Explanations of a couple of Tough Questions

I think the two hardest questions on the sample exam were the ones below. They were intended to be challenging even to the top students in the class. Here are brief explanations of the answers.

Problem 22. This is a revealed preference problem.

When prices are $(5, 1)$, Vanessa chooses $(6, 3)$ which costs her $5\times 6+1\times 3=$ \$33. Notice that at these prices she could have afforded the other bundle, $(5, 7)$ which would cost only $5 \times 5 + 1 \times 7 = \$32$. (You might want to draw a picture to show this.) Therefore we can conclude that for Vanessa, the bundle $(6, 3)$ is revealed preferred to the bundle $(5, 7)$. If Vanessa satisfies WARP, and since she chooses $(5, 7)$ at the new prices, p_x and p_y, then it must be that she can't afford the preferred bundle $(6, 3)$ at the new prices. At the new prices, she can just afford $(5, 7)$ so her income at these prices must be $5p_x + 7p_y$. At these prices, the cost of the bundle $(6, 3)$ is $6p_x + 3p_y$. Therefore if she satisfies WARP, it must be that $6p_x + 3p_y > 5p_x + 7p_y$. (i.e. the old bundle must cost more than her income at the new prices.) We can simplify this inequality by subtracting $5p_x$ and $3p_y$ from each side. When we do this, we have $p_x > 4p_y$, which is equivalent to Answer option a.

Problem 26. This one is fairly tough, but you have experience with similar problems involving overtime wages. In this case, instead of getting a higher wage for working overtime, Debra gets a lower after-tax wage if she works more than 50 hours.

I suggest that you first draw Debra's budget set with r on the horizontal axis and c on the vertical axis. Note that since she earns \$10 an hour and pays no taxes until her income reaches \$500, for the first 50 hours that she works, she earns \$10 per hour after taxes, while for every hour after that she is taxed at 60% so she earns only \$4 after taxes. You should be able to show that her budget set is bounded by two line segments, one running from $(100, 0)$ to $(50, 500)$ and one running from $(50, 500)$ to $(0, 700)$. How do we find her choice from this budget? There are three possible cases. 1) She has an indifference curve that is tangent to the line segment segment $(100, 0)$ to $(50, 500)$. 2) She has an indifference curve that is tangent to the line running from $(50, 500)$ to $(0, 700)$. 3) She doesn't have an indifference curve tangent to either of these line segments, but the highest indifference curve that she can reach just touches the kink in her budget at the point $(50, 500)$. (You should be able to draw a picture of each of these possible cases.)

How do we find out which case it is? The key thing to do to understand the argument is to draw diagrams showing the budget set and also how an equilibrium would look in each of the 3 cases.

Lets see whether Case 1) applies. Every point on the line segment from $(100, 0)$ to $(50, 500)$ satisfies the equation $c + 10r = 1000$. If there is a tangency between her indifference curve and this line segment, it must be that the point of tangency is the combination of income and leisure that she would choose if she received \$10 an hour regardless of how many hours she worked. (Use a diagram to convince yourself of this.) Solving for the point at which $c + 10r = 1000$ and where her indifference curve has slope equal to -10, we find that $r = 33.3$ and $c = 666.66$. But this point is not in the line segment from $(100, 0)$ to $(50, 500)$. Thus there is no tangency between an indifference curve and this line segment.

What about case 2)? All of the points on the line segment running from $(50, 500)$ to $(0, 700)$ satisfies the equation $c + 4r = 700$. An indifference curve will be tangent to this line segment if at some point on this segment, the slope of the corresponding indifference curve is -4. But if $c + 4r = 700$ and if the slope of the indifference curve is -4, it must be that $r = 700/12 = 58.33$ and $c = 466.66$. But this point is not on the line segment running from $(50, 500)$ to $(0, 700)$ and so there is no point on this line segment where an indifference curve is tangent to the line segment.

We have eliminated cases 1 and 2. The only remaining possibility is that Debra's best affordable outcome is at the kink, where she works exactly 50 hours. To check that this is the right answer, sketch how the curve would have to look. Notice that this will be the best solution if the slope of her indifference curve at the kink is somewhere between -4 and -10. In fact, when she consumes 50 hours of leisure and 500 units of consumption, her MRS must be $-c/2r = -5$, which is between -4 and -10.