

# The MM Theorems in the Presence of Bubbles\*

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## Abstract

The Miller-Modigliani dividend irrelevance proposition states that changes in dividends that are offset one-for-one by changes in proceeds from net new issues of securities—so that investment and earnings are unaffected—do not affect equity valuations. Under conditions, a related proposition extends the irrelevance result to settings that allow investment levels to vary as dividends are changed.

Recently these irrelevance propositions have been questioned by DeAngelo and DeAngelo, who asserted that dividend payout rules, like investment plans, can be suboptimal. Therefore, in their view, in general settings there is no valid irrelevance proposition; low dividend payouts give rise to low valuations. We observe that these assertions can be sensibly evaluated only in settings that allow bubbles, which were excluded in the discussions of Miller-Modigliani and DeAngelo-DeAngelo.

As is well known, in standard settings if bubbles can exist at all, there exists a continuum of equilibrium paths indexed by initial values of the bubble component of asset values. We show that along some of these equilibrium paths the Miller-Modigliani dividend irrelevance result obtains. In other equilibria, however, DeAngelo-DeAngelo's conclusion that dividend decisions are relevant to equity values is correct.

The Miller-Modigliani [5] theorem, or theorems, on the irrelevance of corporate dividend policy in a frictionless environment forms the basis of the modern theory of corporate finance. Recently these propositions have been questioned by DeAngelo and DeAngelo ([3], [4]). Specifically, DeAngelo-DeAngelo claimed that the dividend irrelevance proposition is true only in environments that are simplified in a way that is not generally appreciated. They asserted that in more general settings dividend policy is relevant in exactly the same sense as investment policy is relevant: there

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exist feasible dividend policies that do not maximize the net present value of the dividend stream. Still more specifically, they asserted that if a firm pays dividends at levels that are too low relative to its dividend-paying capacity, the value of the firm will be strictly below the maximum attainable value, just as if the firm had adopted a negative-NPV investment project. A similar point was made by Ross [6].

In another strand of the economics-finance literature on asset pricing, analysts take the view that paying too low a level of dividends does not result in undervaluation of the firm; the value of a firm with given current capital is the same under low or zero future dividends as high future dividends (for example, Black [1]). Instead, paying low dividends results in a bubble, so that the value of the firm strictly exceeds the net present value of its future dividends. This paper explores the relation between these two contradictory assertions. We do this by contrasting the effects of the dividend decision in settings that do and do not allow for bubbles, in contrast to the practice of excluding bubbles without discussion, as in most of the literature.

A side issue that must be dealt with is that there are (at least) two distinct dividend irrelevance results: one which Miller-Modigliani actually proposed and another that is commonly attributed to them. The following section begins with a review of the dividend irrelevance proposition as Miller-Modigliani originally formulated it. Their result is proved under restrictive assumptions; we leave it to the reader to judge whether these assumptions render the result vacuous, as DeAngelo-DeAngelo asserted.

Section 2 presents a related result, which we call the dividend-investment irrelevance proposition because it asserts that, under conditions, the value of the firm is invariant to hypothetical changes that involve both dividend payments and investment levels. Despite the fact that the dividend-investment irrelevance proposition is frequently attributed to Miller-Modigliani, they never claimed it. In fact, they went to a considerable length to dissociate themselves from it:

It is true that the literature abounds with statements that in some “theoretical” sense, dividend policy ought not to count; but either that sense is not clearly specified or, more frequently and especially among economists, it is (wrongly) identified with a situation in which the firm’s internal rate of return is the same as the external or market rate of return.

([5], p. 414). We take the view that it is worthwhile distinguishing between these versions of the dividend-irrelevance proposition. Both are correct under assumptions, but the required assumptions are different in the two cases. Also, the extent of validity of the DeAngelo-DeAngelo criticisms is different in the two cases.

## 1 Dividend Irrelevance

Let  $X_t$  equal the earnings of a firm at date  $t$ , and let  $I_t$  equal the firm’s investment level. Here  $I_t$  is defined to include increases in net holdings of financial assets as well

as plant and equipment and the like. Let  $D_t$ ,  $R_t$  and  $S_t$  equal the dividends paid by the firm, share repurchases and new issues of shares, respectively. All variables are assumed to be nonnegative. Following Miller-Modigliani and DeAngelo-DeAngelo, debt is not considered. Then we have the basic identity

$$I_t \equiv X_t + S_t - D_t - R_t, \quad (1)$$

so that the net increase in assets  $I_t$  equals the difference between cash inflows ( $X_t + S_t$ ) and cash outflows ( $D_t + R_t$ ). Define free cash flow  $F_t$  as earnings less investment. Then eq. (1) can be written as

$$F_t = D_t + R_t - S_t, \quad (2)$$

so that free cash flow is by definition equal to net cash transferred by firms to stockholders.

DeAngelo-DeAngelo's exposition of this point is misleading at best. In the abstract to [4], DeAngelo-DeAngelo stated that in Miller-Modigliani's model "[I]rrelevance obtains, but in an economically vacuous sense because the firm's opportunity set is artificially constrained to payout policies that fully distribute free cash flow". This assumption, it is held, plays a central role in producing dividend irrelevance: "Irrelevance is hard-wired into MM (1961) by assumptions that shrink the feasible set to optimal policies by forcing 100% distribution of [free cash flow] in every period" (p. 294). On the contrary, we see here that the equality between free cash flow and cash transferred to stockholders ( $D_t + R_t - S_t$ ) is an unavoidable consequence of an accounting identity, and does not reflect any assumption on the part of Miller-Modigliani. If the firm pays higher dividends it necessarily decreases assets (or increases liabilities), thereby increasing free cash flow by an equal amount.

Miller-Modigliani took investment and earnings as fixed, and they also set share repurchases equal to zero. Therefore we have that free cash flow is fixed. We have

$$\bar{F}_t \equiv D_t - S_t. \quad (3)$$

The net cash flow to investors, equal to dividends less the proceeds from new issues of shares, is identically equal to free cash flow, and is therefore also held fixed by assumption. In this setting hypothetical variations in dividends are necessarily offset one-for-one by variations in the proceeds from new issues.

Eq. (3), incidentally, shows that DeAngelo-DeAngelo were correct in characterizing Miller-Modigliani as assuming that dividends equal at least 100% of free cash flow (with  $S_t$  nonnegative,  $D_t$  necessarily equals or exceeds  $F_t$ ). This result reflects exclusion of share repurchases. However, Miller-Modigliani's dividend irrelevance result does not depend on this simplification: it is easily checked that reversing the suppression of  $R_t$  or, equivalently, relaxing the assumption that  $S_t$  is nonnegative, does not alter their result. The value at date  $t$  of the firm equals

$$\frac{K_{t+1} + D_{t+1} + R_{t+1} - S_{t+1}}{1 + r}. \quad (4)$$

Here  $K_t$  denotes the capital stock, including financial assets. Using (1), there results

$$\frac{K_{t+1} + X_{t+1} - I_{t+1}}{1 + r}. \quad (5)$$

Whether or not  $D_t$  exceeds  $F_t$ , none of the terms in (5) are affected by dividends, implying dividend irrelevance ( $K_{t+1}$  is unaffected by current dividends because Miller-Modigliani assumed it to equal the discounted value of future dividends, which they held fixed). This is the Miller-Modigliani dividend-irrelevance proposition as Miller-Modigliani stated it.

There is another way to see that DeAngelo-DeAngelo did not characterize the Miller-Modigliani setting correctly. They asserted that Miller-Modigliani assumed away earnings retention, and that their conclusion of dividend irrelevance depends critically on this exclusion:

In effect, MM assume away the value-relevant payout/retention decision ....When MM's assumptions are modified to allow retention, a firm can reduce its value by paying out less than the full value of [free cash flow], and so payout policy matters and investment policy is not the sole determinant of value. With retention allowed, a firm is no longer constrained to an optimal payout policy as an automatic by-product of its investment decision, and irrelevance fails because some feasible payout policies do not distribute the full present value of [free cash flow] to currently outstanding shares

(p. 294). Taking free cash flow as fixed, as Miller-Modigliani did, is not the same as ruling out retained earnings or taking retained earnings as fixed. Retained earnings equals earnings less dividends; Modigliani-Miller took earnings as fixed and dividends as under the firm's control, implying that under their assumptions the firm can alter retained earnings.

DeAngelo-DeAngelo presented a three-date example in which, they claimed, it is possible for the net dividend payout to be suboptimal. This conclusion is incorrect. In the Miller-Modigliani setup free cash flow is taken as fixed at the terminal date, just as at earlier dates. It follows that variations in dividends are matched one-for-one by inverse variations in new share issues; since investors value firms at discounted dividends less new issues, the dividend irrelevance proposition applies equally at the terminal date as at earlier dates. It might be objected that at the terminal date by definition firms cannot issue new shares. If so, the implication of the identity (3) is that in the setting Miller-Modigliani specified firms cannot vary dividends, not that dividend payout is suboptimal.

## 2 Dividend-Investment Irrelevance

The key assumption required for the dividend-irrelevance result reported in the preceding section was that free cash flow is taken as fixed. Miller-Modigliani made this assumption in order to derive the dividend-irrelevance proposition in a setting that assumes nothing about the investment opportunities open to the firm. Dividend irrelevance requires only the assumption that investors value net cash flows, so that they are indifferent between internal and external sources of finance for investment.

As noted in the introduction, a different dividend irrelevance proposition is often attributed to Miller-Modigliani. This proposition depends on the assumption that the universe of relevant investment opportunities can be restricted to those that have zero net present value. This assumption is less restrictive than it might otherwise appear. Negative-NPV investment projects are excluded in order to focus attention on the question of whether dividend policy affects firm values; there is no controversy about the fact that firms adopting negative NPV projects decrease their values. Whether strictly positive NPV investment projects exist or not is largely a semantic question. It is often argued that positive NPV projects exist only by virtue of ownership of a factor of production that gives rise to the positive NPV opportunity but for some reason is not being valued. Valuing the services of that factor and charging the associated factor payment against revenue reduces the NPV of the project to zero. It becomes clear that assuming that the firm has access to an unlimited supply of zero-NPV investment opportunities amounts only to agreeing to neglect available but suboptimal investment choices.

We assume that the firm's earnings are generated by its assets  $K_t$ . The return on capital is  $r$ , so that  $X_t = rK_{t-1}$ . Taking  $r$  to be fixed independent of  $K_t$  implies that investment has zero net present value when discounted at rate  $r$  regardless of the dividend-investment policy. Also, it is assumed for simplicity that new issues  $S_t$  equal zero, since the role of  $S_t$  in the model to be presented would be the same as in the model in the preceding section. As above, share repurchases are assumed to be zero.

Finally, Miller-Modigliani's assumption that free cash flow is fixed is relaxed. We have the identity

$$D_t \equiv X_t - I_t \equiv F_t, \tag{6}$$

so that firms pay out 100% of free cash flow as dividends. In this setting variations in dividends cause equal and opposite variations in contemporaneous investment. Dividend changes affect future earnings through their effect on investment, and therefore also future capital:

$$K_{t+1} = K_t + I_{t+1} = (1 + r)K_t - D_{t+1}. \tag{7}$$

Assume that the initial level of the firm's assets is  $K_0$ . Ascertaining the effects of varying dividend payout policy involves determining the sequence of investment levels

$I_1, I_2, \dots$ . In this section it is assumed that investment is constrained by an assumed given capital stock  $\overline{K}_T$  at some future date  $T$  (of course,  $\overline{K}_T$  can be assumed equal to zero). Since we have

$$K_T = K_0 + I_1 + \dots + I_T \quad (8)$$

and investment is constrained period-by-period by investment, eq. (8) induces a constraint on dividends. Note, incidentally, that as eq. (8) indicates, fixing  $K_T$  imposes a restriction on the sum of investment levels, not on the present value of investment as is stated in some formulations.

Using these relations it is easy to derive the present-value relation:

$$K_0 = \sum_{t=1}^T \frac{D_t}{(1+r)^t} + \frac{K_T}{(1+r)^T}. \quad (9)$$

First, let  $p_{t-1}$  equal the unit value of capital at date  $T-1$ . Since capital earns return  $r$ , we have

$$p_{T-1}K_{T-1} = \frac{D_T + K_T}{1+r}. \quad (10)$$

Substituting eq. (7) for  $t+1 = T$  in eq. (10) and simplifying, there results  $p_{T-1} = 1$ . Continuing, we must have  $p_0 = \dots = p_{T-1} = 1$ , implying eq. (9).

Eq. (9) says that the value of the firm equals its current assets regardless of its dividend policy: all dividend-investment policies consistent with  $K_T = \overline{K}_T$  and eq. (8) produce the present-value relation (9). Investment levels not in this set result in a different terminal value of the firm, but it remains true that the initial value of the firm remains equal to the present value of its dividends plus the discounted sale price. Therefore the proposition that the value of the firm is independent of its dividend policy does not require any restriction on admissible levels of dividends.

As noted, ruling out new security issues and share repurchases implies that dividends equal free cash flow, where the latter equals earnings less investment. Thus the irrelevance result just stated can be equally well characterized as dividend irrelevance or investment irrelevance. Accordingly, we term the result of this section “dividend-investment irrelevance” to distinguish it from the Miller-Modigliani “dividend irrelevance” result discussed in Section 1.

We return to DeAngelo-DeAngelo’s three-date example discussed in the preceding section. It will be recalled that DeAngelo-DeAngelo interpreted this example as establishing the feasibility of suboptimal dividend policy. In contrast, the analysis of this section has the value of firm equaling its current capital  $K_0$  under any dividend policy. Relaxing Miller-Modigliani’s specification that free cash flow is fixed, as we are doing in this section, implies that the criticism offered in the preceding section no longer applies. However, it is clear that in DeAngelo-DeAngelo’s setting summed discounted dividends can fall short of current capital value only because

DeAngelo-DeAngelo ignored the discounted terminal value of capital in the present-value calculation, despite the fact that in the case of interest this value depends on assets and future dividends. They did not explain why doing so is acceptable.

### 3 Infinite-Time Settings

The assumptions required above to derive the dividend-investment irrelevance proposition were that (1) investments financed by retained earnings generate the same return as is incorporated in the discount rate, and (2) time is finite. Here we relax the second assumption. If time is infinite the simple backward recursion that demonstrates that equity is fully valued under any dividend plan is no longer available: in the relation

$$p_t K_t = \frac{D_{t+1} + p_{t+1} K_{t+1}}{1 + r} \quad (11)$$

expressing the equality between the rate of return on capital and  $r$ , there is no initial condition  $p_T = 1$  to start the recursion. In the infinite time case there exists no elementary argument that assures that equity is fully valued under any dividend regime and, in fact, there exist dividend payout rules under which equity is undervalued. As DeAngelo-DeAngelo emphasized, these payout rules are off the equilibrium path, precisely because they result in undervaluation.

This is easiest to demonstrate in settings in which bubbles cannot occur, so we begin by making that assumption. Substituting eq. (7) in eq. (11), there results

$$(p_t - p_{t+1})K_t = \frac{(1 - p_{t+1})D_{t+1}}{1 + r}. \quad (12)$$

Suppose now that a firm is considering paying zero dividends forever. Then eq. (12) reduces to  $p_t = p_{t+1}$ , all  $t$ . Further, exclusion of bubbles implies that  $p_t = 0$ , all  $t$ . Paying zero dividends forever implies that the firm's capital increases at rate  $r$  forever, implying that the firm grows faster than the economy (the condition for exclusion of bubbles is that the equilibrium interest rate exceeds the growth rate of the economy; see Tirole [7] or Blanchard and Fischer [2]). Thus paying zero dividends forever is infeasible. This does not invalidate DeAngelo-DeAngelo's argument; competitive equilibrium does not require that actions off the equilibrium path be feasible.

Suppose now that bubbles can occur. In most settings this assumption results in a continuum of equilibrium paths indicated by the magnitude of the bubble. Therefore we need to postulate a mechanism that chooses one of the possible equilibria. It turns out that the validity of the DeAngelo-DeAngelo argument depends on which mechanism we choose. First, we can simply assume that bubbles are zero, in which case DeAngelo-DeAngelo's argument obviously carries over to the present case. However, another selection mechanism would postulate that different possible dividend paths

result in different bubbles. Specifically, suppose that firm values are a common multiple of current capital, so that dividend paths that result in lower fundamental values are associated with bubbles that are higher by exactly the amount of the undervaluation. In that case the values of firms (the sum of the fundamental and the bubble) are invariant to the dividend payout rule. This is the setting implicitly assumed by Black [1], where it is presumed that the firm can set dividends equal to zero without adversely affecting the value of the firm's equity.<sup>1</sup> This specification seems, and is, arbitrary and unmotivated, but the point is that it is a legitimate equilibrium path.

## 4 Conclusion

We have contended that analysis of the dividend irrelevance proposition is best carried out in a setting that allows for an infinite future. In such a setting equity values do not necessarily equal the summed discounted values of dividends: bubbles can occur. It is known that in settings where bubbles occur, equilibrium paths are indexed by initial values of bubbles. Depending on these initial values, the Miller-Modigliani dividend irrelevance proposition may or may not obtain: if one restricts attention to what can be called Black equilibrium paths, then dividend irrelevance obtains, so that all dividend policies are equivalent from the point of view of management and investors.

On what can be called DeAngelo equilibrium paths, however, this proposition is no longer true: if firms fail to pay out full value in dividends, their stock values will be lower than they would be if they did pay out full value. In such cases, just as DeAngelo-DeAngelo argued, equilibrium dividend payout behavior involves paying out full value. Continuing to repeat DeAngelo-DeAngelo's conclusion, in this case there is a complete symmetry between investment policy and dividend policy. As regards investment, firms have available to them negative-NPV projects, but in equilibrium they will reject these in favor of zero-NPV projects. As regards dividends, firms have available to them suboptimal payout policies, but they will reject these in favor of dividend policies that pay out full value.

So far as we can see, there are no persuasive grounds for preferring Black equilibria to DeAngelo equilibria, or vice-versa. Precisely because both are equilibria, they presume rational behavior and cleared markets, so the two equilibrium concepts would seem to be on an equal footing. In any case, it is not likely to be fruitful to engage in informal judgments to the effect that one set of equilibrium paths is more realistic than the other. A better strategy might be to look into the possibility of reducing the equilibrium set via formal equilibrium refinements. For example, the presumption in

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<sup>1</sup>Black's statement was "Under the assumptions of the Modigliani-Miller theorem, a firm has value even if it pays no dividends. Indeed, it has the same value as if it paid dividends." There is no question that this statement is correct as applied to the model of Section 1, which describes the result that Miller-Modigliani actually stated. It is not certain that Black would have endorsed it to apply to the model of Section 2.



all of the literature under discussion that firms can and do precommit to dividend paths is open to question.

The bottom line here is that DeAngelo-DeAngelo's analysis is correct: dividend policy, like investment policy, is in general not irrelevant. It is true that one can limit one's analysis to settings in which dividend policy is irrelevant (Black equilibria), but there does not appear to be any obvious rationale for doing so.

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