Economic Theory manuscript No.
(will be inserted by the editor)

Liquidity and Liquidation*

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March 20, 2006.
First draft June 20, 2000.

Abstract. The manager of a firm that is selling an illiquid asset has discretion as to the sale price: if he chooses a high (low) selling price, early sale is unlikely (likely). If the manager has the option to default on the debt that is collateralized by the illiquid asset, the optimal selling price depends on whether the manager acts in the interests of owners or creditors. We model the former case. In equilibrium the owner will always offer the illiquid asset for sale at a strictly higher price than he paid, and will default if he fails to sell. As a result, upon successful sales the illiquid asset changes hands at successively higher prices.

We also consider a generalization of the model which permits sellers to finance sales using either debt or preferred stock, or both. This allows derivation of an optimal capital structure.

JEL Classification Numbers: G33, G32, D82, D83, G12.

Key words. Illiquidity, Liquidation, Illiquid asset, Bankruptcy costs, Optimal capital structure, Agency costs, Asset pricing, Search.

* We are indebted to seminar participants at the University of California, Los Angeles; University of California, Santa Barbara; Utah State University; University of Miami; Federal Reserve Bank of Atlanta; Federal Reserve Bank of San Francisco and Federal Reserve Bank of Kansas City. We have received helpful comments from Tom Cooley.

1. Introduction

When do firms use debt primarily rather than equity to finance their assets? The answer often given is that they do so when the assets collateralizing the debt are liquid, so that they have approximately the same value in other uses as in their current use. Debt is efficient because in the case of default the debtor can transfer the collateral to the creditor, who can convert it to cash with little loss of value. Airlines are the clearest case: Northwest Airlines can readily issue debt secured by aircraft because these aircraft are worth as much to American Airlines as to Northwest. (This proposition must be qualified because of the possibility of an industry-wide downturn, which has obvious implications for liquidity; these are analyzed by Shleifer and Vishny [9].) When, on the other hand, the assets are specific to their current use the creditor can liquidate them only subject to a major loss of value. In that case, it is argued, equity finance will be preferred to debt finance (Williamson [12]). Firms that produce computer software are an example.

The conclusion of this argument is correct, but the reasoning is at best incomplete. The Miller-Modigliani proposition about the equivalence of debt and equity finance remains valid even if the bondholders take a loss in the event of default. The yield on the debt can easily be set at the time of issue to reflect the possibility of this loss but, as the Miller-Modigliani reasoning makes clear, a higher yield on debt is not by itself a reason to avoid its use. In the standard exposition of capital structure theory it is not the existence of default that invalidates the Miller-Modigliani propositions, but the existence of costly bankruptcy.

The reason illiquid assets are financed by equity is that use of (defaultable) debt involves an agency problem. This agency problem in the present setting is what makes default costly. Analyzing this agency problem requires a fully explicit model of illiquidity. Such a model was supplied by Krainer and LeRoy [4]. Illiquidity there was identified with costly search and matching. Illiquidity in that analysis corresponds to asset specificity in Williamson and Shleifer-Vishny's discussions.

The agency problem arises when the firm's managers, unable to operate assets profitably, try to liquidate them. The fact that the assets are illiquid means that the managers have discretion over the liquidation strategy: they can set a low price and expect to find a buyer relatively quickly, or the opposite. Assuming that the managers work on behalf of the equity holders, and (unrealistically) that the relation between equity holders and managers is free of principalagent problems, the managers will choose a liquidation strategy that works to the benefit of the equity holders and to the detriment of the bondholders. With a liquid asset, in contrast, there is less discretion over pricing. In the limiting case of perfect liquidity, which is equivalent to perfect competition, the seller is a price taker. The managers have no discretion about price, and thus there is no agency problem.

The liquidation strategy that is optimal for the equity holders depends on the size of the debt. For any level of debt, the equity holders will try to sell the illiquid asset for a price that exceeds that level. If they succeed in doing so they can pay off the debt and pocket the difference, while if they fail to sell they will default. There is no point in trying to sell the asset for less than the amount of the debt, since doing so is dominated by immediate default. The resulting high prices increase the probability of default, implying losses to bondholders, but by assumption the manager ignores these losses.

The problem just outlined is formally similar to the well-known distortion of investment incentives that is usually used to illustrate agency problems involving equity and defaultable debt. When default is likely, managers know that if a large investment project succeeds the gains will accrue to the equity holders, whereas if it fails the costs are borne by the debt holders. Managers working for the equity holders may implement such projects even when they have negative net present values. The Savings and Loan crisis of the 1980s is the standard example: savings banks, most of which were under water due to interest rate changes, undertook very risky investment projects using insured deposits.

It is likely that the context of asset liquidation, examined here, is empirically a more important example of the agency problem involving owners and creditors than investment. This is so because firms undertaking large investment projects are typically (although not always, as the Savings and Loan example indicates) successful. For such firms default is a remote possibility, implying that the agency problem just outlined does not result in a major distortion. In contrast, default is not a remote possibility for firms liquidating assets: they often liquidate assets precisely in order to forestall bankruptcy. To the extent that default is imminent, the agency problem is likely to be important quantitatively.

The model is described informally in Section 2. Sections 3 and 4 derive an equilibrium when the pricing strategy is chosen to be optimal from the point of view of equity. To do so we assume initially that the sale is financed using an annuity alone, so that the only equity the buyer has is that resulting from the surplus represented by the excess of his valuation of the firm's assets over the reservation valuation implied by the same price. Ex ante the agency problem induced by debt finance reduces the value of the firm by an amount equal to the capitalized value of the agency costs. In Section 5 we devise a different financial instrument, one which has some similarities to preferred stock, and show that this instrument is not subject to the agency problem.

The final exercise in the paper applies these ideas to construct a model of optimal capital structure. Debt finance is subject to the agency problem just outlined, but is tax advantaged relative to equity. We modify the model by allowing the seller of an illiquid asset to offer a package incorporating both debt and equity. Depending on the amount of preexisting debt and parameter values, the seller may use either all debt, all equity, or a combination of the two.

2. The Model: Informal Description[1]

There exists a single indivisible productive asset which we call a factory. There is an infinite number of potential owner-managers of this factory. These agents have common endowment of a background good, common discount factor and are risk neutral. One of these owner-managers has the factory as part of his endowment. For simplicity, we will assume that the initial owner-manager also plays the role of the bank, which finances subsequent purchases. The other agents will buy the factory from the bank, each in turn having the opportunity to become the current owner-manager. The current owner-manager of the factory has a *match*, with *fit* ε, if he is able to operate the factory with operating profit of ε per period. The fit continues period after period, implying constant profit of ε per period, until the match is broken, an event that occurs with probability that is constant and is independent over time. When the match is broken the owner-manager receives profit equal to zero per period thereafter, and will sell it.

The bank initially does not have a match. Therefore the bank immediately offers the factory for sale. The other potential owner-managers can operate the factory with varying degrees of profitability. Determining profitability involves costly search. In order to model this process expeditiously, we assume that each period one and only

[1] The model described below is similar in essential respects to that of Krainer and LeRoy [4], to which readers should refer for general discussion. See also Krainer [3].

one potential owner-manager has the opportunity to determine costlessly the profit potential of the factory under his management.[2] The seller must sell to this agent immediately or forego forever the opportunity to sell to him.

Potential buyers know their fit at the time they decide whether or not to buy the factory, but the seller does not know the fit and cannot compel or induce the buyer to reveal it to him. The seller does know that the fit ε is distributed uniformly on $[0, 1]$. The seller posts a take-it-or-leave-it price; this price is chosen to balance high revenue if the factory is sold against a high probability of selling the factory.[3]

The endowments of potential owner-managers are not sufficient to allow them to pay the purchase price of the factory in cash (here we depart from Krainer and LeRoy [4]). The buyer finances 100% of the purchase price using an annuity that is issued in favor of the bank, which is risk-neutral (and should be thought of as a stand-in for a competitive banking system rather than as a monopolistic bank). The issuer has the option to default on the annuity, in which case the factory is turned over to the bank. In the event of default there is no recourse for the bank beyond taking over the factory. We introduce the more realistic combination of debt and equity finance in Section 5. Since the results in Section 4 are qualitatively unchanged by including equity finance, we exclude it here for simplicity.

The bank offers the factory for sale until, sooner or later, one of the potential owner-managers determines that he has a fit that is high enough to justify buying the factory, which he therefore does. The new owner-manager operates the factory for as long as his match persists, paying the bank a fixed payment each period. Eventually the new owner-manager loses his match and offers the factory for sale. Unless he defaults, the current seller must continue to make payments on the annuity until the factory is sold. If the current seller sells the factory for a higher price than he paid, then pays off his loan and

[2] An alternative specification would be to allow n potential buyers at each date. Then, for example, the factory might be assumed to be sold via a sealed-bid auction, with the highest bidder receiving the factory at the price bid by the second-highest bidder. The seller's role would be to set a reservation price. In this setting the optimal reservation price is approximately independent of the number of bidders, therefore equaling the sale price derived below in the case of $n = 1$. This invariance of the reservation price to the number of bidders is standard in the auctions literature (Riley and Samuelson [6]). With the reservation price independent of n, it is evident that taking $n > 1$ would not materially alter the analysis.

[3] Riley and Zeckhauser [7] showed that a single take-it-or-leave it offer is superior to other strategies under general conditions.

keeps the difference. If at any time the seller determines that the payments on the debt are higher than is justified by the expected proceeds of eventual sale, he exercises his option to default on the debt. In that case the factory reverts to the bank, and the process repeats.

The conventions on timing—in particular, when the payment on the debt is made—are important in understanding the equilibrium. Figure 1 is a flow chart depicting the sequence of events from the viewpoint of the bank and the subsequent owner-managers of the factory. As noted, the owner-manager with a match receives profits each period at the same rate, ε, as he received at each of the dates since he bought the factory. He learns each period whether the match continues for another period. If it does, he makes an annuity payment, receives his profit and does nothing else until the next date, at which time the cycle is repeated. If the match does not continue, the owner-manager immediately puts the factory up for sale; doing so dominates defaulting immediately because the convention on timing is such that he can make one attempt to sell the factory before the next annuity payment is due. If the first prospective buyer purchases the factory, then the current owner-manager goes into the next period without the factory. If the first prospective buyer does not purchase the factory, the current owner-manager must decide whether to default without making the annuity payment or to make the annuity payment and attempt again to sell the factory in the next period.

3. A Nash Equilibrium with Increasing Prices[4]

In this section we derive a Nash equilibrium that, although not subgame perfect, has the same equilibrium path as a subgame perfect equilibrium that is described in the following section. The equilibrium to be derived involves each owner-manager of the factory who has lost his match offering it for sale at a price strictly higher than that which he paid. The analytical procedure for solving the model involves (1) conjecturing the decision rule for default, (2) computing buy and sell rules that are best responses to each other and to the assumed default rule, and (3) verifying that the assumed default rule is optimal.

[4] There also exists a continuum of Nash equilibria with constant price. These are indexed by $\overline{p}$, $0 < \overline{p} < 1$. Sellers set annuity payment $\overline{p}$, buyers buy if $p = \overline{p}$ and $\varepsilon \geqslant \overline{p}$, and not otherwise, and sellers default upon failing to sell the factory at the first attempt. These equilibria are not subgame perfect, and are therefore implausible. See the March 11, 2003 version of this paper for fuller discussion.

The conjectured default rule is that any owner-manager other than the bank (which has no debt) will take advantage of his one free opportunity to try to sell the factory. Upon failing to do so, he defaults.

3.1. Equilibrium Selling Rule of Non-Initial Owner-Managers

We begin by determining the behavior of the owner-managers other than the bank. The behavior of the bank is considered in the following subsection.

All potential owner-managers are risk neutral and have discount rate β. Let $v(\varepsilon, p)$ be the net value to its owner-manager of a factory with fit ε and annuity payment of p per period. Risk neutrality implies that $v(\varepsilon, p)$ is given by

$$v(\varepsilon, p) = \beta(\varepsilon - p) + \beta\pi v(\varepsilon, p) + \beta(1-\pi)z(p), \tag{1}$$

where π is the probability that the match will continue into the next period, assumed given and constant, and $z(p)$ is the value of a factory with annuity payment p for which the owner-manager does not have a match. Here $z(p)$ is based on the prospective sale price of the factory. As the notation indicates, equation (1) reflects the assumption that the factory can be sold only after the match has been lost.[5]

Assume without loss of generality that the seller quotes the price of the factory in terms of the implied annuity payment y. Then the expected profit on the sale of the factory is the capitalized value of $y - p$, or

$$\mu(y)\lambda(y)(y - p), \tag{2}$$

where $\mu(y)$ is the probability that the factory will be sold to the first prospective buyer, to be determined. The coefficient $\lambda(y)$ is the value per unit of an annuity secured by the factory, also to be determined. The unit value $\lambda(y)$ of the annuity depends on y because y affects the probability of a subsequent default. With probability $1 - \mu(y)$ the owner-manager will fail to sell the factory, in which case by assumption he will default, which has zero value.

[5] The analogous specification in the labor search literature is that a worker can search for a new job only after he has been laid off. Assuming that the owner of the factory cannot offer it for sale while operating it profitably can be defended only as an unrealistic simplification. In [2] we analyze a more realistic, but also more complex, model in which profitability follows a random walk, and in which the owner of the factory can offer it for sale at all levels of profitability.

The seller will set y to maximize (2), and $z(p)$ equals the resulting maximum:

$$z(p) = \max_y [\mu(y)\lambda(y)(y - p)]. \tag{3}$$

Define $p' = \phi(p)$ as the maximizing value of y:

$$p' = \arg\max_y [\mu(y)\lambda(y)(y - p)] \equiv \phi(p). \tag{4}$$

From the definition of p', $z(p)$ is given by

$$z(p) = \mu(p')\lambda(p')(p' - p). \tag{5}$$

Finally, we need to evaluate $\lambda(p')$, the unit value of the annuity payment. The current owner-manager (with annuity payment p) sells the factory in exchange for the capitalized value of $p' = \phi(p)$ per period. The new owner-manager will eventually lose his match and try to sell the factory for p'', where $p'' = \phi(p')$. If he succeeds, which occurs with probability $\mu(p'')$, the annuity will have value $\lambda(p'')$ per unit of payment. If he fails he will default, in which case the annuities terminate and the factory is returned to the bank. It follows that $\lambda(p')$ is given by

$$\lambda(p') = \beta + \beta\pi\lambda(p') + \beta(1 - \pi)\mu(p'')\lambda(p''). \tag{6}$$

3.2. Equilibrium Selling Rule of Bank

The bank has no annuity payment, so p does not enter the bank's decision problem. Also, because the bank will continue trying to sell the factory until it succeeds in doing so (rather than default, as the non-initial owner-managers do), its problem is

$$\max_y [\mu(y)(y\lambda(y) + c(y)) + \beta(1 - \mu(y))m]. \tag{7}$$

The term $c(y)$ is given by

$$c(y) = m \sum_{n=1}^{\infty} \beta^{n+1} \left\{ \begin{array}{c} \sum_{j=1}^{n} \binom{n-1}{j-1} \pi^{n-j}(1-\pi)^j \\ \times \left(\prod_{k=1}^{n-1} \mu(\phi^k(y)) \right) (1 - \mu(\phi^j(y))) \end{array} \right\}. \tag{8}$$

Here $\phi^k(y) \equiv \phi(\phi(...\phi(y)))$, and m, the maximized value of (7), is the value of the factory to the bank prior to sale.

The term $c(y)$ is the component of the current value of the factory accounted for by the defaults of subsequent owners. It represents $m\beta^{n+1}$, the discounted value of the factory conditional on the first default occurring at date $n+1$, multiplied by the relevant probability—the term in braces—summed over n. Here j indexes the number of successful sales that occur before the first default. The term $c(y)$ is a determinant of the optimal initial sale price because different sale prices affect the probability distribution of subsequent defaults: the higher the initial sale price the sooner default will occur, and therefore the greater the present value of the discounted collateral. At the optimum this effect just offsets at the margin the effect of a higher sale price on (1) the size of the annuity payment, (2) the value of the annuity, and (3) the probability of sale.

Note that the bank takes as given the functions $\phi(y)$ and $\mu(y)$ describing the behavior of the non-initial owner-managers derived in the preceding section.

3.3. Equilibrium Buying Rule

We next express the probability of sale $\mu(p)$ in terms of $z(p)$. From (1) the value of the factory for which the owner-manager has a match with fit ε is

$$v(\varepsilon, p) = \frac{\beta(\varepsilon - p) + \beta(1-\pi)z(p)}{1-\beta\pi}. \tag{9}$$

Now define $\varepsilon^*(p)$ as the reservation fit as a function of the annuity payment, so that a prospective buyer acquires the factory in exchange for an annuity with payment p if $\varepsilon > \varepsilon^*(p)$, and not otherwise. If we set the buyer's outside option equal to zero, then:

$$v(\varepsilon^*(p), p) = \frac{\beta(\varepsilon^*(p) - p) + \beta(1-\pi)z(p)}{1-\beta\pi} = 0, \tag{10}$$

implying

$$\varepsilon^*(p) = p - (1-\pi)z(p). \tag{11}$$

This equation, incidentally, implies that an agent may buy the factory even if $\varepsilon < p$, since he will have an asset worth $z(p) > 0$ when he loses the match. If $\varepsilon^*(p) < \varepsilon < p$, the prospect of eventually losing the match and selling the factory outweighs the loss incurred while the match continues, so the prospective buyer will buy the factory.[6]

[6] An owner-manager with fit $\varepsilon^*(p) < \varepsilon < p$ prefers to offer the factory for sale immediately rather than wait until the match is lost. In the interest of

As noted above, the fit ε is assumed to be distributed uniformly on the unit interval. From equation (11), the probability of sale is therefore given by

$$\mu(p) = prob(\varepsilon \geq \varepsilon^*(p)) = 1 - \varepsilon^*(p) = 1 - p + (1-\pi)z(p). \tag{12}$$

3.4. Solving the Model

The behavior of the non-initial owners is characterized by the following equilibrium decision rules:

- Buy rule: the buyer facing a price that corresponds to annuity payment p buys if $\varepsilon \geq \varepsilon^*(p)$, and not otherwise.
- Sell rule: the seller with annuity payment p offers the factory for sale at a price that corresponds to the annuity payment $\phi(p)$.
- Default rule: upon failing to sell the factory at the first try, the seller defaults for all $p > 0$.

The bank sets the initial sale price at the value of y that maximizes (7), and continues trying to sell the factory until it succeeds.

The equilibrium behavior of the non-initial owner-managers does not depend on that of the bank. Therefore we can characterize the behavior of non-initial owner-managers without reference to that of the bank. The equations of equilibrium take the form of recursive functional equations for the functions z, λ, and ϕ:

$$z = \Psi_z(z, \lambda, \phi) \tag{13}$$
$$\lambda = \Psi_\lambda(z, \lambda, \phi) \tag{14}$$
$$\phi = \Psi_\phi(z, \lambda, \phi). \tag{15}$$

Computing these equations involves first assigning parameter values (we chose $\beta = 0.8$ and $\pi = 0.9$ for illustration) and guessing the functions z_0, λ_0, and ϕ_0. Then we computed functions $\lambda_i = \Psi_\lambda(z_{i-1}, \lambda_{i-1}, \phi_{i-1})$, $z_i = \Psi_z(z_{i-1}, \lambda_{i-1}, \phi_{i-1})$ and $\phi_i = \Psi_\phi(z_{i-1}, \lambda_{i-1}, \phi_{i-1})$ for $i = 1, 2, \ldots$ until the convergence criterion was met. The convergence criterion was $\|\phi_i(p) - \phi_{i-1}(p)\| < .0001$, $\|\lambda_i(p) - \lambda_{i-1}(p)\| < .0001$, and $\|z_i(p) - z_{i-1}(p)\| < .0001$, assuming the supremum norm

simplification, we rule out the possibility of offering the factory for sale until the match is broken.

throughout. In the experiments described below we found that there was no difficulty obtaining convergence.[7]

Given the functions ϕ and μ computed from the behavior of non-initial owner-managers, we then solved the decision problem of the bank. This consists of maximizing (7) and setting p_1 according to:

$$p_1 = \arg\max_y \{\mu(y)(y\lambda(y) + c(y)) + \beta(1 - \mu(y))m\} . \qquad (16)$$

3.5. Equilibrium

The equilibrium path of the economy in the computed example is easily characterized using the equilibrium policy function. The bank, not having a match, immediately offers the factory for sale at a price that implies an annuity payment of p_1, continuing to offer it at this price until it is sold. When the second owner loses his match he offers the factory for sale at a price that implies an annuity payment of $p_2 = \phi(p_1)$. We have that $\phi(p) > p$, and also that $\phi(p)$ is increasing (see the appendix for a proof of the latter result).

If the first prospective buyer decides to buy the factory, the second owner will pay off his annuity, and will also receive a payment equal to the capitalized value of $p_2 - p_1$. The third owner will operate the factory, paying p_2 each period, until he loses his match, at which time he offers it for sale at price implying an annuity payment of $p_3 = \phi(p_2)$, and so on. If, upon losing his match, the second owner fails to sell the factory to the first prospective buyer, he will default on the annuity. This implies returning the factory to the bank, which will offer the factory for sale, again at price p_1. Thereafter the cycle will repeat.

It remains to verify that, under the equilibrium values of functions $\phi(p)$ and $\varepsilon^*(p)$, each non-initial owner-manager does in fact optimally choose to default upon failing to sell the factory on the first try, as assumed in deriving the equilibrium. It is optimal for non-initial owner-managers to default if $z(p) < p$, in which case the value of the factory is less than one annuity payment. Figure 2 plots $z(p)$ and p under the assumed parameter values. It shows that the critical value of p above which default occurs is 0.34. Since $p_1 = 0.69$ and

[7] We do not have a proof that Ψ_z, Ψ_λ and Ψ_ϕ define a contraction. The problem is that $z(p)$ enters multiplicatively in these functions, resulting in nonmonotonicity. However, we can prove existence of equilibrium and the value functions via the Schauder fixed point theorem. The theorem also gives a computational check for uniqueness which is satisfied in the examples below.

$p_{j+1} > p_j$ for all j, we see that all non-initial owner-managers will in fact behave as conjectured.

Since sale on the first try is increasingly improbable for the second, third or fourth owners, it is highly unlikely that any given chain will involve more than three or four owners before collapsing (Table 1 shows the transition probabilities and the distribution of the length of a chain for the parameter values specified above). Of course, since time is infinite a chain of any given length will occur infinitely often with probability 1.

3.6. Interpreting the Seller's Optimal Policy Function

The policy function $\phi(p)$ is easily interpreted. As Figure 3 indicates, $\phi(p)$ is very close to $(p+1)/2$. To see why, observe that, from a Taylor expansion of (12) around $p' = 1$, we have

$$\mu(p') \cong 1 - p', \tag{17}$$

since $z(1) \cong 0$ and, since $\mu(1) \cong 0$, also

$$\lambda(p) \cong \frac{\beta}{1 - \beta\pi}, \tag{18}$$

from (6). From (3) the seller's problem is therefore approximately that of choosing y to maximize

$$\frac{(1-y)\beta(y-p)}{1-\beta\pi}, \tag{19}$$

the first-order condition for which is

$$y = \frac{p+1}{2}. \tag{20}$$

This approximation is based on the presumption that non-initial owner-managers of the factory will fail to sell on the first try, and will therefore default. The approximation is more accurate the higher the value of p, since $\mu(p)$ is close to zero when p is near 1 but, as Figure 3 indicates, the approximation is fairly accurate for all values of p.

The bank chooses a selling price of 0.69. This figure is substantially higher than the limiting value of $\phi(p)$ as p approaches zero, which is 0.47. The reason is that the non-initial owner-managers choose low sales prices to induce subsequent owner-managers to do the same, the idea being to reduce the probability of default and consequent termination of the annuity. The bank has less incentive to reduce the probability of default because it is aware that it will recover the factory in the event of the default of a subsequent owner-manager.

For any p the optimal sale price (20), set to optimize the interests of the current owner-manager, exceeds the sale price that would be preferred by the bank. At the extremely high prices that occur after several successful sales, even buyers with fairly—but not extremely—high values of ε will decide against buying the factory. The bank would prefer that default be avoided by selling to such buyers, but by assumption it cannot influence the seller's pricing decision.

The possibility of default plays a central role in generating the successively higher selling prices that occur in equilibrium. To see this, we show that if default were not possible the annuity payment would be a sunk cost, implying that all sellers would face a problem identical to that of the bank. Because any sale results in a perpetuity if subsequent owners do not default, (3) is replaced by

$$z(p) = \mu(p')\frac{\beta}{1-\beta}(p'-p) + (1-\mu(p'))\beta(z(p)-p), \tag{21}$$

Here $z(p)$ separates into the difference between two terms: a term $\overline{z}$ reflecting the unencumbered value of the factory and a term $\beta p/(1-\beta)$ reflecting the value of the perpetuity. To verify this, substitute $\overline{z} - \beta p/(1-\beta)$ for $z(p)$ in (21). The terms in p drop out and (21) reduces to

$$\overline{z} = \mu(p')\frac{\beta p'}{1-\beta} + (1-\mu(p'))\beta\overline{z}, \tag{22}$$

so the value of the factory to the owner who has lost his match is a weighted average of its discounted value if sold and its discounted value if not sold, with the relevant probabilities as weights, irrespective of the preexisting annuity payments.

If debt were non-defaultable, the bank would sell the factory for a price implying an annuity payment p_1 that equals the value of p' that maximizes (22). The optimized value p_1 is 0.59. This results in a sale package with value equal to $p_1\beta/(1-\beta) = 2.36$. This compares to $p_1 = 0.69$, which has value equal to $0.69(\lambda(0.69)) = 2.08$ when debt is defaultable. Correspondingly, the value m of the factory to the bank is 1.59 when debt is defaultable, and 1.71 when it is not defaultable. The difference between these two figures reflects the expected discounted present value of the distortion caused by the default option on the debt.

4. Subgame Perfect Equilibrium

The equilibrium described in the preceding section is not subgame perfect. It was derived under the assumption that owner-managers with annuity p will default upon failing to sell the factory at the first try, for all $p > 0$. For sufficiently low p this behavior is suboptimal. As noted above, the optimal default rule is to default if the value of the factory based on the prospect of its future sale, $z(p)$, is less than the annuity payment p, and not otherwise. Since $z(p)$ is strictly decreasing in p, $z(1) = 0$, and $z(0) > 0$, it follows that there exists a critical value of p, which we call p^*, defined by

$$z(p^*) = p^*, \tag{23}$$

such that the utility-maximizing default rule is

- Default if $z(p) < p^*$, and not otherwise. In the example, $z(p) = p$ for $p = 0.34$, so the optimal default rule is
- Default whenever $p > 0.34$, and not otherwise.

We need to show that the optimal decision rules for buying and selling in the Nash equilibrium of Section 3, in which agents default for all $p > 0$, continue to be optimal under the modified default rule. Since in the example $p_1 = 0.69 > p^* = 0.34$ and $p_{j+1} > p_j$ for all j, we see that all non-initial owners are in the default region under both default rules. Therefore their optimal buying and selling behavior is unaffected by the change in the default rule.

To verify subgame perfection, is necessary to show that the alteration of the assumed default rule of non-initial owners will not induce the bank to alter its sale price. It is immediate that there cannot exist an equilibrium in which the bank sells at a price in the no-default region. This is so because if the second owner never defaults on the annuity, it is in fact a perpetuity, therefore a sunk cost. That being the case, existence of the perpetuity cannot affect the sale price chosen by the second owner. It follows that he will choose the same price as the bank, p_1. Offering the factory at p_1 has a return of zero since the second owner receives an annuity with zero net payments if the factory is sold and defaults if the factory is not sold. But then an offer of p_1 cannot be optimal, since the second owner can raise the offer price and get a positive return if the factory is sold and default if the factory is not sold (contradicting the joint hypothesis that the second owner offers p_1 and that p_1 is in the no-default region).

It remains to show that the original equilibrium remains an equilibrium under the modified default rule. This requires showing that the optimized value of the factory $\widehat{m}$ if sold in the no-default region is less than m, its value if sold for p_1, so that the hypothesized equilibrium selling rule on the part of the bank is in fact optimal. In the example this turns out to be trivial: $\widehat{m}$ is clearly less than the value of the factory based on the assumption the bank can sell it for price p^* with probability 1. But that value is less than m:

$$\widehat{m} < \frac{\beta}{1-\beta} p^* = 1.35 < 1.59 = m. \tag{24}$$

It follows that the bank will not alter its selling rule under the modified default rule. Accordingly, the equilibrium price sequence $(p_1, p_2, ...)$ calculated in Section 3 is a subgame perfect equilibrium price sequence under the buy and sell rules

- Buy rule: the buyer facing price p buys if $\varepsilon \geq \varepsilon^*(p)$, and not otherwise.
- Sell rule: the seller with annuity p offers the factory for sale at price $\phi(p)$.
- Default rule: upon failing to sell the factory at the first try, the seller defaults for all $p > p^*$, and does not default otherwise.

5. Optimal Capital Structure

We have seen that when purchase of an illiquid capital asset is financed using defaultable debt, the purchaser has an incentive to set a high price when the time comes to sell. Because of this distortion, the value of the asset prior to sale is lower than it would otherwise be. The current seller would like to recapture the lost value by inducing the buyer to sell for a lower price than he or she would choose if unconstrained when he or she loses his or her match. Up to now we have assumed, implausibly, that there is no way for the seller to do so. In this section we add a new financial instrument that we will call preferred stock.[8] The seller now is assumed to offer the factory in exchange for a payment package consisting of nonnegative amounts of new defaultable debt and preferred stock, where the proportions of each are chosen by the seller.

What we call preferred stock is a financial asset that, like debt, pays a fixed per-period sum to the bank for as long as the buyer's

[8] There exists a variety of financial assets that could be substituted for preferred stock without qualitatively affecting the equilibrium. Choice among these is a matter of personal preference. Similarly, the asset we refer to as preferred stock may alternatively be characterized as a lease.

match continues. Upon failure of the match, however, the issuer of the preferred stock is permitted to terminate payments without surrendering control of the factory, in contrast to the case with debt. This specification is a stylized counterpart of the real-world provision of preferred stock that its issuer can suspend preferred dividends indefinitely without declaring bankruptcy (although in the real world no payments of common dividends are permitted until the preferred stockholders have been brought current). Thus preferred stock is junior to debt. In the model we assume that when the match ends the payments on the preferred stock are suspended permanently, implying that the preferred stock becomes worthless.[9] Therefore its existence prior to loss of the match has no effect on the financing package that the seller offers to prospective buyers, again in contrast to the case with debt.

The advantage of financing with preferred stock is that its use induces the current buyer to set a low sale price when he loses his match, which increases the value of the package to the current seller. It is easy to see that in the absence of other changes in the model, financing packages will consist of 100% preferred stock. With no debt to be serviced following failure of the match, all sellers face the same decision problem. Therefore they will choose the same selling price, implying that the asset price inflation that takes place when transactions are financed using defaultable debt no longer occurs.

The preferred stock payment d optimally asked by the bank is 0.83. It is easy to check that the value of the factory to the bank, equal to $\beta d/(1-\beta\pi)$, equals the value calculated above, $p_1\beta/(1-\beta)$ when debt is not defaultable. This is as expected because both non-defaultable debt and preferred stock avoid the distortion implied by defaultable debt.

We do not see many firms with capital structures consisting of 100% equity, and the reason is well known: corporate profits taxes. Because bond interest is a cost—in contrast to dividends, which are a distribution of earnings—it is paid out of pretax earnings. This difference in the tax treatment of bonds and stock leads firm managers to prefer debt financing, other things equal. Thus if the setting just described is modified by including taxation of corporate earnings, firms will trade off the tax advantage of debt against its disadvantage in distorting subsequent pricing decisions. The maintained assumption

[9] This is unrealistic, as it is unlikely that failure of a match as incorporated in the present model corresponds to any contractible event in the real world. For our purpose this is not a serious problem, since the role of preferred stock is simply to illustrate one of many possible financing arrangements that are free of agency problems.

here is that all securities that are tax advantaged distort the resale decision, so that corporations have no way to avoid the trade-off modeled below. The fact that real-world corporations place heavy reliance on defaultable debt suggests that this specification is realistic.

For some parameter values one would expect to derive an interior maximum, determining a capital structure that maximizes $z(p)$. In the remainder of this section we will sketch this derivation. We will see that including preferred stock modifies the model of Section 4 only in detail; rather than repeating the derivation we will therefore specify only the new material.

Upon loss of his match, the seller proposes a package consisting of a per-period debt payment of y as before, but also a dividend on preferred stock of d per period. The probability of sale now depends on both y and d, so it is denoted $\mu(y,d)$. The value of the newly issued component of the annuity is $\lambda(y)(y-p)$ as before, whereas the value of the preferred stock equals $\beta d/(1-\beta\pi)$. Therefore we have

$$z(p) = \max_{y \geq p, d \geq 0} \left\{ \mu(y,d) \left[\frac{\beta d}{1-\beta\pi} + \lambda(y)(y-p) \right] \right\}. \qquad (25)$$

Here p equals the per-period annuity paid to preceding owners as before; preferred stock payments to preceding owners do not figure in the calculation because these payments cease as soon as the current owner loses his match.

Define $\phi(p)$ and $\psi(p)$ as the values of y and d, respectively, that maximize $z(p)$:

$$\phi(p) \equiv \arg\max_y \left\{ \mu(y,\psi(p)) \left[\frac{\beta\psi(p)}{1-\beta\pi} + \lambda(y)(y-p) \right] \right\} \qquad (26)$$

$$\psi(p) \equiv \arg\max_d \left\{ \mu(\phi(p),d) \left[\frac{\beta d}{1-\beta\pi} + \lambda(\phi(p))(\phi(p)-p) \right] \right\}, \qquad (27)$$

so that $z(p)$ is given by

$$z(p) = \mu(\phi(p),\psi(p)) \left[\frac{\beta\psi(p)}{1-\beta\pi} + \lambda(\phi(p))(\phi(p)-p) \right]. \qquad (28)$$

The problem of the bank is computed in a similar fashion.

To derive an expression for $\mu(y,d)$, we modify the derivation of $\mu(y)$ above. The value $v(\varepsilon,y,d)$ to a buyer with fit ε of a factory with debt payment y and preferred stock payment p satisfies

$$v(\varepsilon,y,d) = \beta(\varepsilon-y)(1-t) - \beta d + \beta\pi v(\varepsilon,y,d) + \beta(1-\pi)z(y), \qquad (29)$$

where t is the tax rate on corporate earnings. If $\varepsilon^*(y,d)$ is defined as the reservation fit, we can derive

$$\varepsilon^*(y,d) = y + \frac{d}{1-t} - \frac{1-\pi}{1-t} z(y) \tag{30}$$

by setting $v(\varepsilon^*(y,d), y, d)$ equal to zero and solving for $\varepsilon^*(y,d)$. Therefore we have

$$\mu(y,d) = prob(\varepsilon \geq \varepsilon^*(y,d)) = 1 - \varepsilon^*(y,d) = 1 - y - \frac{d}{1-t} + \frac{1-\pi}{1-t} z(y). \tag{31}$$

The right-hand side of (31) is the analogue of (12) above.

Introducing preferred stock does not materially complicate solving the model. Note that, from (25) and (31), the maximand is quadratic in d, implying that for given p, d can be easily determined analytically. Figures 4 and 5 show the value of the factory and the optimal decisions and equilibrium path for different levels of p when $\beta = 0.7$, $\pi = 0.3$ and $t = 0.05$.

Figure 6 displays computed optimal capital structures. The lower line of the diagram displays the market value of debt (not the per-period payments as in Figure 5) as a function of preexisting annuity per-period payments p, while the upper line displays the value of debt plus preferred stock. As the diagram indicates, for low levels of p the optimal financing package consists almost entirely of preferred stock, while for higher levels of p there is less use of preferred stock and more use of debt. For values of p greater than 0.72 no preferred stock is issued. This pattern makes sense: for high values of p the distortion incurred by debt financing has negligible present value since the seller is almost sure to default upon losing his match regardless of the current financing package. Therefore the tax advantage of debt while the match lasts renders it more attractive. In contrast, for low levels of p the prospective value of the factory upon resale accounts for a higher fraction of its current value, implying that the price distortion is more important quantitatively.

Note the discontinuity in the optimal capital structure as p approaches zero: the bank chooses a financing package with more debt than non-initial owners with low values of p. This difference reflects the fact that the bank loses his annuity but also recovers the factory upon default of subsequent owners, whereas non-initial owners lose their annuities without any offset. Because of this difference, the present value of the distortion implied by debt finance is lower for the bank than for non-initial owners. Accordingly, the bank makes more use of debt.

It is useful to compare the determinants of optimal capital structure as represented in this section with the received theory from the corporate finance literature (as in, for example, Ross, Westerfield and Jaffe [8]).

In the corporate finance literature firms trade off the tax advantage of debt against the expected present value of bankruptcy costs, which increases with the level of debt, in choosing an optimal capital structure.[10] A difficulty with this line is that the effect of a debt increase on the present value of the tax shield appears to be much larger than its effect on the present value of bankruptcy costs, at least for capital structures in line with those typically observed. The model of this section suggests part of the explanation: in the received literature bankruptcy costs are too narrowly construed. In the model of this paper bankruptcy costs as usually interpreted are zero: in the event of default the factory is assumed to revert to the bank without any diminution of value. The role of bankruptcy costs is played by mispricing of the factory, which results in a value loss whether or not there is a default.

The suggestion is that if the costs of debt finance were broadened to include the pricing distortion that debt finance induces, as well as the parallel distortion of investment, it might be easier to rationalize existing levels of debt and equity as optimal capital structures. Of course, it would be very difficult to determine the quantitative importance of the distortions analyzed theoretically here.

6. Conclusion

We have presented a model in which a corporation's management liquidates assets in the fashion that is optimal for the owners, despite the losses that this liquidation strategy imposes on creditors. Here we consider the incentives the various players have to adopt alternative possible liquidation strategies.

Ex ante it is the owners of fixed resources that have an incentive to adopt a different regime of corporate control. The owners of valuable fixed resources consist of, first, the agent who has the factory as part of his initial endowment and, second, the agents who will have an early opportunity to buy the factory (we are assuming here that the agents are numbered as to the order in which they will arrive, and

[10] There is a loose end here: bankruptcy costs can be avoided if the creditors replace debt with equity prior to bankruptcy. Therefore the costs of such financing changes place an upper bound on bankruptcy costs (see Haugen and Senbet [1] and Wang, Young and Zhou [11] for related discussion).

that this numbering is public knowledge). These agents each own an asset the value of which depends on the control regime. In any regime the value of this asset is well determined, and it fluctuates over time as the random components of the model are realized.

Because equity values depend on the control regime, the agents—especially the bank—have an interest in adopting indentures that constrain the prices at which they and subsequent owners sell the factory. Of course, each agent, being in a different position from the others, would prefer a different indenture (for example, the n-th potential buyer would prefer an indenture that requires the bank and the first $n-1$ owners to give away the factory), so it is far from obvious what indenture provisions, if any, would be adopted. As a problem in cooperative game theory, study of this question is not a proper part of the present paper.

We have interpreted the model presented above as illustrating the conflict of interest between owners and creditors. Other applications of the model are possible: the model provides a possible explanation of credit chains. As another example, corporate takeovers require that the buyer perform "due diligence": a detailed examination of the corporation being acquired. The expenses involved in due diligence, being considerable, imply that only one or a very small number of purchasing groups will be involved in the takeover of a large firm. Corporate finance models rarely take explicit account of these expenses, despite the fact that in the absence of such expenses the takeover market could reasonably be assumed to be competitive for all but the largest takeovers. The model presented in this paper can be interpreted as providing a setting in which the expenses of due diligence are represented by assuming that only one buyer can perform due diligence per period, implying an explanation of why the takeover market is not competitive.

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7. Appendix: Proof that ϕ is increasing

Theorem 1: The value and policy functions z, λ, and ϕ satisfy:

1. z, λ, $\phi \geq 0$ for all p, and $z(1) = 0$.
2. $-z'$, $-\lambda'$, $\phi' \geq 0$ for all p.

Proof: We use a proof by induction. Recall:

$$\begin{bmatrix} z_i(p) \\ \lambda_i(p) \\ \phi_i(p) \end{bmatrix} = \Psi(z_{i-1}, \lambda_{i-1}, \phi_{i-1}) \tag{32}$$

$$= \begin{bmatrix} \max_y [\mu_{i-1}(y)\lambda_{i-1}(y)(y-p)] \\ \frac{\beta}{1-\beta\pi}\left(1 + (1-\pi)\mu_{i-1}(\phi_{i-1}(p))\lambda_{i-1}(\phi_{i-1}(p))\right) \\ \arg\max_{y\in[0,1]}\left[\mu_{i-1}(y)\lambda_{i-1}(y)(y-p)\right] \end{bmatrix} \tag{33}$$

1. Let z_{i-1}, λ_{i-1}, $\phi_{i-1} \geq 0$ for all p. Then

$$\mu_{i-1}(p) = 1 - p + (1-\pi)z_{i-1}(p) \geq 0.$$

Hence $\lambda_i \geq 0$ for all p. Further, the principle of optimality implies:

$$z_i(p) \geq \mu_{i-1}(p)\lambda_{i-1}(p)(p-p) = 0. \tag{34}$$

Hence $z_i \geq 0$ for all p and $z_i(1) = 0$. Finally, $\phi_i(p) \geq 0$ since $\phi_i(p) = y \in [0,1]$.

2. Let $-z'_{i-1}$, $-\lambda'_{i-1}$, $\phi'_{i-1} \geq 0$ for all p and assume z_{i-1}, λ_{i-1}, ϕ_{i-1} are continuously differentiable. Then:

$$\mu'_{i-1}(p) = -1 + (1-\pi)z'_{i-1}(p) < 0 \tag{35}$$

Hence:

$$\lambda'_i(p) = \frac{\beta(1-\pi)}{1-\beta\pi}\phi'_{i-1}(p)\begin{pmatrix} \mu_{i-1}(\phi_{i-1}(p))\lambda'_{i-1}(\phi_{i-1}(p)) + \\ \mu'_{i-1}(\phi_{i-1}(p))\lambda_{i-1}(\phi_{i-1}(p)) \end{pmatrix} \leq 0 \tag{36}$$

Next, the solution y^* is on the interior of $[0,1]$ for $p \neq 1$ since the corner solutions are inferior to at least one point on the interior. Hence we can apply the envelope theorem to get:

$$z'_i(p) = -\mu_{i-1}(\phi_i(p))\lambda_{i-1}(\phi_i(p)) \leq 0. \tag{37}$$

For $p = 1$, we have $\mu_{i-1}(1) = 0$ and hence $z'_i(1) = 0$, which is non-negative.

Finally, Milgrom and Shannon [5] give conditions for monotone comparative statics using the 'supermodularity' results of Topkis [10]. They show that if only one decision variable y exists and is a function of one parameter p, then y is non-decreasing in p if and only an increase in p increases the marginal value of y (known as 'increasing differences'). Here:

$$\frac{\partial z_i(p;y)}{\partial y \partial p} = -\mu'_{i-1}(y)\lambda_{i-1}(y) - \mu_{i-1}(y)\lambda'_{i-1}(y) \geq 0.$$

Hence $\phi'_i(p) \geq 0$. Hence we have shown that if Properties (1) and (2) hold for z_{i-1}, λ_{i-1}, ϕ_{i-1}, then they hold as well for z_i, λ_i, and ϕ_i. We have shown via the Schauder fixed point theorem (proof available on request) that Ψ has at least one fixed point [z, λ, ϕ]. Further, the fixed point can be obtained by iterating on Ψ starting from the lower bound of $-z_0 = -\lambda_0 = \phi_0 = 0$, which are non-decreasing, non-negative functions. Thus since the space of functions which satisfies Properties (1) and (2) is a complete space, Properties (1) and (2) hold for [z, λ, ϕ].

8. Appendix: Figures and Tables

State:	Transition Probabilities:				state
owner	original	buyer 1	buyer2	buyer3	prob.
original	0.687	0.313	0	0	0.196
buyer 1	0.084	0.9	0.016	0	0.683
buyer 2	0.092	0	0.9	0.008	0.111
buyer 3	0.096	0	0	0.9	0.010

Table 1. State transition matrix and probability distribution.

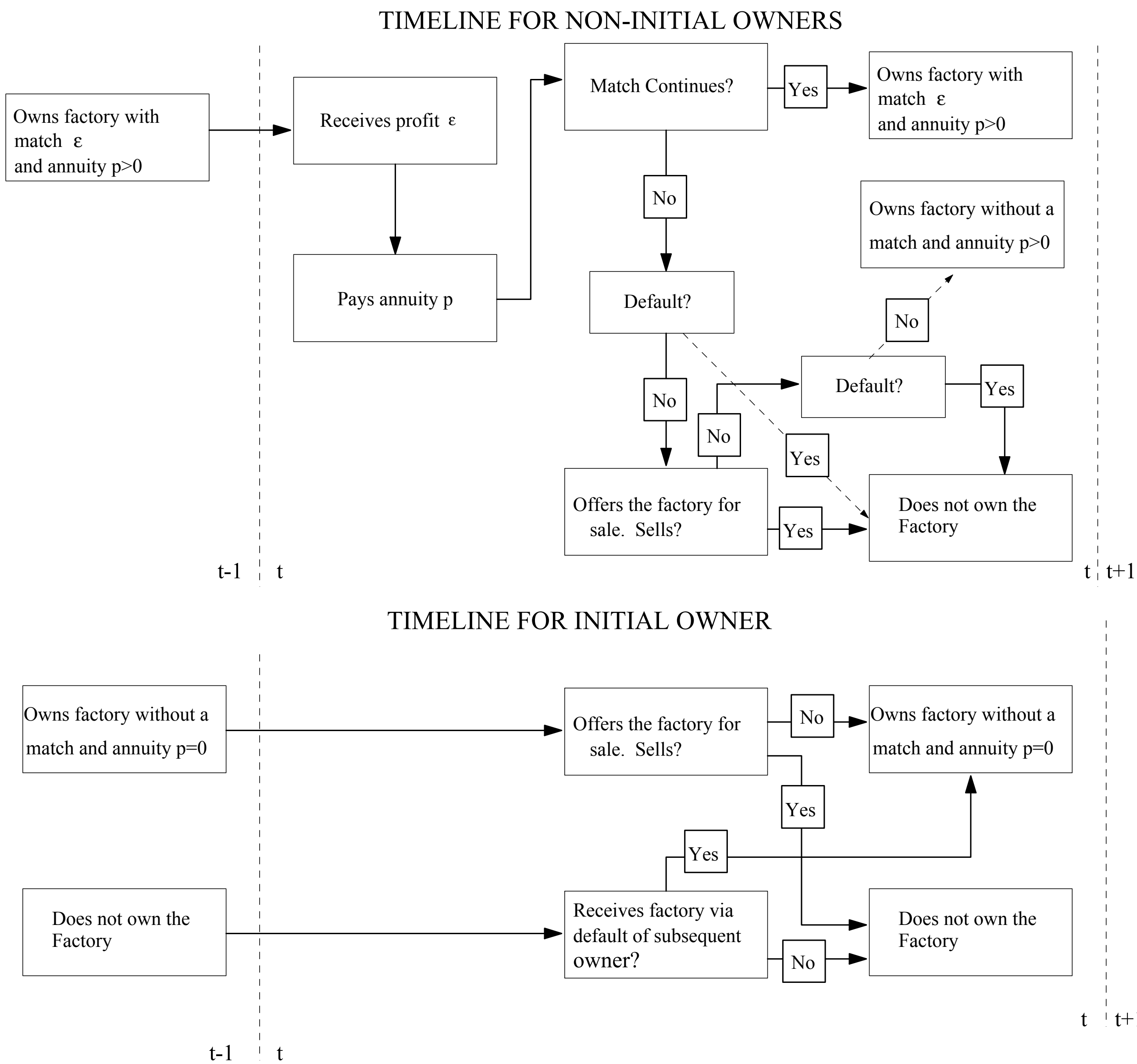

Fig. 1. Timeline and decision tree for first and subsequent owners.

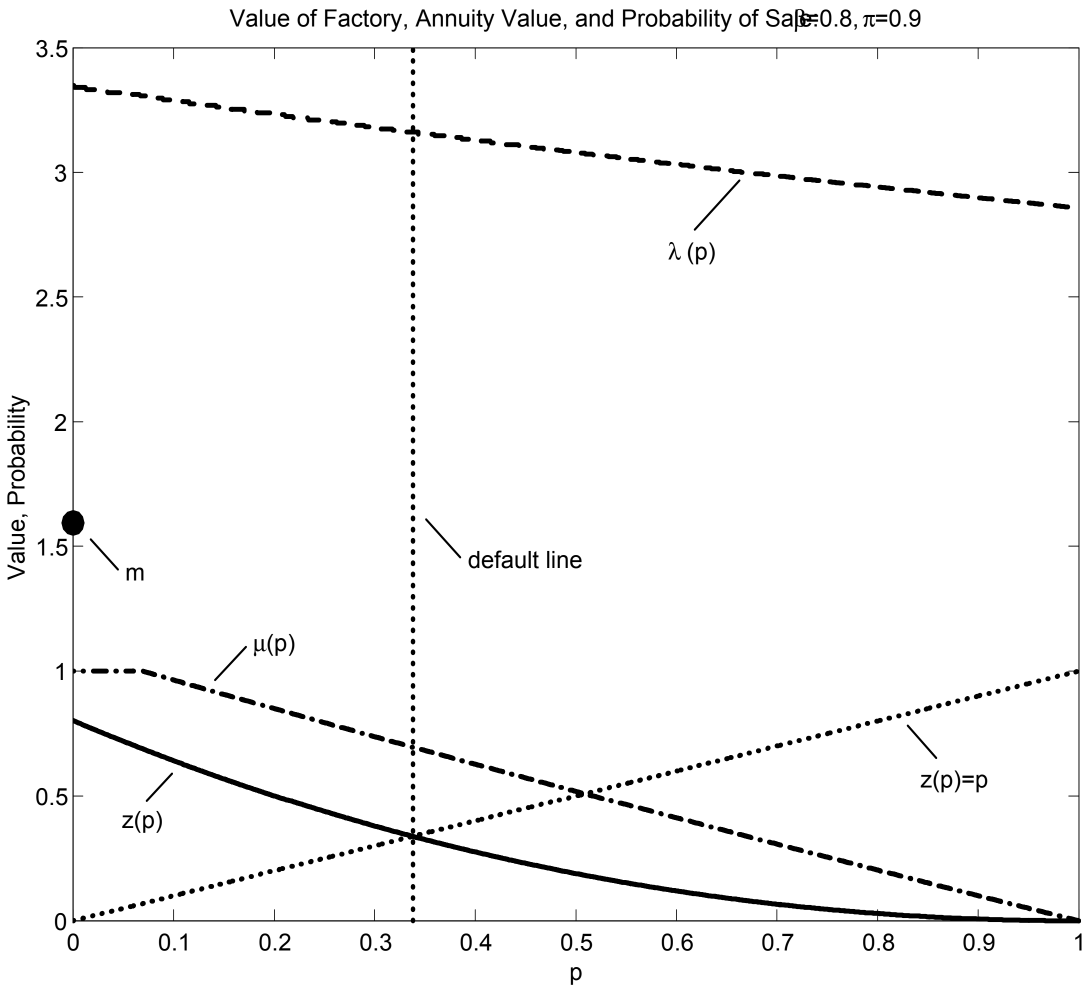

Fig. 2. Value of factory, annuity value, and probability of sale.

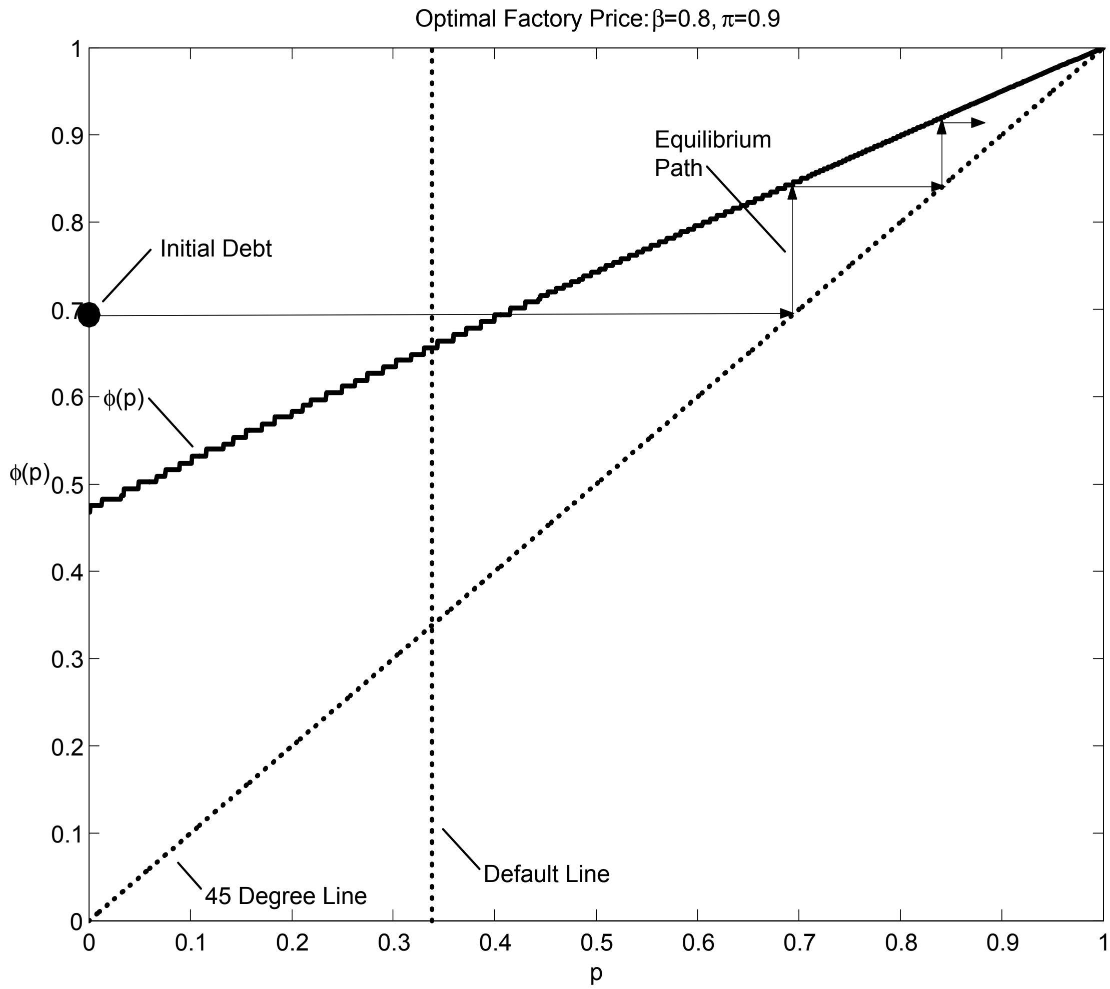

Fig. 3. Optimal factory price and equilibrium path.

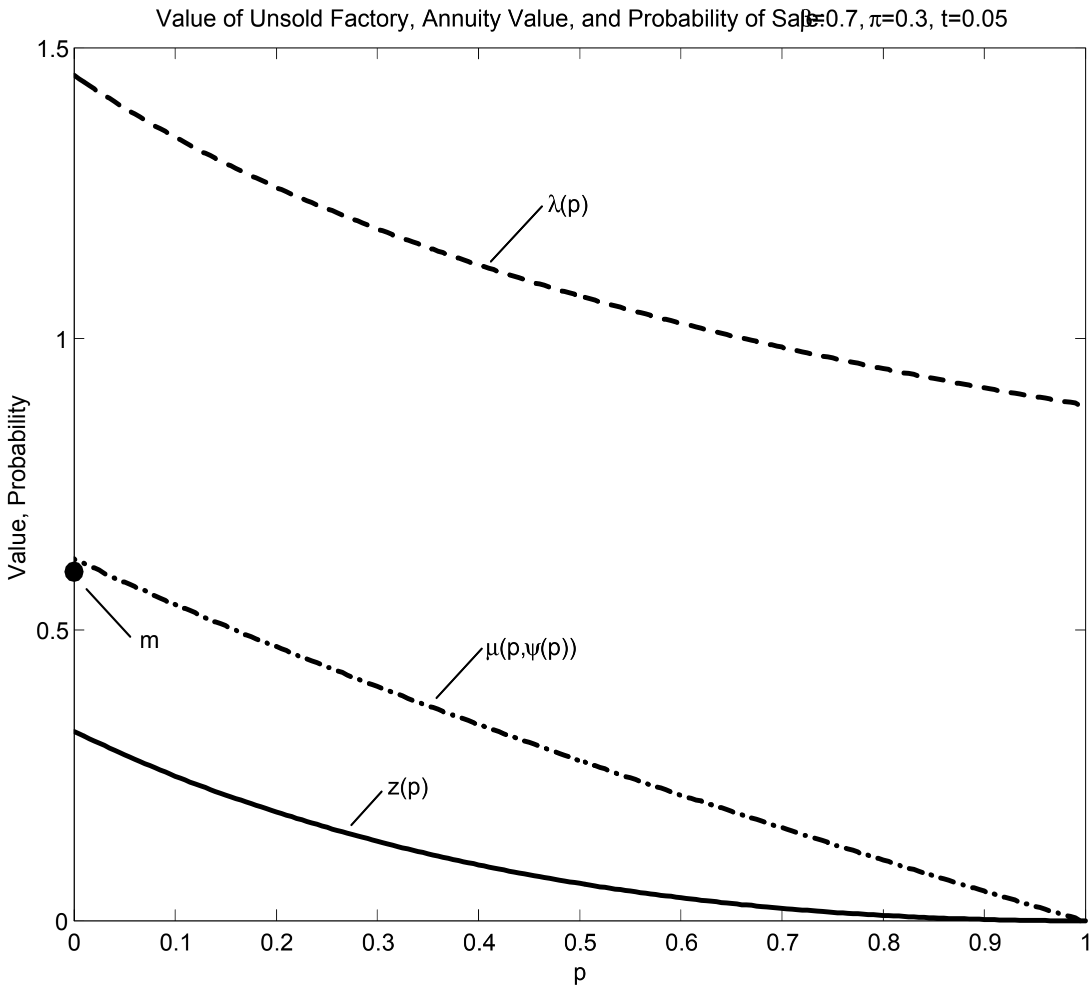

Fig. 4. Value of Factory, annuity value and probability of sale, with both debt and equity offered.

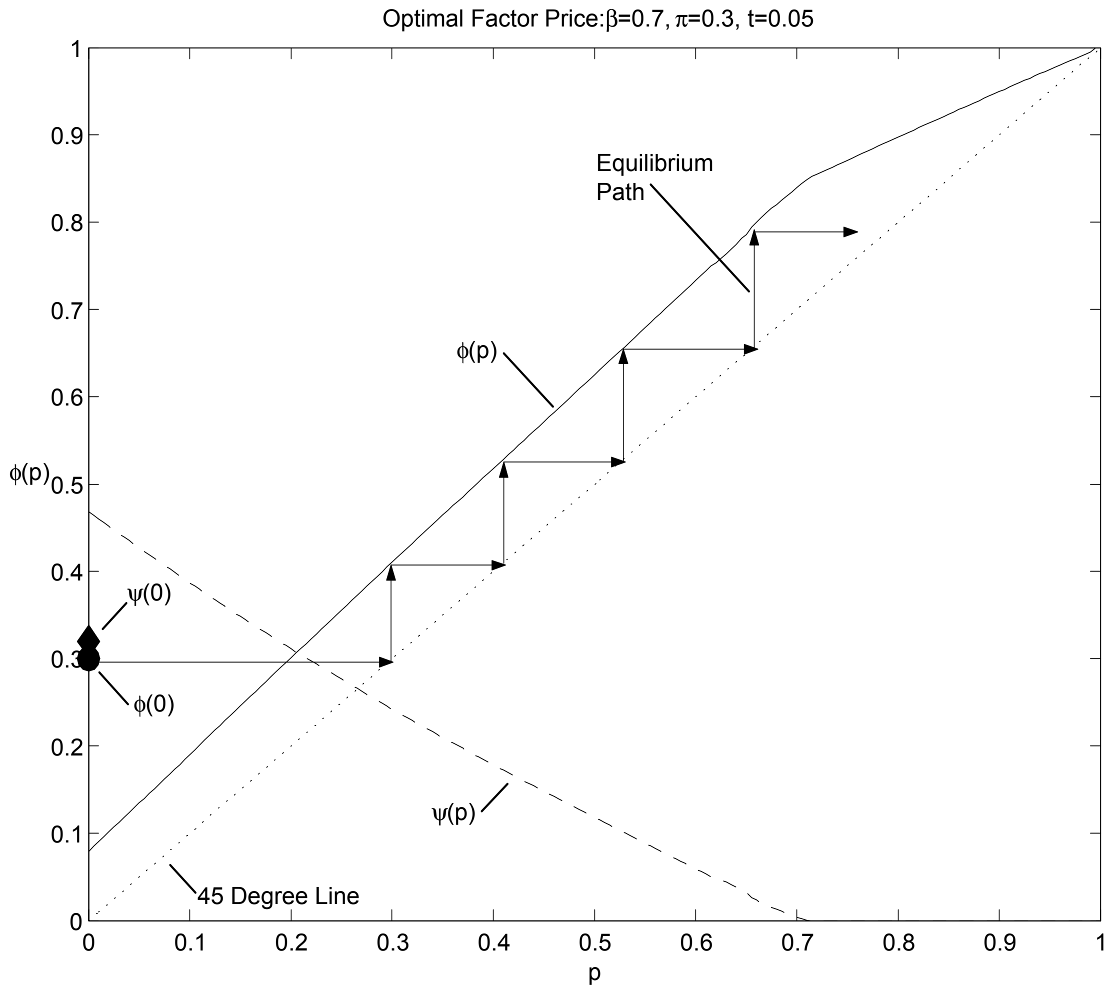

Fig. 5. Debt and equity offers and equilibrium path.

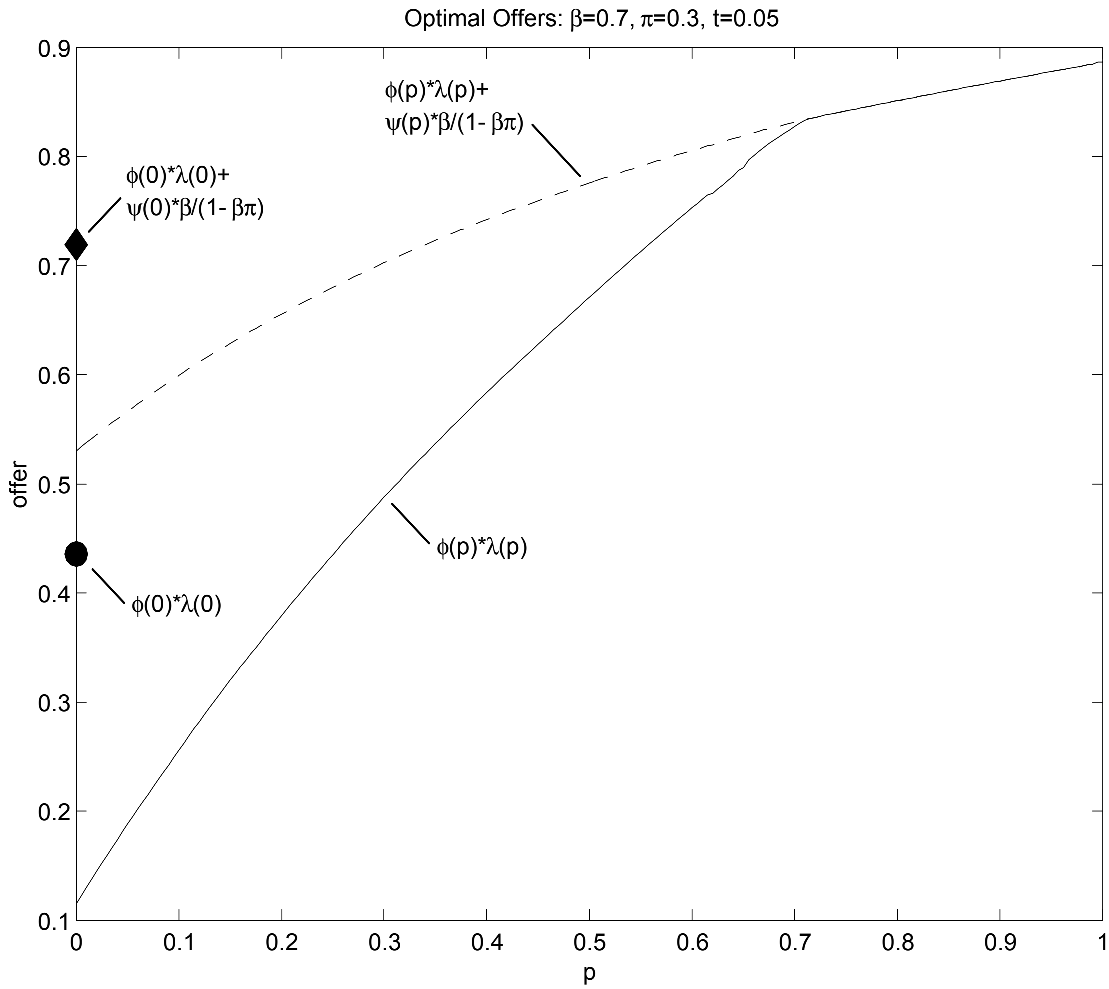

Fig. 6. Optimal capital structure. Equity offer is the difference between the dashed and solid line.