	0.618	0.622	0.624	0.616	0.610	0.615	0.618	0.548
v^0-part of (17)	0.188	0.178	0.175					
μ^0-part of (17)	0.0	-0.00022	-0.00030					
a-part of (17)	-0.143	-0.158	-0.163	-0.142	-0.137	-0.141	0[b]	-0.091
Combined effect of shocks: QFORM1 in (14)&(17)	0.044	0.020	0.011	0.045	0.050	0.047	0.188	0.097
Capital term: $\Omega\ (\pi_{c1k}-\pi_{c2k})$ QFORM2 in (14)	-0.006	-0.005	-0.004	-0.006	-0.005	-0.006	-0.006	-0.007
Marginal Welfare Effect: $(dW_0/d\xi)/(\eta\ \bar{u})$	**0.038**	**0.015**	**0.007**	**0.040**	**0.045**	**0.041**	**0.182**	**0.090**
Discrete Shift from $\iota^e=0$ to $\iota^e=74\%$ (Consumption-Value)	**0.23%**			**0.05%**	**0.01%**	**0.25%**	**1.52%**	**0.67%**
Optimal ι^e (not shown if >2)	122%			125%	140%	131%	>>200%	>>200%

Notes: Empty cells have the same value as in Column 1. Col.1 (benchmark) is based on the VAR estimates in Table I and the macroeconomic parameters of Tables VI and VIII. Col.2-3 consider different policy (ι^e) parameters for the same calibration. Col.4-5 assume CRRA with lower risk aversion. Col.6 assumes Epstein-Zin utility with unit elasticity of substitution. Col.7 assumes state-contingent debt that eliminates the effects through $d\pi_{c2a}/d\xi$, and Col.8 assumes a zero initial government debt. The "discrete gains" row shows the welfare effects in terms of consumption-equivalents of moving from zero equity to an unlevered portfolio of claims on capital (from $\iota^e=0$ to $\iota^e=1/\lambda\approx74\%$).

[a] Indicates the ι^e-value at which (14), (17), and their components are evaluated.

[b] By assumption, as explained in the text.

Table IX: Welfare Effects with Alternative Data Sets

Column:	(1)	(2)	(3)
Specification from:	Table II 1932-96	Table III 1932-96 with wages	Table IV ECM
Risk Aversion η	24.45	24.71	16.45
Equity share ι^e for marginal analysis	0	0	0
$\pi_{c1,v}-\pi_{c2,v}$	*as in Table VIII, Col.1*		
$\pi_{c1,\mu}-\pi_{c2,\mu}$			
$\pi_{c1,a}-\pi_{c2,a}$			
v^0-part of (17)	0.197	0.203	0.264
μ^0-part of (17)	0.000	0.000	0.000
a-part of (17)	-0.168	-0.174	0.249
Combined effect of shocks: QFORM1 in (14)&(17)	0.030	0.029	0.513
Capital term: Ω ($\pi_{c1k}-\pi_{c2k}$) QFORM2 in (14)	-0.005	-0.004	-0.044
Marginal Welfare Effect: $(dW_0/d\xi)/(\eta\ \bar{u})$	**0.025**	**0.025**	**0.469**
Discrete Shift from ι^e=0 to ι^e=74% (Consumption-Value)	**0.10%**	**0.10%**	**2.61%**
Optimal ι^e (not shown if >2)	68%	68%	>>200%

Notes: Col. 1-3 show welfare results obtained when one combines the covariance estimates implied by Tables II to IV with the benchmark calibration.

Figure 1: The correlation of wage income and capital income.