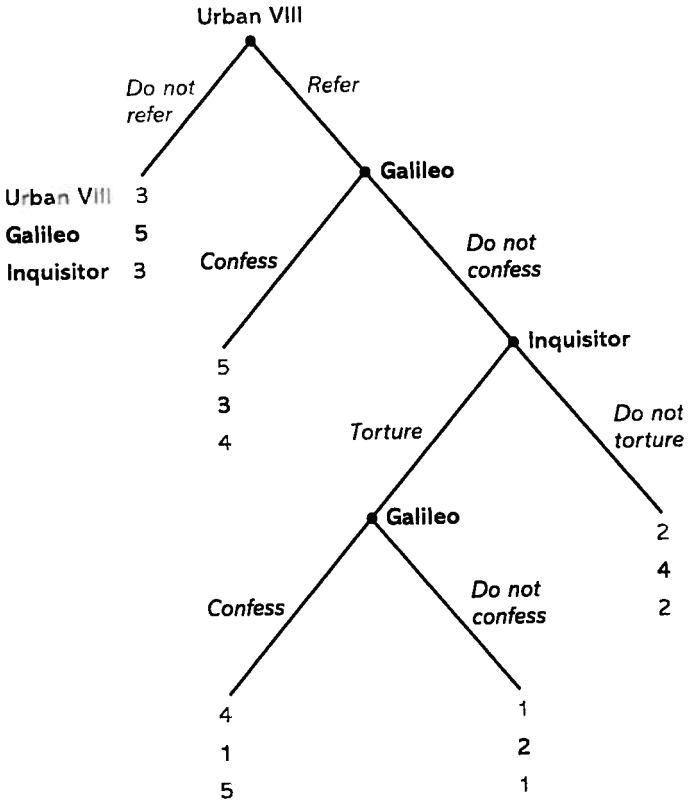
EXERCISES

1. Return to the situation described in Chapter 2 in which Galileo Galilei might be confronted by the Inquisition. Let us describe what actually transpired. First, Pope Urban VIII referred Galileo to the Inquisition, and he was brought to trial on April 12, 1633. After verbal persuasion from the commissary general of the Inquisition, Galileo confessed that he had gone too far in supporting the Copernican theory in one of his books (even though he hadn't). Galileo was then given an "examination of intention," which involves showing the instruments of torture to the accused. The final hearing by the Inquisition was held on June 22, 1633, at which time the 69-year-old Galileo pleaded for mercy because of his "regrettable state of physical unwellness." With the threat of torture and imprisonment lurking in the background, the Inquisitors forced Galileo to "abjure, curse, and detest" his work. Galileo complied in every way and was convicted and sentenced to life imprisonment and religious penances. Due to his age (and possibly his fame), the sentence was commuted to house arrest. He was allowed to return to his villa near Florence, where he would remain for the last years of his life. That is history, and now we turn to our simple modeling of it. The extensive form game in Figure 2.3 is reproduced here.

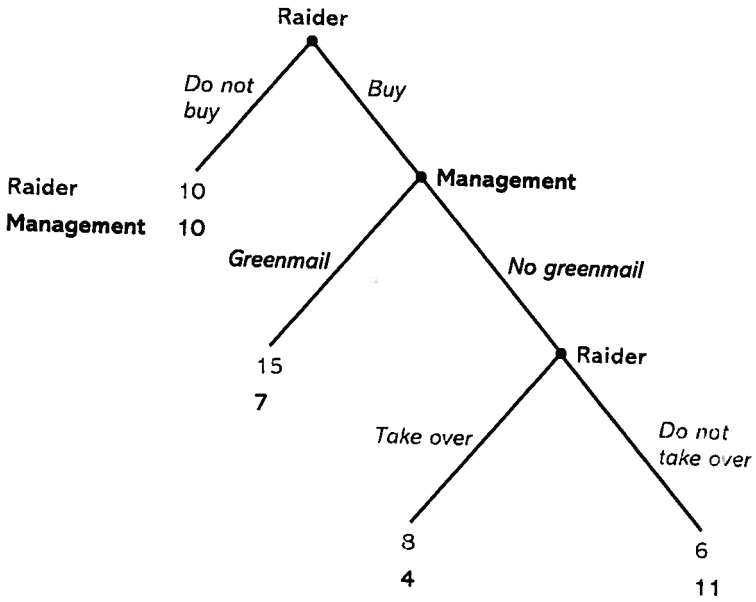
Galileo Galilei and the Inquisition



a. Find all Nash equilibria. (*Hint:* First derive the strategic form game.)
b. Find all of the subgame perfect Nash equilibria.
c. For each Nash equilibrium that is not an SPNE, explain why it is not a SPNE.

2. There were still pirates in the 1980s, although they tended to appear in corporate boardrooms rather than the open seas. These swashbuckling financiers would engage in a "hostile takeover" by acquiring a company through the purchase of shares on the open market and against the will of the target company's existing management (thus making the takeover "hostile"). Such investors were known as "raiders" and included people such as T. Boone Pickens, Sir James Goldsmith, Henry Kravis, and Victor Posner. All this was fictionalized in the movie *Wall Street*, with Michael Douglas portraying the raider Gordon Gekko, who famously espoused "Greed is good." The time was full of jocular jargon, as management could consume a "poison pill" by taking on a costly financial structure that would make it difficult to consummate a hostile takeover. In some cases, a raid could be fought against by buying a raider's shares back at a premium; this tack became known as "greenmail," a takeoff on blackmail. To get a gist of the strategizing that occurred between a raider and management, consider the figure below. The raider makes an initial stock purchase, in response to which management decides whether to buy the shares back at a premium (pay greenmail) or not. If no greenmail is paid, then the raider decides whether to purchase additional shares in order to take control of the target company.

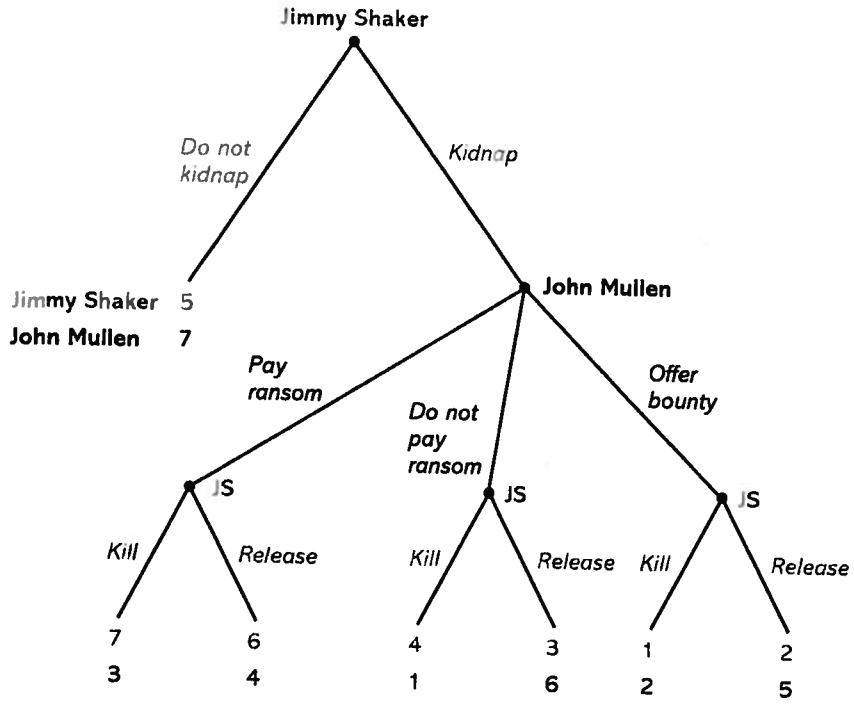
Greenmail



   a. Find all subgame perfect Nash equilibria.
   b. Find a Nash equilibrium that is not an SPNE, and explain why it is not a SPNE.

3. Return to the Kidnapping game from the film *Ransom* (first discussed in Chapter 2), which is reproduced here. Solve for all subgame perfect Nash equilibria.

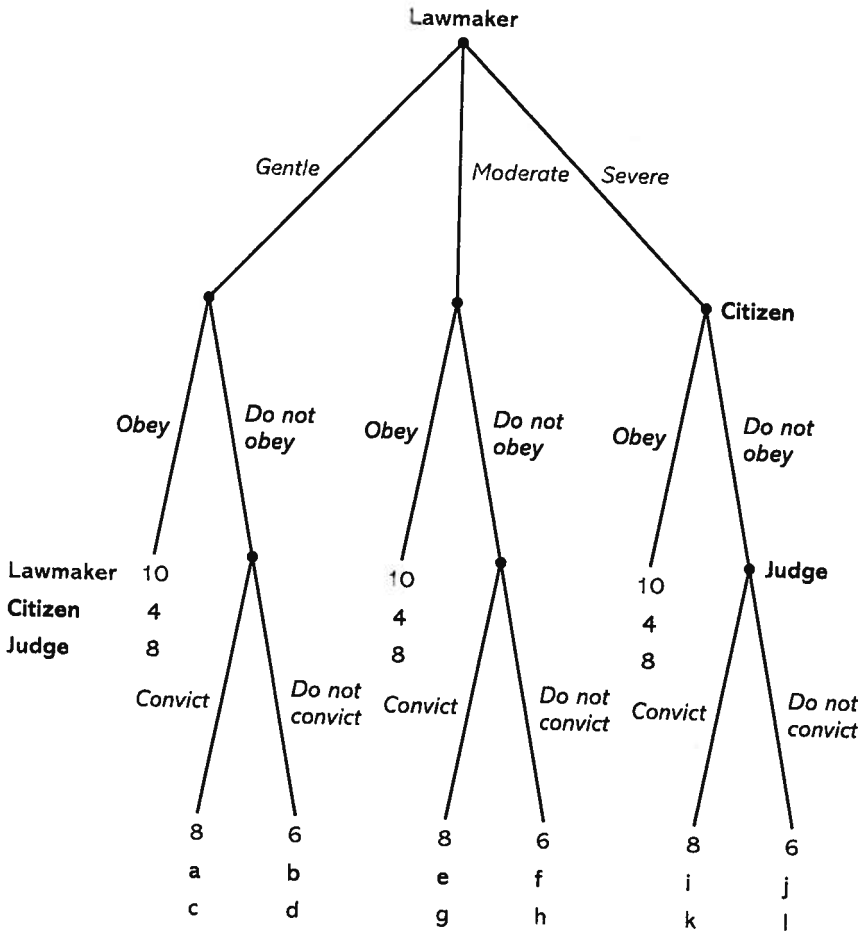
Extensive Form for the Film *Ransom*



4. Benjamin Franklin once said, "Laws too gentle are seldom obeyed; too severe, seldom executed." To flush out what he had in mind, the following game on page 298 has three players: a lawmaker, a (typical) citizen, and a judge. The lawmaker chooses among a law with a gentle penalty, one with a moderate penalty, and a law with a severe penalty. In response to the law, the citizen decides whether or not to obey it. If she does not obey it, then the judge decides whether to convict and punish the citizen. Using SPNE, find values for the unspecified payoffs (those with letters, not numbers) that substantiate Franklin's claim by resulting in a lawmaker's choosing a law with a moderate penalty.

5. Nobel Laureate Thomas Schelling once proposed a solution to the problem of how a kidnappee can induce his kidnapper to release him after the kidnappee has learned the identity of the kidnapper. Let's return to the kidnapping scenario, but instead have the players be Guy (kidnapper) and Orlando (kidnappee). The problem is that one would expect Guy to be inclined to kill Orlando once Orlando sees Guy's face, since then Orlando, if released, would be able to help the police capture Guy. The situation is as depicted on page 299. Guy starts off by deciding whether to kidnap Orlando. Orlando then decides whether to reveal some incriminating details about himself that are unknown to the rest of the world. (Perhaps Orlando stole funds from his church or had an affair unbeknownst to his wife.) Then Guy decides whether to kill or release Orlando. If he releases Orlando, then Orlando has to decide whether to inform the police of his kidnapper's identity. If he does, and if Orlando revealed his dirty secret to Guy, Guy must then decide

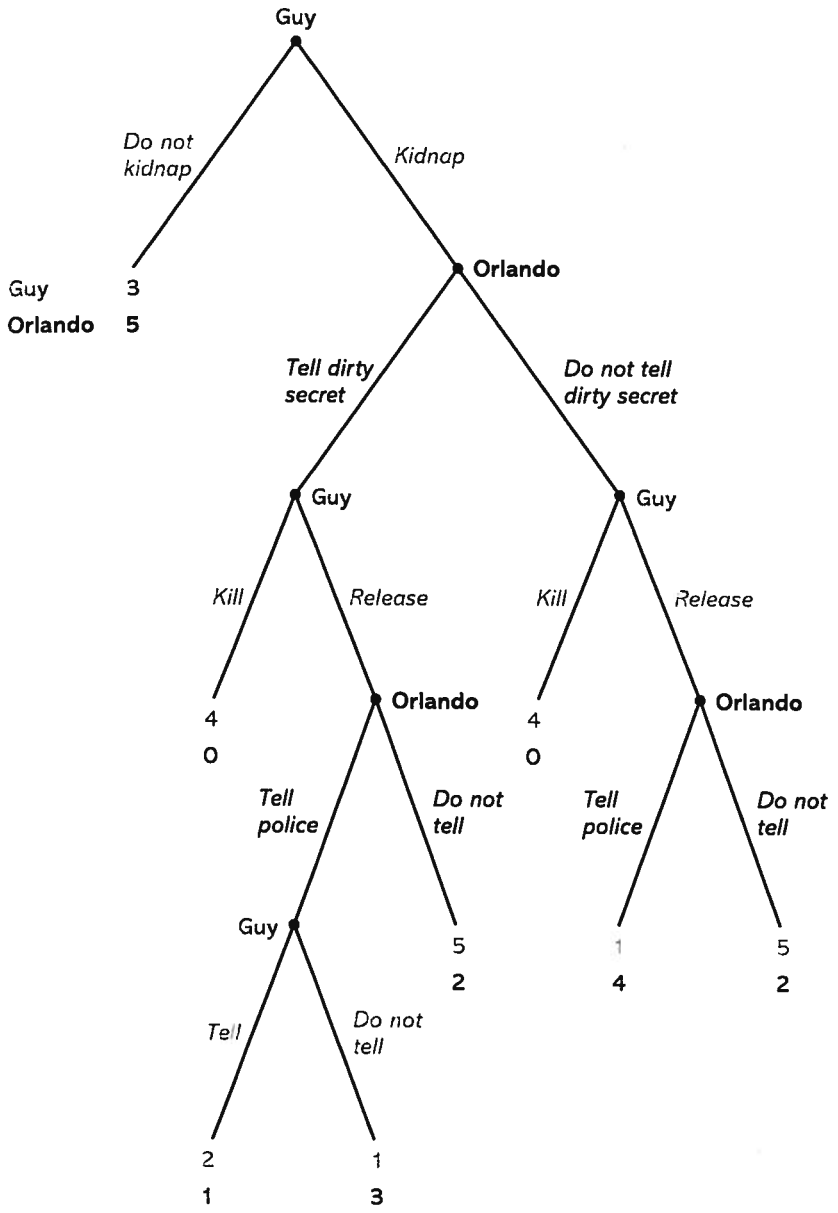
Punishments: Severe or Gentle?



whether to share that secret with the world. Find the unique SPNE, and you'll find Schelling's proposed solution.

6. In 1842, the *Sangamo Journal* of Springfield, Illinois, published letters that criticized James Shields, the auditor of the State of Illinois. Although the letters were signed "Rebecca," Shields suspected that it was state legislator Abraham Lincoln who penned the letters. As shown in the figure on page 300, Shields considered challenging Lincoln to a duel, and, as history records, Shields did challenge Lincoln. In response to a challenge, Lincoln could avoid the duel, or, if he chose to meet Shields's challenge, he had the right to choose the weapons. We will also allow Lincoln to decide whether to offer an apology of sorts. (Actually, it proved to be a bit more complicated than that, so allow me some poetic license here. An "apology of sorts" means making some remarks that could provide an honorable retreat for Shields—something which Lincoln ultimately did.) If he decides to go forward with a duel, then Lincoln has four choices: propose

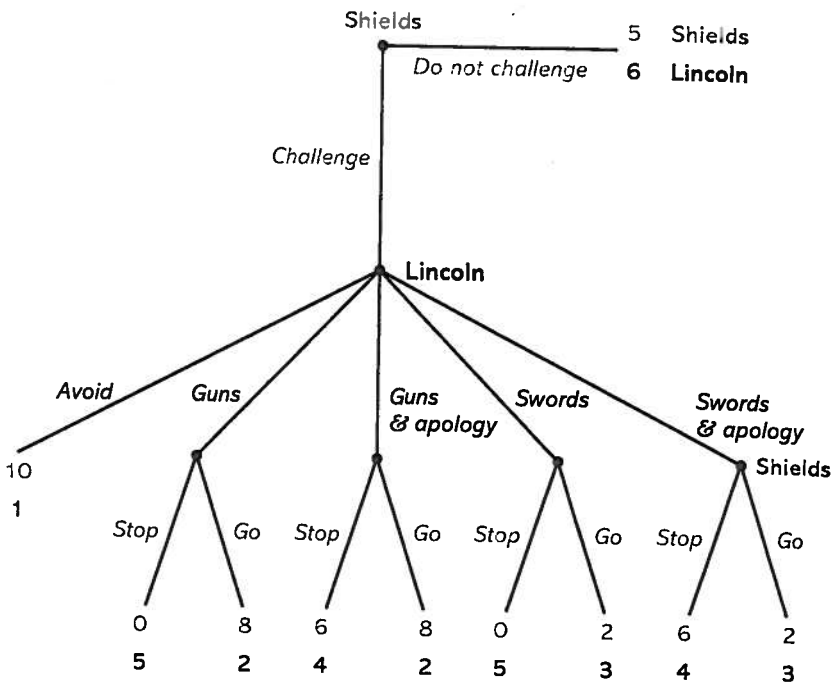
Revised Kidnapping Situation



guns, propose guns and offer an apology, propose swords, and propose swords and offer an apology. (Shields was known to be a good shot, so Lincoln chose cavalry broadswords of the largest size, as it gave the 6-foot, 4-inch Lincoln a sizable advantage against the much shorter Shields.) In response to any of the four choices, Shields must decide to either go forward with the duel or stop the duel. (In the latter case, Shields accepts Lincoln's apology if, indeed, Lincoln offered one.) Find all subgame per-

fect Nash equilibria. As a closing note, Lincoln once said, "If all the good things I have ever done are remembered as long and as well as my scrape with Shields, it is plain I shall not be forgotten."

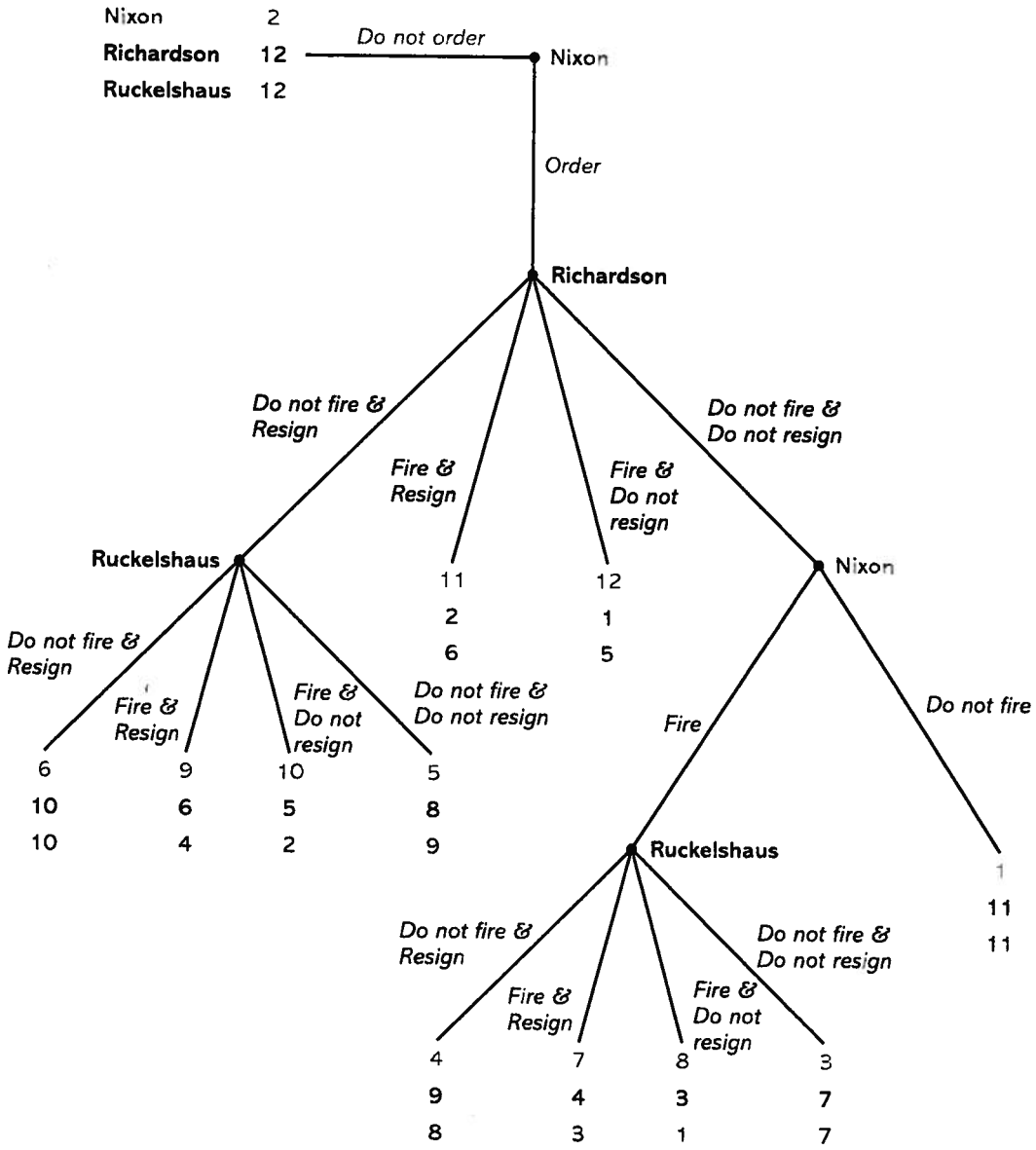
Lincoln–Shields Duel



7. An infamous event that came to be known as the Saturday Night Massacre took place during the second term of the presidential administration of Richard Nixon. Though no one was fired *upon*, many were effectively fired *from* their high-level positions in the federal government. The Nixon White House was in the midst of covering up crimes committed by close aides to the president. As part of the investigation, Attorney General Elliot Richardson (who was not part of the cover-up) named Harvard Law professor Archibald Cox as a special prosecutor. During the investigation, President Nixon was acutely concerned with Cox's investigation and contemplated ordering Richardson to fire Cox (expressed as the initial decision node in the figure on page 301. When Nixon's intent was expressed to Richardson, the latter conveyed that if he did fire Cox, he might feel compelled to resign, but also that he might be inclined not to fire Cox and, in that case, might also resign. Richardson's four possible combinations of firing Cox or not and resigning or not are depicted in the extensive form. If Richardson did choose to resign and not fire Cox, then Nixon would still be left with the matter of getting rid of Cox. And if Richardson chose not to fire Cox and did not resign, then Nixon would have to decide whether to fire Richardson. Upon Richardson's departure, Deputy Attorney General William Ruckelshaus would assume the position of acting attorney general and would face the same four options as Richardson. If Ruckelshaus also chose to resign and not fire Cox, then Solicitor General Robert Bork would become acting attorney general, and again, he would have the same four choices. To simplify matters, we'll not model Bork, even though

what happened was that Richardson refused to fire Cox and resigned and Ruckelshaus did the same, at which point Bork came in and did fire Cox and did not resign. Find all subgame perfect Nash equilibria.

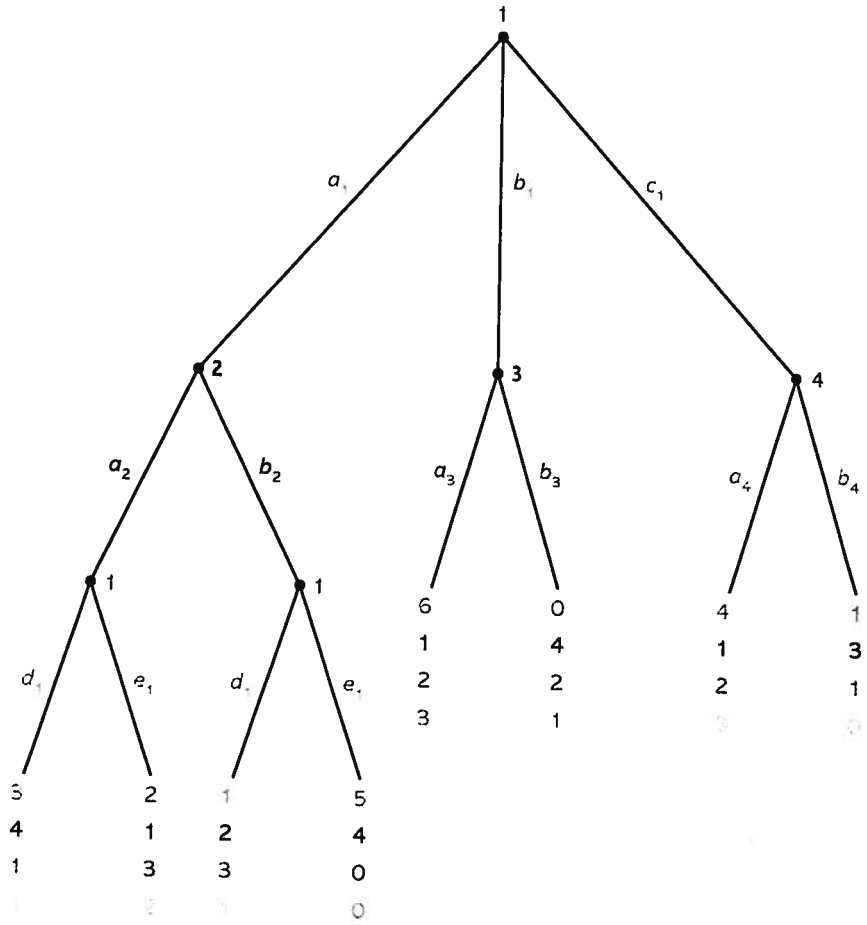
Saturday Night Massacre



8. Seven goblins are deciding how to split 100 galleons. The goblins are named Alguff, Bogrod, Eargit, Griphook, Knadug, Ragnuk, and Uric, and they've been rank-ordered in terms of magical power, with Alguff the weakest and Uric the strongest. The game starts with Alguff, who proposes an allocation of the 100 galleons coins, where an allocation is an assignment of an amount from {0,1, . . . , 100} to each goblin and where the sum across goblins equals 100. All goblins then vote simultaneously, either "yea" or

"nay," on the allocation. If at least half of them vote in favor of the allocation, then it is made and the game is over. If less than half vote for the proposed allocation, then the other goblins perform a spell on Alguff and transform him into a house elf for a week. In that event, it is Bogrod's turn to put forth an allocation for the remaining six goblins. Again, if at least half vote in favor, the allocation is made; if not, then Bogrod is made into a house elf for a week and it is Eargit's turn. This procedure continues until either an allocation receives at least half of the votes of the surviving goblins or all but Uric have been transformed into house elfs, in which case Uric gets the 100 galleons. Assume that the payoff to a goblin is $-1{,}000$ if he is made into a house elf and that it equals the number of galleons if he is not. Using the solution concept of SPNE, what happens? (Focus on subgame perfect Nash equilibria in which a goblin votes against an allocation if he is indifferent between voting for it and against it.)

9. Consider the four-player game displayed below. Find all subgame perfect Nash equilibria.



10. Their rich uncle left 100 pounds of gold to Todd and Steven. The negotiating process for allocating the treasure between them was also laid out in their uncle's will. They have three rounds by which to come to an agreement. In an odd (even) round, Todd (Steven) is required to propose an allocation. (Isn't it clever how Todd moves in odd rounds and Steven moves in even rounds?) In response to a proposal, the other nephew can accept or reject it. If he accepts the proposal, the process is ended and the proposed allocation is made. If he rejects the proposal, the game moves to the next round. Failure to agree by the end of the third round means that all of the gold goes to charity, so none of it lands in the pockets of Todd and Steven. Furthermore, at the end of each round in which an agreement has not been reached, a fraction $1 - d$ of the allotment of gold is given to charity, where $0 < d < 1$. Thus, there are $100d$ pounds of gold at the beginning of round 2 (after an agreement was not reached in the first round) and only $100d^2$ pounds of gold at the beginning of round 3 (after an agreement was not reached in the first two rounds). In other words, there is a cost to delaying agreement and, of course, a cost to ever failing to agree. Each nephew's payoff equals the number of pounds of gold he ends up with, so neither cares about the other or about their uncle's favorite charity. For notational purposes, assume that a proposal in round t is a value for x_i, where x_i is the share of the remaining amount of gold for Todd and, therefore, Steven's share is $1 - x_i$. Note that $0 \leq x_i \leq 1$ and thus is any number between 0 and 1 inclusive. Find an SPNE.

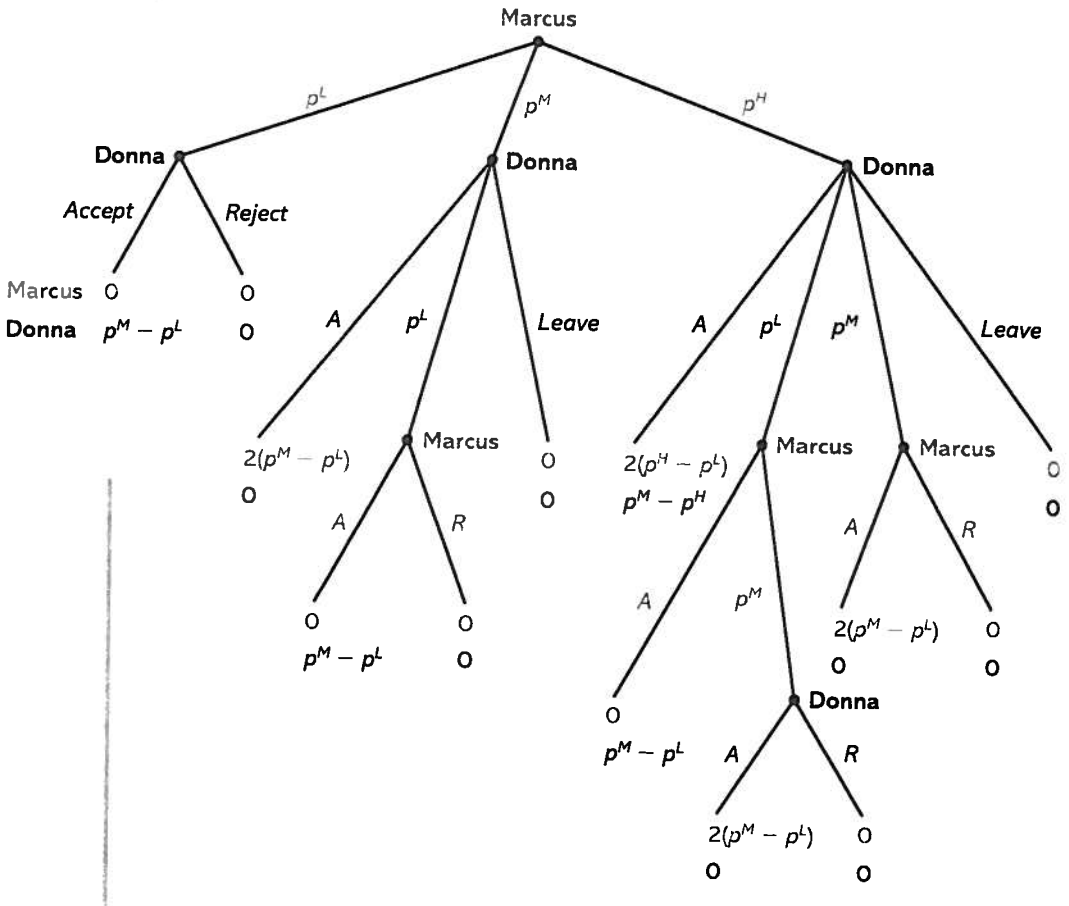
11. Consider the following passage from *Midnight in the Garden of Good and Evil*:[12]

> *There's a woman here, a grande dame at the very apex of society and one of the richest people in the Southeast, let alone Savannah. She owns a copper mine. She built a big house in an exclusive part of town, a replica of a famous Louisiana plantation house with huge white columns and curved stairs. You can see it from the water. Everybody goes 'Oooo, look!" when they pass by it. I adore her. She's been like a mother to me. But she's the cheapest woman who ever lived! Some years ago she ordered a pair of iron gates for her house. They were designed and built especially for her. But when they were delivered she pitched a fit, said they were horrible, said they were filth. "Take them away," she said, "I never want to see them again!" Then she tore up the bill, which was for $1,400—a fair amount of money in those days. The foundry took the gates back, but they didn't know what to do with them. After all, there wasn't much demand for a pair of ornamental gates exactly that size. The only thing they could do was to sell the iron for its scrap value. So they cut the price from $1,400 to $190. Naturally, the following day the woman sent a man over to the foundry with $190, and today those gates are hanging on her gateposts where they were originally designed to go. That's pure Savannah. And that's what I mean by cheap. You mustn't be taken in by the moonlight and magnolias. There's more to Savannah than that. Things can get very murky.*

Using backward induction, can you explain where the foundry went wrong?

12. The haggling game from Chapter 2 is reproduced here. Solve for all subgame perfect Nash equilibria for which a player chooses *accept* whenever that is an optimal action. That is, if a player's payoff is maximized by either choosing *accept* or choosing some other action, he or she chooses *accept*.

Haggling at the Auto Dealer



13. A scientist works in a lab with four summer interns and, as it is the end of the summer, he anticipates the head of the lab will ask him which of them he wants to retain. In terms of quality, the scientist and the head of the lab agree that intern A is better than intern B who is better than intern C who is better than intern D. The scientist would like to keep as many interns as possible and, given any number, the highest-quality ones. However, due to funding restrictions, the head of the lab wants to limit the number of interns that are retained. The head of the lab initially tells the scientist to select two interns to retain. After the scientist chooses two interns, the head of the lab decides whether to allow the scientist to retain a third intern. Thus, the sequence of moves is: 1) the scientist chooses two interns (both of whom are then retained); 2) the head of the lab decides either to allow the scientist to choose a third or not; and possibly 3) if the head of the lab chose to allow the scientist to retain a third intern, the scientist chooses a third intern to retain. There are 10 possible outcomes in terms of the number of interns that are retained at the end of the game (two or three) and who are they. These outcomes and the associated payoffs for the head of the lab and the scientist are shown in the accompanying table.

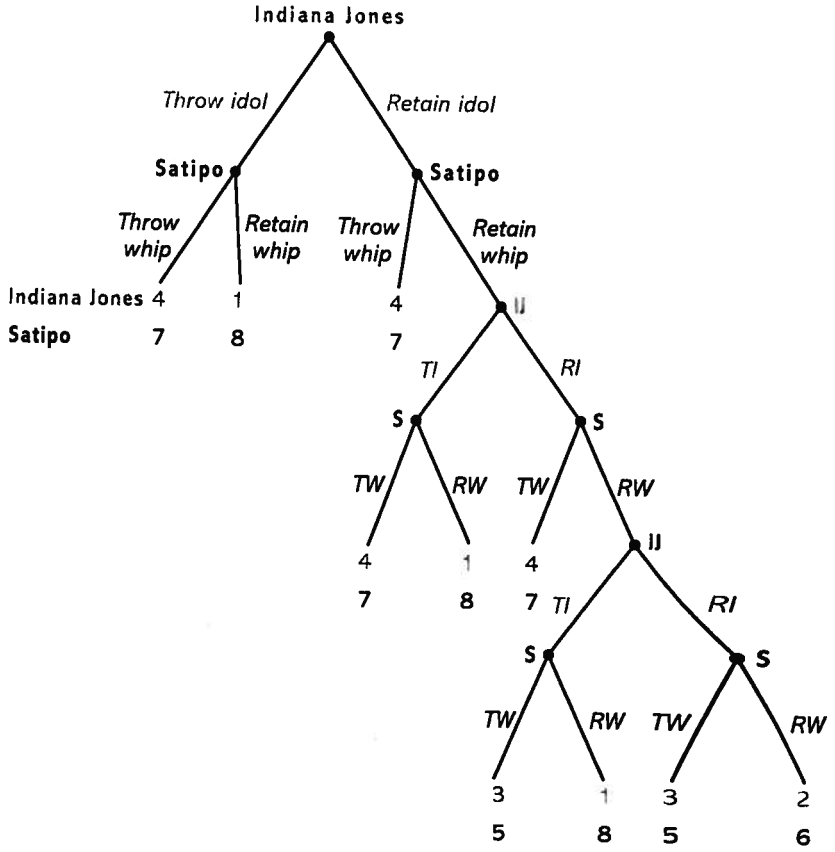
a. Write down the extensive form of the game (though you can exclude payoffs) and describe what a strategy looks like for the scientist, and for the head of the lab.
b. What does SPNE predict as to whether the scientist ends up with two or three interns? Which interns are selected? (Hint: Watch the episode entitled "Games" from the fourth season of the television series *House*.)

Interns retained	Payoff–Head of Lab	Payoff–Scientist
A, B	10	6
A, C	9	5
A, D	7	3
B, C	8	4
B, D	4	2
C, D	2	1
A, B, C	6	10
A, B, D	5	9
A, C, D	3	8
B, C, D	1	7

14. An instructor for a class that meets on Mondays, Wednesdays, and Fridays announces on Friday that there will be a pop quiz next week. The students have to decide what day to study. In terms of performance, it is best to study the evening before the pop quiz. Thus, on each of the evenings before the possible day of the quiz, students decide whether or not to study. (Note that a student may end up studying more than one evening if at first she thinks the test is on, say, Wednesday and thus studies Tuesday evening, but it turns out the quiz is on Friday in which case she'll study Thursday evening as well.) The instructor, who is an unpleasant sort, wants to minimize students' grades and thus would most like to have it on a day when the students did not study the evening before. Everything else the same, he prefers to give it earlier in the week. Find the SPNE.
15. In the opening sequence of the film *Raiders of the Lost Ark*, Indiana Jones is at the Temple of the Chachagoyan Warriors and holds the golden idol of which he was in search. He and his not-to-be-trusted assistant Satipo are in the sacred cave on either side of a wide cavernous pit. The escape route is on Satipo's side and Indy needs Satipo to toss him his whip so that he can wrap it around a beam above the pit and swing to the other side. At the same time, a thick stone slab is gradually closing behind Satipo and, once shut, will cut off their escape route. Satipo says to Indy: "No time to argue. Throw me the idol, I throw you the whip." Indy hesitates, which causes Satipo's tone to become frantic: "You have no choice! Hurry!" Not seeing any alternative, Indy tosses him the idol. Satipo tucks the idol away and says to Indy: "Adios, amigo!" In modeling this situation, Indy is deciding whether or not to throw the idol and, in response to what Indy does, Satipo is deciding whether or not to throw the whip. As soon as the whip is

thrown, the game is over as both escape with Indy having the idol. If Indy has thrown the idol and Satipo keeps the whip then the game ends as Satipo departs with the idol. At some point, the game ends because the stone slab has closed the exit. The extensive form is in the accompanying figure. Using SPNE, does Indy escape?

Temple of the Chachagoyan Warriors



16. It is Halloween evening and Nick and Rachel have just returned home after trick-or-treating. They have been dividing up the candy and are now down to four candy bars: Snickers, Milky Way, Kit Kat, and Baby Ruth. The allocation procedure is that Nick gets to choose one of the four candy bars, then Rachel can choose one of the three remaining candy bars, then Nick chooses one of the two remaining candy bars, and finally Rachel gets the remaining candy bar. Nick's preferences are that he most likes Snickers (assigning it a payoff of 5) then Milky Way (payoff of 4), then Kit Kat (payoff of 3), and lastly Baby Ruth (payoff of 1). Rachel most likes Milky Way (payoff of 6), then Kit Kat (payoff of 5), then Baby Ruth (payoff of 4), and finally Snickers (payoff of 2). Using SPNE, how are the candy bars allocated between Nick and Rachel?

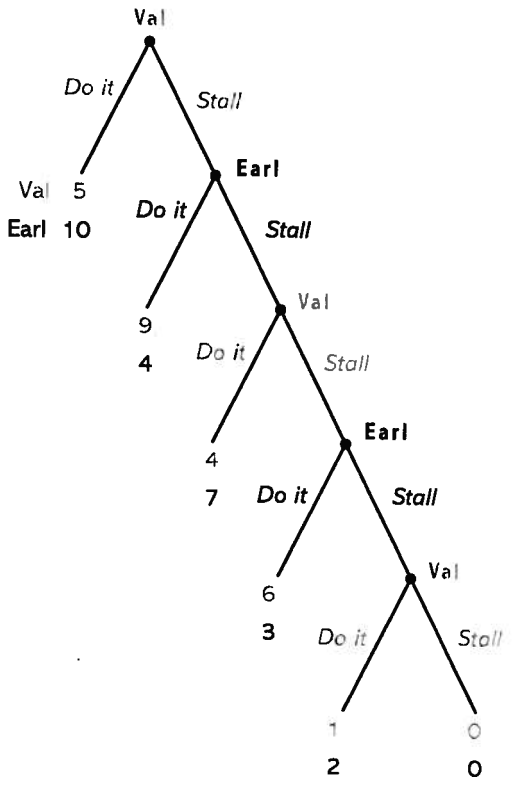
17. Samiyah and DeAndre decide to play the following game. They take turns choosing either 1, 2, or 3. As each number is chosen, it is added to the previously chosen numbers. The winner is the player who chooses a number that brings the cumulative number to 10. For example, if Samiyah chooses

3 and DeAndre chooses 2 (so the cumulative number is 5) and Samiyah chooses 2 and DeAndre chooses 3 then DeAndre wins as his choice of 3 results in the sum equaling 10. Using SPNE, who wins?

18. There have been some burglaries in a neighborhood so the residents of the $n \geq 2$ houses on the block are each deciding whether to install an alarm system. For a resident, the cost of installing an alarm system is $x > 0$ and the cost of his or her house being broken into is $y > 0$. Assume $y > x > y/n$. A burglar will decide which house to break into and is able to determine which houses have alarm systems. The burglar will avoid all homes with an alarm system and randomly choose to break into one of the houses without an alarm system (if any). Thus, if m homes install alarm systems then each of the $n - m$ unprotected homes have a probability of $1/(n - m)$ of being burglarized, which means an expected cost of $(1/(n - m)) \times y$. In deciding whether to buy an alarm system, a resident chooses an alarm system if and only if the cost of the system is less than the expected cost of a burglary. The residents sequentially decide whether or not to install an alarm system. Resident 1 moves first and either installs one or doesn't. After observing what resident 1 did, resident 2 then decides. After observing what residents 1 and 2 chose, resident 3 decides, and so forth.
    a. Find the subgame perfect Nash equilibria.
    b. Find a Nash equilibrium that is not a SPNE.

19. It is Thursday, which means local handymen Val and Earl have to empty the septic tank of a neighbor down the road. Neither wants to be the one to connect the hose and turn on the pump, as there is always a bit of sludge that

Val and Earl from "Tremors"



comes squirting out. As a result, they engage in what is called *mamihlapinatapai* in the Yagán language of Tierra del Fuego, which means: "a look shared by two people with each wishing that the other will initiate something that both desire but which neither one wants to start" (Wikipedia). The longer they wait and look at each other, the longer until they can head to Mo's Tavern and down a cold Bud. The extensive form in the figure describes the situation they are facing. Using SPNE, who will turn on the pump?

20. Return to the American Idol game in Chapter 4.4. Now suppose that Alicia, Kaitlyn, and Lauren move sequentially. Lauren decides whether to wear the shirt with the letter A or the top from Bebe. Having observed Lauren's choice, Kaitlyn decides whether to wear the shirt with the letter C or the top from Bebe. Finally, Alicia chooses between wearing the shirt with the letter E or the Bebe top, while knowing what Lauren and Kaitlyn are wearing. Find the subgame perfect Nash equilibria.

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