

Answers for Midterm, Spring 2009, 100C

Question 1.

Part a) If none of the B's speak language A, then the payoff for an A to learn language B is \$400, since he is able to speak to 400 more people if he learns language A. If none of the A's speak language B, then the gain to a type B from learning language A is \$600. The outcome where nobody learns the other groups language will be a Nash equilibrium only if when nobody knows the other language, nobody finds it worthwhile to do so. This will be the case if $C_A > \$400$ and $C_B > \$600$.

Part b) First we note that it will never be efficient for both groups to learn the other language, since if one group is bilingual, the other group can speak to and understand them. The total cost of making all the A's bilingual is $600C_A$ and the total cost of making all the B's bilingual is $400C_B$. Therefore in order for it to be efficient for the A's to become bilingual, it must be that $600C_A \leq 400C_B$, or equivalently, $C_A \leq (2/3)C_B$. If this is the case, it is more efficient to make the A's bilingual than the B's. But for it to be efficient for the A's to become bilingual rather than have nobody become bilingual, it needs to be that total benefits to making the A's bilingual exceed total costs. Total costs are $600C_A$. The total benefits of making the A's bilingual are $600 \times 400 + 400 \times 600$ since 600 type A's each gain \$400 from being able to talk to the 400 B's and since 400 type B's each gain \$600 from being able to talk to the 600 A's. So benefits exceed costs if $600 \times 400 + 400 \times 600 > 600C_A$. Divide both sides of this inequality by 600 and combine terms and you find $\$800 > C_A$. We see that if $600 < C_A < C_B$, and if $C_A \leq (2/3)C_B$, then it is efficient for the A's to learn language B, even though it is not a Nash equilibrium. The reason that this happens is that in Nash equilibrium, the A's take account only of the benefit to themselves of learning language B, and not of the benefit to the B's of having the A's learn language B.

Part c) If $C_A < 400$, there would be a Nash equilibrium in which the type A's learn language B and the B's do not learn language A. This is true, even if $C_B < 600$, because if the A's are bilingual, then the B's can speak to them and understand them without learning language A.

Part d) If $C_B < 600$, then there is a Nash equilibrium in which the type B's learn language A and the A's do not learn language B. This is the case even if $C_A < 400$, since if the B's are bilingual, the A's do not gain anything by learning language B.

Part e) If $c > \$600$, there is a Nash equilibrium where nobody learns any other language. If nobody is bilingual, the most that a type B or C can gain by learning A is \$600. If $\$600 < c < \800 , there is, however, another Nash equilibrium in which the B's and C's learn language A. If the B's learn A, then by learning to speak A, the C's will be able to speak to and understand an additional 800 people, the 600 A's and the 200 B's. Therefore if the B's learn A, the C's would want to learn A if $c < 800$. Similarly if the C's learn A, then

the B's will gain \$800 by learning A, so they will want to learn A if $c < 800$. Thus there is an equilibrium where the B's and C's both learn A.

Question 3

Part a) Megasoft's profits will be

$$PX - (r_A + r_B)X = 1000X - X^2 - (r_A + r_B)X.$$

These will be maximized when the derivative of profits with respect to X is zero and the second derivative is negative. The derivative of profits with respect to X is zero when

$$1000 - 2X - (r_A + r_B) = 0.$$

The second derivative is -2 , which is always negative. We solve the above equation for X to find that

$$X = 500 - \frac{r_A + r_B}{2}.$$

Part b) Patent holder A has profits equal to its royalty rate times the number of units that Megasoft sells. That is

$$r_A \left(500 - \frac{r_A + r_B}{2} \right) = 500r_A - \frac{r_A^2}{2} - \frac{r_A r_B}{2}.$$

To maximize its profits, Patent holder sets the derivative of this expression with respect to r_A equal to zero (and checks second order condition). This means that

$$500 - \frac{r_B}{2} - r_A = 0.$$

Similar reasoning shows that Patent holder B sets its royalty so as to satisfy the equation

$$500 - \frac{r_A}{2} - r_B = 0.$$

We can solve these two equations in the two unknowns r_A and r_B . The solution is

$$r_A = r_B = 1000/3.$$

Part c) If Megasoft buys companies A and B, it will maximize its profits by choosing X to maximize

$$PX = 1000X - X^2.$$

The derivative of profits with respect to X is equal to zero when $1000 = 2X$ and hence $X = 500$. Thus Megasoft would sell for a price of $P = 1000 - 500 = 500$. Then Megasoft's revenue would be $500 \times 500 = 250,000$. If Megasoft paid a total of V for the patents, then its profits after the buyout would be $250,000 - V$.

To find out how much Megasoft would be willing to pay for the two companies, we need to see what its net revenue is if it has to pay royalties. We found that the patent holders would set royalties $r_A = r_B = 1000/3$. In this case, we see from Part A that Megasoft would choose output $X = 500 - 1000/3 = 166.66$

and the price would be $P = 1000 - 166.66 = 833.33$. Megasoft's profits would then be

$$(1000 - X)X - (r_A + r_B)X = 833.33 \times 166.66 - 666.66 \times 166.66 = 166.66 \times 166.66 = 27775.6.$$

Megasoft would be willing to buy out the patent holders so long as the total amount V that it pays is such that

$$250,000 - V > 27775.6$$

, or equivalently, so long as

$$V < \$222,224.4.$$

I didn't ask this, but you may want to compare this amount with the total royalties that the patent holders would get if they didn't sell out to Megasoft. If the patent holders don't sell, we see from part b that they each set royalty rates of 333.33 per unit. From Part a, we see that the number of units sold will be $500 - 666.66/2 = 166.66$. So the profit of each royalty holder will be $333.33 \times 166.66 = 55552.8$ and total profits of the two royalty holders will be 111,106, which is much less than the amount V that Megasoft would be willing to pay to buy them out.

Part d) Suppose that Megasoft buys one of the two companies and the other continues to charge royalties. Megasoft's profits would be $100X - X^2 - rX$ where r is the royalty rate. Megasoft would maximize its profits when $100 - 2X - r = 0$, which means

$$X = 500 - \frac{r}{2}.$$

The royalty holder would now choose r to maximize

$$rX = 500r - \frac{r^2}{2}.$$

To do this, it would set $r = 500$. Therefore Megasoft would sell $500 - 500/2 = 250$ units and the price would be $1000 - 250 = 750$. So Megasoft's profit would be

$$PX - rX = (750 \times 250) - (500 \times 250) = 62500.$$

We saw earlier that if Microsoft did not buy out either patent holder, its profits would be 27775.6. Megasoft would be better off buying one of the patent holders at price V than buying neither of them if $62500 - V > 27775.6$. This is equivalent to $V < 34724.4$. So the most that Macrosoft would pay to buy out a single royalty holder is 34724.4. We note that this is less than the amount that each royalty owner makes if neither sells out.