Name _____

Midterm Exam–Discussion of Answers

1) Player 1 and Player 2 play a game in which each has two possible strategies, A and B. They choose their strategies simultaneously. If both play strategy A, they each get a money payoff of 5. If both play strategy B, they each get a money payoff of 2. If one player plays A and the other player plays B, the player who played A gets a money payoff of 1 and the player who played B gets a money payoff of 4.

A) Depict this game in extensive form.

Almost everybody got this right. The only wrong answers were some people did not show the information set properly. See chapter 2 to find out how to do this.

B) Depict this game in strategic form, showing the money payoffs to Players 1 and 2. How many Nash equilibria, if any, does this game have? If there are any, what are they?

Almost everybody got this. See chapter 2 to find out how to do this.

C) Suppose that Player 1 and Player 2 are good friends and each cares about the other's money payoffs as well as his own. The utility of Player 1 is given by $m_1 + am_2$ and the utility of Player 2 is $m_2 + am_1$, where $a > 0$. (a does not have to be an integer.) Write down the game matrix in strategic form, showing the utility payoffs for Players 1 and 2.

Where Player 1 chooses the row and Player 2 chooses the column, the payoff matrix is

	Strategy A	Strategy B
Strategy A	5+5a,5+5a	4a+1,4+a
Strategy B	4+a,4a+1	2+2a, 2+2a

D) For what values of a does this game have the same Nash equilibria as when $a = 0$? Are there any values of a for which this game has fewer Nash equilibria than when $a = 0$? Explain.

When $a = 0$ there are two Nash equilibria. In one Nash equilibrium they both play A. In the other they both play B.

If $a \neq 0$, these two strategy profiles will both be Nash equilibria if $5+5a \geq 4+a$ and $2 + 2a \geq 4a + 1$. The first of these inequalities holds if $a \geq -1/4$ and the second will hold if $a \leq 1/2$. So outcomes where both do A and where both do B, which are the Nash equilibria when $a = 0$ will still be Nash equilibria when $-1/4 \leq a \leq 1/2$. If $a > 1/2$, then the outcome where both do B will no longer be a Nash equilibrium. If $a < -1/4$, the outcome where both do A will no longer

be a Nash equilibrium. (It also turns out that the outcomes where one player does A and the other does B will not be Nash equilibria for any value of a.)

2) If a strategy profile is a Nash equilibrium in a game with N players, it must be that no player is using a strictly dominated strategy. True or False? If true explain why it is true. If false, give an example of a Nash equilibrium in which at least one player is using a strictly dominated strategy.

This statement is true. To get any points for the problem, you have to explain why it is true. Assertions that rational players will not play dominated strategies do not explain why it is that in Nash equilibrium, no player can be using a strictly dominated strategy.

If a strategy profile is a Nash equilibrium, it must be that, given the actions of the other players, every player's strategy maximizes this player's payoff over all strategies that this player might choose. If a strategy is strictly dominated for some player, then there is another strategy that gives this player a higher payoff no matter what strategy the others choose. Therefore a strictly dominated strategy cannot possibly maximize this player's payoff given what the others are doing.

3) If a strategy profile is a Nash equilibrium in a game with N players, it must be that no player is using a weakly dominated strategy. True or False? If true explain why it is true. If false, give an example of a Nash equilibrium in which at least player is using a weakly dominated strategy.

This statement is false. To get credit for this question you have to produce an example of a game in which there is a Nash equilibrium in which at least one player uses a weakly dominated strategy.

One example of such a game is the following, where Player 1 chooses the row and Player 2 chooses the column. In this game the strategy profile in which each chooses Strategy B is a Nash equilibrium, but Strategy B is weakly dominated for both players.

	Strategy A	Strategy B
Strategy A	1,1	0,0
Strategy B	0,0	0,0

4) A game has three players. Player 1 has three possible strategies, a, b, and c. Player 2 has three possible strategies, x, y, and z. Player 3 has 2 possible strategies, v and w. The payoffs are given in the table below. In each case, the first entry in the table is the payoff to Player 1, the second entry is the payoff to Player 2, and the third entry is the payoff to Player 3.

Circle the payoffs representing best responses for each player. What are the Nash equilibria of this game if there are any?

This game had two Nash equilibria. In one Nash equilibrium Player 1 plays a, player 2 plays x and player 3 plays w. In the other, Player 1 plays b, player 2 plays y and player 3 plays w.

5) Alice and Bob like to go to the movies together, but they have differing tastes in movies. Tonight there are two movies at different theaters. Alice prefers Movie A to Movie B. Bob prefers Movie B to Movie A. Both prefer an outcome where they go to the same movie to going to their favorite movie alone. Each prefers going to his favorite movie alone to going to the other movie alone. Suppose that Bob moves first and that Alice knows Bob's choice when she makes her choice. A) Write out an extensive form representation of this game. Include numerical payoffs that are consistent with this story.

B) In a strategic representation of the game, what are the possible strategies for Bob? What are the possible strategies for Alice?

C) Write down a game matrix for Bob and Alice describing the game in which Bob chooses a movie first and Alice chooses a movie, having observed Bob's choice. Find all of the Nash equilibria for this game. Is there a Nash equilibrium in which both go to Movie A?

The answers to this problem were discussed extensively in class and can be found in the power points of the class lecture notes.

6) Three hunters go into the woods seeking game. Each of them must choose whether to hunt for stag or hunt for hare. They make their choices simultaneously, without knowing the choices made by the others. If two or more hunters choose to hunt for stag, they will kill a stag and they will split the stag equally among those who hunted for stag. If two hunters split the stag, each gets a payoff of 6. If three hunters split the stag, each gets a payoff of 4. If any hunter chooses to hunt for hare, he will certainly catch a hare and get a payoff of 3. If one hunter chooses to hunt for stag, and the other two hunters hunt for hare, the one who hunted for stag gets a payoff of zero and the two who hunted for hare will each get 3.
A) Find all of the Nash equilibria for this game (if there are any).

There are two Nash equilibria. In one of them, all three hunters hunt stag. In the other, all three hunt hare.

B) What strategy profile(s) would maximize the sum of the payoffs to the three hunters? Do any of the Nash equilibria maximize the sum of the three hunters' payoffs?

Total payoff is maximized if any two of the hunters hunt stag and the other hunts hare. Neither of the Nash equilibrium achieves the maximal total payoff.