# Dividend Policy and Income Taxation[*]

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May 1, 2008

## Abstract

The effects of dividend and capital gains taxes on optimal dividend payout policy are analyzed in the context of a one-good model (so that capital consists of stored units of the consumption good). The aftertax discount factor is assumed to adjust to taxes to bring about equality between the discounted value of the firm's aftertax dividend stream under the optimal dividend policy and the number of units of capital the firm is operating. A standard result—that the Miller-Modigliani dividend irrelevance proposition applies in the presence of taxes if the dividend tax rate equals the capital gains tax rate (and if capital gains are taxed as they accrue)—is demonstrated. The analysis is extended to deal with unequal tax rates. The two major results are (1) allocating retained earnings to share repurchases has the same tax implications as allocating retained earnings to new investments, and (2) either of these will be optimal if and only if the tax rate on capital gains is lower than that on dividends.

JEL codes G1, G3.

The Miller-Modigliani [14] dividend-irrelevance principle asserts (among other propositions) that, in the absence of frictions, corporate dividend policy does not affect firm value. This is so because if investment is held constant, as Miller-Modigliani assumed, then by an identity a change in dividends is offset one-for-one by a change in proceeds from new security issues. Assuming that investors value firms by discounting payments to stockholders net of proceeds of new issues, firm value is unaffected by the dividends change.

A related dividend-irrelevance proposition, often incorrectly attributed to Miller-Modigliani, may apply if investment is not held constant: variations in future dividends do not affect firm value provided that retained earnings are invested in zero-net-present-value projects. This invariance is held to occur because the direct effect on firm value of an increase in current dividends increase is exactly offset by lower

future dividend growth due to lower retained earnings. Even in the absence of frictions, this dividend-irrelevance proposition is not correct without further restrictions, as has been pointed out by DeAngelo and DeAngelo [5], [6].

In the presence of frictions these results certainly fail, and there may exist a unique optimal dividend policy. Available results in this general case are much less complete than in the case where frictions are excluded. In this paper we analyze dividend policy in the presence of taxes on dividends and capital gains. The goal is to provide conclusions that are the counterpart for the case of positive taxes of the Miller-Modigliani results for zero taxes.

# 1 The Equilibrium Condition

It will be assumed that dividends and capital gains are subject to taxation at rates $t_d$ and $t_g$. For simplicity it is assumed that capital gains are taxed as they accrue, not when they are realized. We adopt a one-good deterministic framework. Investors are assumed to be able to transfer capital freely to and from firms without restriction, but subject to whatever taxes apply (an equivalent assumption would be that capital can be physically converted to the consumption good, and vice-versa, one-for-one without restriction other than that implied by taxes). In this setting firms are completely identified by the number of capital goods they operate, and this magnitude uniquely determines their value.

An implication of the one-good assumption is that the equilibrium market value of firms measured in units of the consumption good is numerically equal to the number of capital goods in place. The validity of this assertion is obvious in the absence of taxes, but less so in their presence. Suppose instead that, as many analysts have asserted (Auerbach [1] or McGrattan and Prescott [13], for example) that in equilibrium the capital held by firms is valued at $1 - t_d$ units of the consumption good per unit of capital (that is, Tobin's $q$ equals $1 - t_d$), or at some level between this value and 1. This is held to occur in equilibrium because when firms transfer wealth to stockholders via dividend payments the dividend recipients must pay dividend taxes.

This, however, is a non-sequitur. If Tobin's $q$ were less than 1 then firm managers could generate an arbitrage profit for stockholders by liquidating the firm. Return of the initial paid-in capital to stockholders generates no tax liability, there being no associated income. Past retained earnings also can be returned to stockholders without taxation, as noted in the first paragraph of this section. Thus each unit of capital transferred to stockholders generates an arbitrage profit of $1 - q$. This arbitrage opportunity cannot occur in equilibrium, implying $q = 1$ despite the presence of taxes.

The argument applies in the same way if one considers that creation of new firms. If $q < 1$ stockholders incur an immediate capital loss when they transfer new capital to firms. Again, this cannot occur in equilibrium.

The fact that $q = 1$ in equilibrium in the presence of taxes implies that we cannot identify the (pretax) return to capital with the (aftertax) rate of return at which

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aftertax returns are discounted: the latter must be lower than the former if the discounted value of dividends is to equal the number of units of capital. We have to take at least one of these as endogenous, since Tobin's $q$ will generally not equal 1 if these parameters are specified independently. In equilibrium the condition $q = 1$ enforces the requirement that the latter must be lower than the former by exactly the magnitude that implies that the discounted value of aftertax cash flows equals the number of units of capital that generated these cash flows.

Firm managers are assumed to maximize the market value of equity (corporate debt is assumed equal to zero throughout). In our setting the only decisions the firm manager makes are what proportion of the firm's earnings to pay out in dividends and whether to allocate whatever earnings remain after paying dividends to share repurchases or to new investment. The above argument implies that a firm that adopts a dividend policy that is optimal in the assumed tax regime will have value equal to the number of capital goods that it operates. Correspondingly, if the firm were to adopt a suboptimal dividend policy its capital would be valued at less than one consumption good per unit. This would occur because the firm would be subjecting its stockholders to higher taxes than necessary, and stockholders will take this into account in valuing the firm's shares. More precisely, an equilibrium dividend payout rate is such that (1) the pretax and aftertax returns to capital are such that the capital of firms that adopt that dividend payout rate is valued at one consumption good per unit, and (2) firms cannot increase the value of their capital above one consumption good per unit by deviating from the equilibrium dividend payout rate.

It is assumed throughout that firm managers can precommit to future dividend and share repurchase policies, implying that investors can readily determine the consequences for firm values of actions off the equilibrium path. This specification cannot be defended on grounds of realism. However, adopting it allows us to sidestep some problems that would divert the analysis from the line that appears most useful. For example, without the assumption it could be objected that in the presence of a well-functioning capital market firm managers that proposed a suboptimal dividend or share repurchase program would be replaced as part of a corporate takeover. Allowing for this possibility would not add anything interesting because the new managers would face the same decision problem as the former managers.

It is assumed that dividends as a fraction of earnings are bounded below by zero and above by 1. Removing the upper bound without otherwise modifying the model would lead to problems. In the presence of a very high capital gains tax, firms could induce capital losses by setting dividend payouts that exceed earnings. Applying the capital gains tax to these losses would result in payments from the government to stockholders. This cannot occur under US tax law: capital losses can be netted against capital gains, but cannot generate a payment from the government if they exceed capital gains. Dealing with this asymmetry between the tax treatment of capital gains and losses would complicate the model unnecessarily. It is easiest to circumvent such difficulties by imposing the upper bound on dividends.

For the same reason we neglect depreciation. If capital depreciates and dividends are bounded above by gross earnings (i.e., earnings before allowing for depreciation), managers can generate a capital loss by paying dividends that are higher than net earnings. If the capital gains tax rates were applied to capital losses, a transfer from the government to stockholders would occur. Our preference is to avoid going down this road.

As noted, the equilibrium condition $q = 1$ implies that the four parameters that describe the environment that firms face in our setting—the dividend tax rate, the capital gains tax rate, the pretax return on capital and the aftertax return on capital—cannot be specified independently. In a full general equilibrium setting, both the pretax and aftertax returns on capital would generally depend on the assumed tax rates, assuming that the tax rates are exogenous. Determining the magnitude of these effects would involve specifying, at a minimum, the aggregate production function and individuals' preferences. The production technology affects equilibrium factor intensities, and therefore pretax returns on capital, while saving behavior affects the aftertax discount rate. Except in the context of an example to be presented below, we will not take this circuitous route, instead assuming that the aftertax return on capital accommodates to the pretax return on capital and the assumed tax rates.

We now restate the equilibrium definition incorporating the convention that the aftertax rate of return is determined endogenously. Firms pay out proportion $\delta$ of their earnings as dividends, $0 \leq \delta \leq 1$. Denote $r(\delta)$ as the aftertax discount rate that sets the discounted value of aftertax dividends per unit of capital to one under whatever tax regime is in place, assuming a dividend payout rate of $\delta$. Let $\bar{\delta}$ be the equilibrium dividend payout rate, implying that the equilibrium aftertax discount rate is $r(\bar{\delta})$. In general, as just noted, $\bar{\delta}$ and $r(\bar{\delta})$ will depend on the pretax return on capital and the tax rates on dividends and capital gains. The individual firm, as a price taker, sets its dividend payout rate to maximize the value of the firm, treating $r(\bar{\delta})$ as fixed. This specification reflects the assumption that each firm is small. In this setting $\delta = \bar{\delta}$ is an equilibrium if for discount rate $r(\bar{\delta})$ the value of an individual firm is less than or equal to one for all values of $\delta$.

The framework just set out implies that firms' dividend behavior is determined jointly with the value of the aftertax discount rate as a Nash equilibrium. That being so, the question arises whether it appropriate to use causal language as we did above in characterizing the effect of dividend policy on firm values. If firms' dividend policy is endogenous and determined jointly with firm values, it is not obvious that language impling that the former can be taken to be a cause of the latter (at least under the treatment of causation proposed in LeRoy [11], [12]) is appropriate. In the most general Nash equilibrium there is no generally applicable way to analyze the effect on the equilibrium of one player's deviation from an equilibrium strategy, barring further specification. This is so because the other players' Nash equilibrium strategies are generally no longer best responses to the deviation, so there is no reason to rule out alterations in the behavior of other agents in response to the deviation.

It is possible to argue that in the present context there is in fact enough structure to justify a causal interpretation of dividend policy: because individual firms are assumed to be vanishingly small relative to the aggregate of firms, it seems reasonable to assert that a deviation from optimal dividend behavior on the part of one firm will reduce that firm's value, but will not affect economy-wide aggregates. If statements of this sort are accepted, then the conclusion from the present exercise will be that, if $t_d \neq t_g$, then dividend policy does affect firm value, so the Miller-Modigliani invariance proposition does not extend to this case. If, on the other hand, one prefers to avoid causal language in this setting, the conclusion would be that when $t_d \neq t_g$ firms' equilibrium dividend policies are singletons, as opposed to the $t_d = t_g$ case in which all admissible dividend policies are equilibrium policies.

## 2   A Basic Framework

The general practice in extending the Miller-Modigliani analysis to the case where taxes are positive is to work with static or two- or three-date models. Such settings require that firms pay a liquidating dividend at the terminal date (however, see DeAngelo and DeAngelo [5], where it appears that firms do not necessarily pay a liquidating dividend at the terminal date). A setting more suitable for the present purpose is to assume that the relevant cash flows are perpetuities. We will assume that each unit of capital yields $g$ units of output each period forever. Here $g$ is taken as given independently of the tax rates; this simplification allows us to avoid a general equilibrium analysis, as observed above.

To fix notation we begin by reviewing the case where there are no taxes. Suppose that a firm owns $k$ units of capital, each of which has price $q$. If $q$ is constant over time, as occurs in equilibrium in the deterministic steady states that we will examine, the value $qk$ of the firm obeys

$$qk = \frac{qk' + d'}{1 + r},\tag{1}$$

where $k'$ is the next-period number of capital units the firm owns, $d'$ is next-period dividends, and $r$ is the discount rate. Dividends are given by

$$d' = g\delta k,\tag{2}$$

where $\delta$ is the fraction of earnings paid out as dividends. Assuming that retained earnings $g(1 - \delta)k$ are used to acquire new capital, the next-period capital stock $k'$ obeys

$$k' = k(1 + g(1 - \delta)).\tag{3}$$

Substituting (2) and (3) into (1) results in

$$q = \frac{g\delta}{r - g(1 - \delta)}. \tag{4}$$

It is easily checked that $q > (=, <)$ 1 as $g > (=, <)$ $r$, so the equilibrium condition $q = 1$ is equivalent to $g = r$.

Equation (4) is recognized as the Gordon [7] model: high (low) values of $\delta$ result in a high (low) initial level of dividends, but a low (high) growth rate $g(1 - \delta)$ of dividends. The two effects offset in present-value terms. This makes sense: if retained earnings generate the same return as the return implicit in the discount factor, then retaining earnings instead of paying them out as dividends does not affect value.

## 3   Dividends vs. Share Repurchases

We now assume that dividends are taxed at rate $t_d$ and capital gains are taxed at rate $t_g$. These are personal taxes, and are constant over individuals and income levels. There are no corporate taxes. As noted in the introduction, it is assumed that capital gains are taxed as they accrue, not when they are realized. If firms pay out less than 100% of their earnings in dividends, they can use the earnings that remain either to repurchase shares or to acquire new capital. In this section it is assumed that firms allocate all earnings not paid out in dividends to repurchasing their own shares, while in the following section it is assumed instead that firms use retained earnings to acquire new capital. We could, of course, combine the two cases, but that would require an expansion of notation.

If $t_d > 0$ and $t_g > 0$, $g = r$ results in $q < 1$ for any $\delta$. As stated in the introduction, this cannot be an equilibrium. It follows that we cannot carry over the assumption $g = r$ to the positive-tax case: if both tax rates are strictly positive, aftertax returns $r$ are strictly lower than pretax returns $g$.

We will continue to take $g$ to be exogenous, but will specify that $r$ is determined endogenously so as to allow $q = 1$ under equilibrium dividend policy. As indicated above, it is assumed that equilibrium consists of a dividend payout rate $\bar{\delta}$ such that when all other firms adopt a dividend payout rate of $\bar{\delta}$, implying that $r$ equals $r(\bar{\delta})$, then it is value-maximizing for the firm being studied also to choose $\delta = \bar{\delta}$.

Suppose that the firm owns one unit of capital and has $n$ shares outstanding, each of which has price $p$ at the present date. We will normalize $n$ also to equal 1, so that $np = p = 1$ in equilibrium (if dividend policy is suboptimal we will have $np = p < 1$). Since the firm does not acquire new capital, it has one unit of capital at the next date. It has $n'$ shares at the next date, each of which has price $p'$. These variables satisfy $n'p' = k = 1$ as a consequence of the assumption that the firm does not acquire new capital.

The analogue of eq. (1) in the presence of taxation is

$$p = \frac{p' - (p' - p)t_g + g\delta(1 - t_d)}{1 + r}. \tag{5}$$

Here $p' - p$ is the accrued capital gain resulting from the share repurchase. The next-period share price $p'$ is given by equating earnings net of dividend payments $g(1 - \delta)$ to the value of share repurchases $(n - n')p'$:

$$g(1 - \delta) = (n - n')p' = p' - 1, \tag{6}$$

using $n = 1$ and $n'p' = 1$. Using eq. (6) to eliminate $p'$ in eq. (5), that equation becomes

$$p = \frac{1 + g(1 - \delta)(1 - t_g) + (p - 1)t_g + g\delta(1 - t_d)}{1 + r}. \tag{7}$$

In equilibrium we must have $p = 1$ by the argument given in the preceding section. Substituting $p = 1$ in eq. (7) and solving for $r$, there results

$$r = g(1 - \delta)(1 - t_g) + g\delta(1 - t_d). \tag{8}$$

If $t_d = t_g = t$, this simplifies to

$$r = g(1 - t). \tag{9}$$

The assumption that dividends and capital gains are taxed at the same rate implies that the aftertax rate of return does not depend on the dividend payout rate. Therefore the Miller-Modigliani dividend irrelevance proposition applies under taxation in this case. This result is well known.

Suppose now that $t_d \neq t_g$. Then, from eq. (8), a dividend payout rate $\bar{\delta}$ can be an equilibrium only if the aftertax return is $r(\bar{\delta})$, given by

$$r(\bar{\delta}) = g(1 - \bar{\delta})(1 - t_g) + g\bar{\delta}(1 - t_d). \tag{10}$$

The other condition that must be satisfied for $\bar{\delta}$ to be an equilibrium is that an individual firm has no incentive to deviate by setting $\delta \neq \bar{\delta}$. Determining whether this is the case involves discounting the aftertax dividends generated by payout rate $\delta$ using the aftertax discount rate $r(\bar{\delta})$. The relevant solution variable is the per-share value $p(\delta, \bar{\delta})$ of the aftertax cash flows. By construction we have $p(\bar{\delta}, \bar{\delta}) = 1$. If $p(\delta, \bar{\delta}) > 1$ for any value of $\delta$ then the firm can increase its market value by deviating in favor of the dividend payout rate $\delta$ from the dividend payout rate $\bar{\delta}$ that (by assumption) is adopted by other firms. Because all firms will have the same incentive, $\bar{\delta}$ cannot be an equilibrium. However, if $p(\delta, \bar{\delta}) \leq 1$ for all $\delta$, then firms have no incentive to deviate from $\delta = \bar{\delta}$.

Evaluating the effects of a deviation in dividend payout policy from the value-maximizing policy depends on how long the deviation is expected to persist. We evaluate $p(\delta, \bar{\delta})$ for $\delta \neq \bar{\delta}$ under the assumption that dividend policy will revert to

the optimal policy in the next period. If so, the share price, $p'$, will revert to its equilibrium value of $1 + g(1 - \delta)$ in the next period (see eq. (6)). In that case, from eq. (7), the value of cash flows satisfies

$$p(\delta, \overline{\delta}) = \frac{1 + g(1 - \delta)(1 - t_g) + (p(\delta, \overline{\delta}) - 1)t_g + g\delta(1 - t_d)}{1 + r(\overline{\delta})}, \tag{11}$$

which can be solved for $p(\delta, \overline{\delta})$:

$$p(\delta, \overline{\delta}) = \frac{1 + g(1 - \delta)(1 - t_g) - t_g + g\delta(1 - t_d)}{1 + r(\overline{\delta}) - t_g} \tag{12}$$

$$= \frac{1 + g(1 - \delta)(1 - t_g) - t_g + g\delta(1 - t_d)}{1 + g(1 - \overline{\delta})(1 - t_g) - t_g + g\overline{\delta}(1 - t_d)}, \tag{13}$$

using eq. (10).

Optimal dividend policy can now be determined. Suppose that $\delta < \overline{\delta}$ implies $p(\delta, \overline{\delta}) < 1$. Under this inequality, eq. (13) reduces to $t_d < t_g$. The interpretation is that $t_d < t_g$ implies $p(\delta, \overline{\delta}) < 1$ for any $\delta < \overline{\delta}$. In this case optimal dividend policy consists of setting $\delta = 1$ for any level of $\overline{\delta}$: since $p > 1$ for $\delta > \overline{\delta}$, $\overline{\delta}$ can be an equilibrium only if $\overline{\delta} = 1$. Thus the firm pays all earnings out in dividends.

In the contrary case, when $t_d > t_g$, then $\delta < \overline{\delta}$ implies $p(\delta, \overline{\delta}) > 1$. In this case no strictly positive value of $\overline{\delta}$ can be an equilibrium, since a firm could always deviate by setting $\delta < \overline{\delta}$ and raise its value. The equilibrium dividend payout policy is $\overline{\delta} = 0$, so that the firm uses all its earnings for share repurchases.

The finding that firms optimally pay out all earnings in dividends if $t_d < t_g$, and only then, has been stated before (e.g., Brennan [3]). However, other analysts have obtained a different result: Auerbach [1], for one, claimed that the dividend irrelevance proposition carries over to nonzero taxes even if $t_d \neq t_g$.

The result implies that a buy-and-hold strategy involving the shares of a firm that allocates all earnings to share buybacks is a pure bubble (see Sethi [16] for further discussion of the fact that share repurchases can induce bubbles). That fact by itself is innocuous: any portfolio strategy in which all gains are reinvested is a bubble, so portfolio strategies that are bubbles are always available in infinite-time settings. Such strategies are usually not optimal; in overlapping generations settings optimal portfolio strategies always involve liquidation of portfolios at the last period of life, assuming bequest motives are ruled out. Discussion of conditions under which equilibrium portfolio strategies can and cannot have bubbles in equilibrium are available elsewhere (Santos and Woodford [15], Huang and Werner [8], for example).

There are also questions of feasibility. Depending on the setting, portfolio strategies involving bubbles may or may not be feasible. Under some conditions such portfolio strategies are clearly not feasible. For example, an agent who engages in a buy-and-hold strategy involving a firm that allocates all its earnings to share repurchases is clearly not feasible. Since no earnings are reinvested, the value of the firm is

constant, yet the value of the investor's position increases at a constant rate. Eventually the agent will own the entire firm, at which point something has to change. Such considerations can be considered only in an equilibrium setting, so we sidestep them here.

## 4    Dividends and the Cost of Capital

The equilibrium dividend payout function,

$$\bar{\delta} = \begin{cases} 1 \text{ if } t_d < t_g \\ \\ 0 \text{ if } t_d > t_g \end{cases} \tag{14}$$

implies that the dividend payout minimizes the effect of taxes on aftertax returns. Specifically, for given pretax return to capital, optimal dividend policy maximizes the aftertax return to capital:

$$r(\bar{\delta}) = \begin{cases} g(1 - t_d) \text{ if } t_d < t_g \\ \\ g(1 - t_g) \text{ if } t_d > t_g \end{cases}. \tag{15}$$

We can reverse the causation between the pretax return on capital $g$ and the aftertax return by taking the aftertax return on capital as exogenous and deriving the equilibrium pretax return on capital ("cost of capital") as a function of tax rates. Inverting the relation (15) results in

$$g(\bar{\delta}) = \begin{cases} r/(1 - t_d) \text{ if } t_d < t_g \\ \\ r/(1 - t_g) \text{ if } t_d > t_g \end{cases}, \tag{16}$$

where $r$ is the given aftertax return on capital. It is seen under this interpretation the equilibrium dividends policy rule minimizes the cost of capital.

The result that the cost of capital is the minimum of the dividend tax rate and the capital gains tax rate depends on the version of accrual taxation specified above. In particular, it depends on the assumption that past retained earnings are not subject to tax when returned to stockholders. An example will make the role of this assumption clear. Suppose that investors transfer 1000 to a firm at date 1, that $g$ equals 0.2, that $t_d = t_g = t = 0.1$, that, initially, the firm pays out all earnings in dividends at dates 2 and 3, and that it liquidates the firm at date 3. The aftertax cash flows are 180 at date 2 and 1180 at date 3, implying an internal rate of return of $g(1 - t)$, or 18%. Now suppose instead that the firm retains all earnings at dates at date 2, implying an aftertax payoff of $-20$ at date 2 since the investor must pay tax on the capital gain of 200. At date 3 the firm pays a liquidating dividend of 1440. Of this, only the

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current dividend of 240 is taxed, so the aftertax cash flow is 1416. It is easily checked that the internal rate of return continues to be 18%. Suppose that, contrary to our specification, dividends paid out of past retained earnings were subject to taxation. In that case the double taxation would invalidate our result that the aftertax return on capital equals $g(1 - \min(t_d, t_g))$.

# 5 Investment

We now assume that the firm allocates earnings remaining after dividend payments to acquisition of new capital rather than to share repurchases. This capital generates income at the same rate $g$ as the preexisting capital. As in the preceding section, it is assumed that capital gains are taxed as they accrue, not when they are realized. In that case the analogue of eq. (5) is

$$qk = \frac{qk' - t_g q(k' - k) + d'(1 - t_d)}{1 + r}, \tag{17}$$

which has eq. (1) as a special case if $t_d = t_g = 0$.

Setting $q = 1$, $k' = (1 + g(1 - \delta))k$ and $d' = g\delta k$ in eq. (17), we have that $r(\bar{\delta})$ is given by

$$r(\bar{\delta}) = g(1 - \bar{\delta})(1 - t_g) + g\bar{\delta}(1 - t_d). \tag{18}$$

For $\bar{\delta}$ to be an equilibrium we must have in addition that $q(\delta, \bar{\delta}) \leq 1$ for all $\delta$, where $q(\delta, \bar{\delta})$ is given by

$$q(\delta, \bar{\delta})k = \frac{(1 + g(1 - \delta)(1 - t_g))k + (q(\delta, \delta) - 1)t_g k + \delta g k(1 - t_d)}{1 + r(\bar{\delta})}, \tag{19}$$

again using eq. (17). Solving eq. (19) for $q(\delta, \bar{\delta})$ results in

$$q(\delta, \bar{\delta}) = \frac{1 + g(1 - \delta)(1 - t_g) - t_g + g\delta(1 - t_d)}{1 + r(\bar{\delta}) - t_g}. \tag{20}$$

Note that eq. (18) is the same as eq. (10), and the right-hand side of eq. (20) is the same as that of eq. (12), implying that $p(\delta, \bar{\delta})$ of Section 3 equals $q(\delta, \bar{\delta})$ of this section. It follows that allocating retained earnings to acquisition of new capital has the same tax consequences as buying back shares: both induce a capital gain in share prices, implying that the effective tax rate for both is $t_g$. Firms are always indifferent between using retained earnings to buy capital or to repurchase shares, and they always prefer either to paying dividends if and only if $t_d > t_g$.

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# 6  Dividends and Taxes in General Equilibrium

It is easy to embed the dividend payout model just presented into a full general equilibrium setting. The obvious specification for this exercise is a standard-issue overlapping generations model. The model to be presented coincides with a special case of the model of Blanchard and Fischer [2], Ch. 3, from which this exposition is drawn, except for the inclusion of dividend and capital gains taxes here. Thus assume that each generation lives for two periods. Young people work and save part of their labor income, consuming the rest. With their savings they buy shares of stock of firms. The capital that they transfer to firms in exchange for these shares is used in production next period. When these agents become old they receive taxable income from their capital—whether firms choose to pay out this income in the form of dividends or share repurchases depends on the tax environment in the manner specified in the preceding sections. Investors also receive a liquidating dividend from the firms, which is not taxable. For simplicity it is assumed that the labor income of workers is not taxed. Finally, tax proceeds are assumed to be dissipated in unproductive consumption by the government.

The utility of generation $t$ is of the form

$$u(c_{1t}, c_{2t+1}) = \ln(c_{1t}) + (1 + \theta)^{-1} \ln(c_{2t+1}), \tag{21}$$

where $c_{1t}$ is the consumption of young agents at date $t$ and $c_{2t+1}$ is the consumption of old agents at date $t + 1$. The budget constraint of generation $t$ is

$$c_{1t} = w_t - s_t \tag{22}$$

$$c_{2t+1} = (1 + r_{t+1})s_t. \tag{23}$$

Here $w_t$ is the labor income of the young, $s_t$ is the saving of the young and $r_{t+1}$ is the aftertax return to capital invested from date $t$ to date $t + 1$.

As above, firms are subject to dividend taxation at rate $t_d$ and capital gains taxation at rate $t_g$. From the preceding section it is clear that the effective tax rate is $t \equiv \min(t_d, t_g)$. Therefore we have

$$r_{t+1} = g_{t+1}(1 - t), \tag{24}$$

where $g_{t+1}$ is the pretax return to capital.

To solve the model, assume first that there are no taxes. In that case logarithmic utility implies that the saving of the young is a constant proportion $1/(2+\theta)$ of labor income:

$$s_t = \frac{w_t}{2 + \theta} \tag{25}$$

regardless of the return on capital $r_{t+1}$. Production is assumed to be of the Cobb-Douglas form:

$$Q_t = K_t^\alpha N_t^{1-\alpha}, \tag{26}$$

implying that the pretax return to capital at $t+1$ is

$$g_{t+1} = \alpha k_{t+1}^{\alpha-1} \tag{27}$$

and labor income at $t$ is

$$w_t = (1-\alpha)k_t^\alpha, \tag{28}$$

where we have converted to intensive units $(k_{t+1} \equiv K_{t+1}/N_{t+1})$. The investment-equals-saving identity is

$$K_{t+1} - K_t = N_t s_t - K_t \tag{29}$$

or, in intensive units,

$$(1+n)k_{t+1} = s_t, \tag{30}$$

where $n$ is the population growth rate $N_{t+1}/N_t - 1$, assumed constant.

The evolution of the capital stock per worker is given by

$$k_{t+1} = \Psi(k_t) = \frac{w_t}{(1+n)(2+\theta)} = \frac{(1-\alpha)k_t^\alpha}{(1+n)(2+\theta)}, \tag{31}$$

from eqs. (25), (28) and (30). The stationary point $k^*$ of this difference equation defines a unique stable steady state.

Assume now that the tax rate is $t > 0$. The aftertax return on saving becomes $r_{t+1} = g_{t+1}(1-t)$. The assumption of logarithmic utility implies that in equilibrium agents will consume and save the same amounts when young in the presence of taxation as they would in its absence. It follows that the consumption of the old decreases one-for-one in response to the tax. Further, the fact that saving is unaffected by the tax implies that the same is true of the equilibrium capital stock at each date. This in turn implies that the pretax return to capital $g_t$ will also not be affected by the tax.

We see that the equilibrium in the presence of taxes is identical to that which obtains in their absence, except that the consumption of the old is reduced by the amount of the tax. Under the adopted specification the assumption adopted above that imposition of the tax does not affect the pretax return on capital is in fact justified in the equilibrium setting. Of course, this outcome reflects the special assumptions imposed in deriving the equilibrium (specifically, logarithmic utility and that tax revenues do not affect utility and are not returned to agents via transfers.

# 7    Nonshiftable Capital

The preceding discussion has dealt exclusively with the setting in which the consumption good can be converted to capital one-for-one without restriction, and vice-versa. This is the appropriate specification if the intent is to restrict analysis to one-good settings, as is frequently the analyst's stated intention in the public finance and macroeconomics literatures. If the free convertibility assumption fails, consumption is a different good from capital and, in general, capital in different industries must be distinguished. Adjustment cost models of investment may apply in this case. Alternatively, the analyst may prefer to specify that capital and consumption are both produced by labor and capital (but used in different factor ratios), as in the two-sector and multi-sector models analyzed in the 1960s.[1] Finally, one can specify that capital goods once allocated to a given industry cannot be transferred to different industries, or to consumption. This setting also was analyzed in the 1960s and earlier.[2] To give some indication of how the analysis of the effects of taxation on dividend policy and valuation is affected by such respecifications, we adopt a simple version of nonshiftable capital. It is assumed that the consumption good can be transformed one-for-one into capital, but cannot be transformed at all in the opposite direction. We will see that most, but not all, of the analysis presented above carries over to this case.

If capital is nonshiftable and $q < 1$ in equilibrium, then firms are no longer indifferent between share repurchases and investment of earnings in new capital: the former is always preferred. This is so because a unit of output owned by the firm that is allocated to share repurchases increases aftertax stockholder wealth by $1 - t_g$ units, whereas a unit of output that is converted to capital increases aftertax stockholder wealth by $q(1 - t_g)$ units of wealth. The former dominates. Consequently, we have that the effective tax rate $t$ is given by

$$t = \min(t_d, t_g), \tag{32}$$

as above, and firms pay out all earnings in dividends if $t = t_d$, while they repurchase shares if $t = t_g$. Also as above, the equilibrium unit value $q$ of capital held by firms equals the discounted aftertax value of earnings:

$$q = \frac{g(1 - t)}{r}. \tag{33}$$

We saw above that under shiftable capital we must have $q = 1$ in equilibrium, implying a functional relation between $g$ and $r$. With nonshiftable capital the equilibrium condition $q = 1$ is replaced by $q \leq 1$, assuming that individual preferences

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[1]Burmeister and Dobell [4] is a good introduction to this literature.

[2]For example, Keynes' analysis of investment using the marginal efficiency of capital is best interpreted as specifying nonshiftable capital both across firms and between firms and individuals (Keynes [9], LeRoy [10]).

satisfy some condition like the Inada restriction so as to rule out the zero consumption that would occur under $q > 1$. With $q \leq 1$ as the equilibrium condition there is no longer a dependence between $g$, $t$ and $r$, so these can be specified independently (but subject to $g(1 - t)/r \leq 1$). It is simplest to think of $g$ as determined by the technology and $r$ as implied by preferences (as would be case with utility function $\sum_t (1 + r)^{-t} c_t$, where $c$ is consumption), but in more general settings more complicated interpretations may be appropriate. Note, incidentally, that with nonshiftable capital if both $t_d$ and $t_g$ are strictly positive, $g$ can be either greater than or less than $r$ in equilibrium, in contrast to the case in the one-good setting, for which $r < g$.

If $q = 1$ in equilibrium, the analysis is unchanged from that presented above. Investors are willing to contribute capital to start new firms, since capital has the same unit value as the consumption good and doing so generates no tax liability. On the other hand, when $q < 1$ they will not contribute capital to new firms, since doing so would entail an immediate loss of value. If $q < 1$ firm owners would prefer to convert capital to consumption so as to obtain a capital gain of $1 - q$ per unit of capital, but by assumption the capital is bolted in place, so they cannot do so.

Depending on the other specifications of the model, either $q < 1$ or $q = 1$ can occur in equilibrium. If $q < 1$ the capital stock will remain constant (assuming that depreciation is excluded). If demand conditions imply positive capital growth, then the equilibrium value of $q$ will be 1 so that investors are willing to contribute however much outside capital to firms demand conditions require.

# 8    Conclusion

Our major result is that in a one-good setting taxation of income-related transfers from firms to investors induces a spread between the pretax and aftertax returns to capital, but does not alter the conclusion that in equilibrium Tobin's $q$ equals 1. In a setting with this property we find that share repurchases have identical tax implications as reinvestment of retained earnings. Also, we derive the conclusion that firms will pay out all earnings in dividends if $t_d < t_g$, and will pay zero dividends if $t_d > t_g$. If $t_d = t_g$ a dividend-irrelevance proposition obtains, as Miller-Modigliani observed.

A noteworthy feature of this analysis is that the equilibrium condition $q = 1 - t$, which frequently appears in the public finance literature, plays no special role here (although, of course, it can occur as a special case). In a one-good setting (i.e., when capital is shiftable in both directions) the equilibrium condition is $q = 1$, so $q = 1 - t$ can only occur when $t = 0$. The purported equilibrium condition $q = 1 - t$ is equivalent to $g = r$, but $g = r$ is not an equilibrium condition under either shiftable or nonshiftable capital except by accident, as has been observed.

These results were derived in a highly stylized setting. We specified the simplest production technology, assuming that capital and consumption are the same good

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(except in Section 7). This setting made it possible to present the central conclusion of this paper in a setting free of distractions. That conclusion is that pretax and aftertax returns to capital cannot be specified independently of the tax regime. Doing so results in the incorrect conclusion that the unit value of a firm's capital does not equal 1 in general, which is inconsistent with equilibrium in the assumed setting. To avoid this outcome it was assumed here that the aftertax discount factor always adjusts so that the unit value of a firm's capital is 1 under optimal dividend payout behavior, and less than or equal to 1 for any dividend payout behavior; in general settings both the pretax and aftertax returns to capital depend on taxes.

The tax environment could be generalized. No account was taken here of debt financing or of corporate taxes. Adapting the present model to deal with this question would not involve any difficulties.

More general production technologies could be specified. Under two-sector capital accumulation models, capital and consumption goods are produced using different technologies, implying that the equilibrium price ratio of capital and consumption is endogenous, and not generally equal to one. Adding adjustment costs would produce the same conclusion. We do not defend the present setting against these more general specifications. Our point is to emphasize that one cannot abstract away from equilibrium considerations in analyzing the effect of taxation on corporate valuation under optimal dividend payout behavior.

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