

Capital Market Efficiency: A Reinterpretation

Stephen F. LeRoy*

University of California, Santa Barbara

Kevin J. Lansing

Federal Reserve Bank of San Francisco

November 30, 2016

Abstract

In Fama’s classic definition, capital markets are efficient if asset prices fully reflect available information. One implementation of this definition identifies efficiency with the condition that future excess returns are unforecastable. The result of this paper is that, in a broad class of models, this condition is satisfied if conditional variances of shocks are constant across states. This result occurs even if agents are risk averse, in contrast to the situation that obtains when efficiency is identified with returns rather than excess returns. A model that exemplifies the result is discussed.

Keywords: *Asset Pricing, Market Efficiency, Excess Returns, Return Predictability, Return Volatility.*

JEL Classification: E44, G12.

Fama [4] defined capital markets as efficient if current asset prices fully reflect available information about future gains (in the context of stocks, the gain is defined as next-period price plus next-period dividends). After some initial confusion about how to give content to “fully reflect” (see LeRoy [8] for discussion), analysts arrived at a consensus that asset prices may be taken to fully reflect available information when the excess returns on each asset equal zero or, if nonzero, are not forecastable (the excess return on an asset or portfolio is its return less the return on a position of equal size in the riskless asset).¹

*University of California, Santa Barbara, CA 93106, (805) 689-2344, leroy@ucsb.edu.

¹Here the focus is on time-series aspects of market efficiency. In much of the finance literature, in contrast, the focus is on the existence of cross-sectional variations in returns relative to a baseline model (usually CAPM) due to factors that may or may not be connected to risk, such as book-market ratios or momentum. Such effects are treated as exogenous, and their existence is taken to reflect something called “mispricing” rather than as establishing the validity of some alternative to the baseline model.

To see the connection between asset prices fully reflecting information and excess returns being nonforecastable, note first that the conditional expectation of the excess return on equity is an optimal (under a mean-square criterion) forecast of future excess returns. It follows that the stated criterion for efficiency is equivalent to the proposition that conditionally expected excess returns on any asset at a given date are equal in all events at that date. If that condition is satisfied, the forecasts of future excess gains constructed by applying the common (across events) conditionally expected excess return to current asset prices—which differ across events—depend only on the information about the current event that is incorporated in prices at that event. Thus all relevant information is incorporated in asset prices. If, on the contrary, the condition is not satisfied the conditionally expected excess return in the current event still constitutes an optimal forecast of future excess gains when applied to current price. However, the fact that the conditionally expected excess return varies over events implies that the forecast incorporates information about the current event over and above the information reflected in the current price of the asset. Thus the current price of the asset does not fully reflect available information.

If financial markets are efficient according to the definition just given, the portfolio consisting of a long position in the risky asset and an equal short position in the riskless asset has an expected gain that, while generally nonzero, does not depend on the current event. The expected return on this portfolio is undefined since its current value equals zero.

The criterion just stated for financial market efficiency can be altered to apply to gains themselves rather than to excess gains. To understand the consequences of this redefinition, consider a model in which returns on risky and riskless assets have forecastable components that are nonzero and equal, so that excess returns on risky assets are nonforecastable (as in the model discussed below). In such a model financial markets would be classified as inefficient under the redefinition. Analysts appear to find this implication objectionable (the point is seldom discussed explicitly). This attitude may reflect the view that forecastable interest rates reflect autocorrelated productivity of capital, and as such their existence should not be considered a violation of market efficiency. Insofar as variations in interest rates are much smaller than variations in excess returns, there is little difference quantitatively between returns and excess returns, so the distinction is not important empirically. But it is important theoretically: it is desirable to have in hand models in which financial markets are efficient, while avoiding the very strong assumptions that are needed to shut down forecastability in interest rates. To achieve this we adopt the definition that associates market efficiency with excess returns rather than with returns.

If in an exchange economy agents are risk neutral, conditionally expected excess returns on all assets are zero, so markets are efficient. They are also efficient under the altered definition under weak restrictions on the behavior of interest rates (this is easily proved in the simplest version of the Lucas [9] tree model, and is true in the model presented below).

If agents are strictly risk averse, equilibrium expected excess returns depend on risk: if risk varies predictably over time, as appears to be the case, in general one would expect risk-averse agents to price assets so that they generate returns that have a predictable component. Otherwise the agents are not compensated for variations in risk, which is inconsistent with their being risk averse. However, it is possible that variations over events in expected returns induced by differences in risk are exactly equal to variations in real interest rates. In that case conditional expected excess returns would be equal across events, so that financial markets would be efficient under the definition discussed above.

This argument suggests that it should be possible to prove that if variation over time in risk is appropriately restricted, financial markets may be efficient even when agents are strictly risk averse. Expected excess returns will be nonzero, but that may be consistent with their being nonforecastable. In this paper we show that this intuition is correct: in a class of financial models that allow strict risk aversion, markets are efficient if and only if certain composite variables have constant conditional variances.² It follows that in this class of models market inefficiency can be connected with time-varying volatilities and with these alone.

It is noteworthy that efficiency of financial markets is associated with a routine regularity condition—constant conditional variances—and not with an esoteric and uninterpretable special condition, as might otherwise be expected. Accordingly, one expects that financial markets will turn out to be efficient in standard asset pricing models with or without risk aversion. It turns out that this conjecture is correct. We discuss an example, presented in detail elsewhere, which is very close to the standard Lucas tree model in which agents are strictly risk averse, but financial markets are efficient.

1 Excess Returns and Volatilities

The framework for our analysis is a class of standard representative agent consumption-based asset pricing models. We do not need to specify the model fully; for example, the dividend/consumption process is not characterized. Our results thus apply to a fairly general set of models.

For any traded asset and any specification of investor preferences, the first-order condition of the representative investor's optimal consumption choice yields

$$1 = E_t(M_{t+1}R_{t+1}^i)/M_t, \tag{1}$$

²Since the definition of efficiency involves natural probabilities, the corresponding condition on conditional variances applies to variances under the natural measure as well. One could generalize the efficiency definition to involve an arbitrary distribution rather than the natural probabilities. In that case the derived condition on variances would still be valid, but would refer to variances under the arbitrary distribution, not those under the natural probabilities.

where E_t is the mathematical expectation operator conditional on information available at time t , M_t is the stochastic discount factor, and R_{t+1}^i is the gross holding period return on asset type i from time t to $t + 1$. With time-separable constant relative risk aversion (CRRA) preferences, we have $M_{t+1}/M_t = \beta (c_{t+1}/c_t)^{-\alpha}$, where β is the subjective time discount factor, c_t is the investor's consumption at t , and α is the risk aversion coefficient.

For a dividend-paying stock, we have $R_{t+1}^s = (d_{t+1} + p_{t+1}^s)/p_t^s$, where R_{t+1}^s is the gross return on stock, p_t^s is the ex-dividend stock price at t and d_{t+1} is the dividend received in period $t + 1$. Eq. (1) can be written as

$$p_t^s/d_t = E_t \left[\left(\frac{M_{t+1}}{M_t} \right) \left(\frac{d_{t+1}}{d_t} \right) (1 + p_{t+1}^s/d_{t+1}) \right], \quad (2)$$

where p_t^s/d_t is the price-dividend ratio (the inverse of the dividend yield) and d_{t+1}/d_t is the gross growth rate of dividends. At this point it is convenient to define the following nonlinear change of variables:

$$z_t^s \equiv \left(\frac{M_t}{M_{t-1}} \right) \left(\frac{d_t}{d_{t-1}} \right) (1 + p_t^s/d_t), \quad (3)$$

where z_t^s represents a composite variable that depends on the stochastic discount factor, the growth rate of dividends, and the price-dividend ratio.³ The investor's first-order condition (2) becomes

$$p_t^s/d_t = E_t(z_{t+1}^s), \quad (4)$$

which shows that the equilibrium price-dividend ratio is simply the investor's rational forecast of the composite variable z_{t+1}^s .

The gross stock return can now be written as

$$R_{t+1}^s = \frac{d_{t+1} + p_{t+1}^s}{p_t^s} = \left(\frac{1 + p_{t+1}^s/d_{t+1}}{p_t^s/d_t} \right) \left(\frac{d_{t+1}}{d_t} \right) = \left(\frac{z_{t+1}^s}{E_t(z_{t+1}^s)} \right) \left(\frac{M_t}{M_{t+1}} \right), \quad (5)$$

where we have eliminated p_t^s/d_t using the first-order condition (4) and eliminated $1 + p_{t+1}^s/d_{t+1}$ using the definitional relationship (3) evaluated at time $t + 1$.

Turning to bonds, we assume that the coupon at time t on a default-free bond initiated at time τ is $\delta^{t-\tau}$, where $\tau \leq t$. This specification of geometrically declining coupons allows parametrization of bonds of differing Macaulay duration ($\delta = 0$ represents a one-period bond, while $\delta = 1$ represents a perpetuity). Also, bonds initiated at different dates can be aggregated using $p_{\tau,t}^b = \delta p_{\tau+1,t}^b$, where $p_{\tau,t}^b$ denotes the price at t of a bond initiated at τ (in units of the consumption good). The gross return R_{t+1}^b from t to $t + 1$ on bonds initiated at any date $\tau \leq t$ can be written as

³This nonlinear change of variables technique is also employed by Lansing and LeRoy [6].

$$R_{\tau,t+1}^b \equiv \frac{\delta^{t-\tau} + p_{\tau,t+1}^b}{p_{\tau,t}^b} = \frac{1 + \delta p_{t+1,t+1}^b}{p_{t,t}^b} \equiv R_{t+1}^b \equiv \frac{1 + \delta p_{t+1}^b}{p_t^b}, \quad (6)$$

where the two rightmost identities incorporate a notational simplification made possible by the fact that the first and second subscripts in the third term are the same.

Starting again from eq. (1) and proceeding in a fashion similar to the treatment of stock prices, the bond price is determined by the following first-order condition:

$$p_t^b = E_t[(M_{t+1}/M_t)(1 + \delta p_{t+1}^b)] = E_t(z_{t+1}^b), \quad (7)$$

where

$$z_t^b \equiv (M_t/M_{t-1})(1 + \delta p_t^b). \quad (8)$$

The gross bond return can be written as

$$R_{t+1}^b = \frac{1 + \delta p_{t+1}^b}{p_t^b} = \left(\frac{z_{t+1}^b}{E_t(z_{t+1}^b)} \right) \left(\frac{M_t}{M_{t+1}} \right). \quad (9)$$

Taking logs and subtracting eq. (9) from eq. (5) yields the following compact expression for the excess stock return:

$$\log(R_{t+1}^s) - \log(R_{t+1}^b) = \log(z_{t+1}^s) - \log[E_t(z_{t+1}^s)] - \log(z_{t+1}^b) + \log[E_t(z_{t+1}^b)]. \quad (10)$$

The first terms on the right-hand side of eq. (10), $\log(z_{t+1}^s) - \log E_t(z_{t+1}^s)$, almost equal the forecast error for $\log(z_{t+1}^s)$, and similarly for $\log(z_{t+1}^b)$. To obtain an exact expression for the forecast errors we need to interchange the log and expectations operators. To do so we approximate the distribution of z by lognormal. If a random variable z_{t+1} is lognormal, then we have

$$\log[E_t(z_{t+1})] = E_t[\log(z_{t+1})] + \frac{1}{2}Var_t[\log(z_{t+1})]. \quad (11)$$

Starting from eq. (10), we assume that the composite variables z_{t+1}^s and z_{t+1}^b are both conditionally lognormal. Making use of eq. (11) to eliminate $\log[E_t(z_{t+1}^s)]$ and $\log[E_t(z_{t+1}^b)]$ yields the following alternative expression for the realized excess return:

$$\log(R_{t+1}^s) - \log(R_{t+1}^b) \quad (12)$$

$$= [\log(z_{t+1}^s) - E_t \log(z_{t+1}^s)] - [\log(z_{t+1}^b) - E_t \log(z_{t+1}^b)] \quad (13)$$

$$- \frac{1}{2}Var_t[\log(z_{t+1}^s)] + \frac{1}{2}Var_t[\log(z_{t+1}^b)] \quad (14)$$

The next step is to take expectations conditional on t in eq. (12). There results

$$E_t[\log(R_{t+1}^s) - \log(R_{t+1}^b)] \quad (15)$$

$$= \frac{1}{2} \{-Var_t[\log(z_{t+1}^s)] + Var_t[\log(z_{t+1}^b)]\}. \quad (16)$$

Here the left-hand side gives the forecastable component of the log excess return one period ahead, and the right-hand side shows that this depends on the conditional variances of z_{t+1}^s and z_{t+1}^b . Finally, take the unconditional variance in eq. (15):

$$\text{Var}\{E_t[\log(R_{t+1}^s) - \log(R_{t+1}^b)]\} \quad (17)$$

$$= \frac{1}{2} \text{Var}\{-\text{Var}_t[\log(z_{t+1}^s)] + \text{Var}_t[\log(z_{t+1}^b)]\}. \quad (18)$$

The left-hand side gives a measure of the predictable variation in excess returns. Eq. (17) shows that if the conditional variances of $\log z^s$ and $\log z^b$ are constant across date- t events (although generally not equal to zero), then excess returns on stock are unpredictable. In that case markets are efficient. If, on the other hand, the conditional variances differ according to the event, then a strictly positive fraction of excess returns are forecastable, so markets are inefficient.

The expression (17) for the variance of the excess return on equity was first reported in Lansing and LeRoy [7].

2 An Example

The restriction on conditional variances derived in the preceding section involves the composite variables z^s and z^b , and therefore is difficult to interpret. An example will support our contention that financial markets are efficient in a variety of settings that are close to standard-issue consumption-based pricing models, owing to the fact that these models avoid exotic characterizations of risk variation. We work with the tree model of Lucas [9], modified to deal with intensive variables like consumption growth, returns and price-dividend ratios rather than extensive variables like consumption levels. The model is presented in Lansing and LeRoy [6].

The representative agent has constant relative risk aversion. As in Lucas' paper the dividend on stock equals the representative agent's consumption, so that asset prices are such as to induce the representative agent to consume his endowment. The log consumption growth rate—equivalently, the log dividend growth rate—is generated as a first-order autoregression with autocorrelation parameter ρ :

$$\frac{c_{t+1}}{c_t} = \mu + \rho \left(\frac{c_t}{c_{t-1}} - \mu \right) + \varepsilon_{t+1}, \quad (19)$$

where ε_t is normal IID. In the special case $\rho = 0$ log consumption follows a random walk, but we are interested in the equilibrium for general ρ , so that log consumption growth contains a forecastable component. Following Campbell and Shiller [3] we solve the model by imposing a log-linearization on the Euler equation.

The equilibrium return on equity is given by

$$E_t[\log(r_t)] = \alpha\rho\left(\frac{c_t}{c_{t-1}} - \mu\right) + \text{constant}. \quad (20)$$

(LeRoy and Lansing [6] eqs. (37) and (B9)). As usual, expected log returns are constant if $\alpha = 0$ or $\rho = 0$. If $\alpha > 0$ and $\rho > 0$ an unusually rapid increase in current consumption ($c_t/c_{t-1} > \mu$) leads agents to anticipate a rapid increase in next-period consumption. Due to risk aversion the state-price of next-period consumption is lower than usual relative to the state price for current consumption. Accordingly, returns are higher. Except in the special cases $\alpha = 0$ or $\rho = 0$, returns contain a forecastable component.

The log one-period risk-free return r_t^f satisfies eq. (19), although with a different constant (Lansing-LeRoy [6], eq. (B11)). Thus whether or not agents are risk neutral the expected log expected excess return on equity $E_t[\log(r_t)] - \log(r_t^f)$ is constant. Financial markets are efficient.

3 Conclusion

The point that autocorrelated volatilities imply forecastable excess returns when investors are risk averse has been stated by several authors (for example, Hansen and Singleton [5], Attanasio [1], Bollerslev, Tauchen and Zhou [2]). Our analysis strengthens the result as usually presented: at least in a large class of models and if special cases like risk neutrality are ruled out, time-varying volatilities are not just sufficient for return forecastability, they are necessary.

Most empirical studies conclude that expected equity returns have a sizeable forecastable component, particularly at longer maturities like 3-5 years. If so, either risk is strongly autocorrelated or a direct implication of equilibrium in standard consumption-based models—the result in Section 1—is incorrect. One is reminded of the equity premium puzzle.

References

- [1] Orazio P. Attanasio. Risk, time-varying second moments and market efficiency. *Review of Economic Studies*, 58:497–494, 1991.
- [2] Tim Bollerslev, George Tauchen, and Hao Zhu. Expected stock returns and variance risk. *Review of Financial Studies*, 22, 2009.
- [3] John Y. Campbell and Robert J. Shiller. The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*, 1:195–228, 1988.
- [4] Eugene F. Fama. Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25:283–417, 1970.
- [5] Lars P. Hansen and Kenneth J. Singleton. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Econometrica*, 91:249–265, 1983.

- [6] Kevin J. Lansing and Stephen F. LeRoy. Risk aversion, investor information and stock market volatility. *European Economic Review*, pages 88–107, 2014.
- [7] Kevin J. Lansing and Stephen F. LeRoy. Examining the sources of excess return predictability. Reproduced, Federal Reserve Bank of San Francisco, 2016.
- [8] Stephen F. LeRoy. Efficient capital markets: Comment. *Journal of Finance*, 31:139–141, 1976.
- [9] Robert E. Lucas. Asset prices in an exchange economy. *Econometrica*, 46:1429–1445, 1978.