strategies assigned positive probability must yield the highest expected payoff, and it is for that reason that a player is content to let a random device determine how she behaves. If that weren't the case, then, after performing the randomization, a player might decide to ignore what was recommended and choose a better strategy. This equilibrium property of indifference is highly useful in solving for mixed-strategy Nash equilibria.

A player's **security strategy** was defined to be that strategy which maximizes his payoff when he makes the highly cautious assumption that, whatever strategy he chooses, the other players will act in the worst way possible for him. For two-player games of pure conflict, a **security solution**, whereby both players use a security strategy, is also a Nash equilibrium. Thus, regardless of whether a player is pessimistic (thinking that the other player has outsmarted him) or optimistic (thinking that he has correctly anticipated the other player's strategy), the proper behavior is the same.

EXERCISES

1. Reproduced below is the telephone game from Section 4.2. Find all Nash equilibria in mixed strategies.

The Telephone Game

		Winnie: *Call*	Winnie: *Wait*
Colleen	*Call*	0,0	2,3
	Wait	3,2	1,1

2. The count is three balls and two strikes, and the bases are empty. The batter wants to maximize the probability of getting a hit or a walk, while the pitcher wants to minimize this probability. The pitcher has to decide whether to throw a fast ball or a curve ball, while the batter has to decide whether to prepare for a fast ball or a curve ball. The strategic form of this game is shown here. Find all Nash equilibria in mixed strategies.

Baseball

		Pitcher: *Fastball*	Pitcher: *Curveball*
Batter	*Fastball*	.35,.65	.3,.7
	Curveball	.2,.8	.5,.5

3. It's spring break and you're traveling south on Interstate 95, heading toward Fort Lauderdale. Do you travel the legal limit of 65 miles per hour, or do you crank it up to 80 and hope that there's no speed trap? And what

about the state police? Do they set a speed trap or instead head into town and find out whether the "Hot and Fresh" neon sign is lit up at the Krispy Kreme? (Ouch, that's a cheap shot!) The police like to nab speeders, but they don't want to set a speed trap if there won't be any speeders to nab. A strategic form for this setting is shown in the accompanying figure. The driver can either go the legal limit of 65 mph or speed at 80 mph. The police officer can set a speed trap or head into town and grab some of those delicious high-carb doughnuts. The best outcome for the driver is that she speeds and isn't caught; the payoff for that case is 70. The worst outcome is that she speeds and is nailed by the police, for which the payoff is 10. If she chooses to drive the legal limit, then her payoff is 40 and is the same regardless of what the state police do. (In other words, the driver doesn't care about the caloric intake of the trooper.) As for the police officer, his best outcome is setting a speed trap and nailing a speeder, giving him a payoff of 100. His worst outcome is sitting out there in a speed trap and failing to write a ticket; this outcome delivers a payoff of only 20. His payoff is 50 when he chooses to go to the Krispy Kreme. Find all Nash equilibria in mixed strategies.

Speed Trap and Doughnuts

		State police officer	
		Speed trap	*Krispy Kreme*
Driver	*80 mph*	10,100	70,50
	65 mph	40,20	40,50

4. A mugger and a victim meet on a dark street. The mugger previously decided whether to bring a gun and, if he did, whether to show it during the robbery. If the mugger does not show a gun—either because he doesn't have one or has one and hides it—then the victim has to decide whether to resist. (Note that if the mugger does have a gun and shows it, then the victim's payoff is 5 regardless of the strategy chosen, because the victim's strategy is what to do *if* no gun is shown.) The strategic form of this situation is shown below. Note that all payoffs have been specified, except for the mugger's payoff when he chooses to have a gun and show it. Find a condition on x, whereby a Nash equilibrium exists in which the mugger randomizes over the two pure strategies *gun, hide* and *no gun* and the victim randomizes over *resist* and *do not resist*.

		Victim	
		Resist	*Do not resist*
Mugger	*No gun*	2,6	6,3
	Gun, hide	3,2	5,4
	Gun, show	x,5	x,5

5. For the game below, find all mixed-strategy Nash equilibria.

Player 2

Player 1		x	y	z
	a	2,3	1,4	3,2
	b	5,1	2,3	1,2
	c	3,7	4,6	5,4
	d	4,2	1,3	6,1

6. Find all Nash equilibria in mixed strategies for the game shown here.

Player 2

Player 1		*Left*	*Middle*	*Right*
	Top	2,2	0,0	1,3
	Middle	1,3	3,0	1,0
	Bottom	3,1	2,3	2,2

7. It is the closing seconds of a football game, and the losing team has just scored a touchdown. Now down by only one point, the team decides to go for a two-point conversion that, if successful, will win the game. The offense chooses among three possible running plays: run wide left, run wide right, and run up the middle. The defense decides between defending against a wide run and a run up the middle. The payoff to the defense is the probability that the offense does not score, and the payoff to the offense is the probability that it does score. Find all mixed-strategy Nash equilibria.

Two-Point Conversion

Offense

Defense		*Run wide left*	*Run up middle*	*Run wide right*
	Defend against wide run	.6,.4	.4,.6	.6,.4
	Defend against run up middle	.3,.7	.5,.5	.3,.7

8. The childhood game of Rock–Paper–Scissors is shown in the accompanying figure. (If you're unfamiliar with this game, see Section 4.2.) Show that each player's assigning equal probability to his or her three pure strategies is a symmetric Nash equilibrium.

Rock–Paper–Scissors

Lisa

		Rock	Paper	Scissors
Bart	Rock	0,0	−1,1	1,−1
	Paper	1,−1	0,0	−1,1
	Scissors	−1,1	1,−1	0,0

9. Each of three players is deciding between the pure strategies *go* and *stop*. The payoff to *go* is $\frac{120}{m}$, where m is the number of players that choose *go*, and the payoff to *stop* is 55 (which is received regardless of what the other players do). Find all Nash equilibria in mixed strategies.

10. A total of $n \geq 2$ companies are considering entry into a new market. The cost of entry is 30. If only one company enters, then its gross profit is 200. If more than one company enters, then each entrant earns a gross profit of 40. The payoff to a company that enters is its gross profit minus its entry cost, while the payoff to a company that does not enter is 60. Find a symmetric Nash equilibrium in mixed strategies.

11. Sadaam Hussein is deciding where to hide his weapons of mass destruction (WMD), while the United Nations is deciding where to look for them. The payoff to Hussein from successfully hiding WMD is 5 and from having them found is 2. For the UN, the payoff to finding WMD is 9 and from not finding them is 4. Hussein can hide them in facility X, Y, or Z. The UN inspection team has to decide which facilities to check. Because the inspectors are limited in terms of time and personnel, they cannot check all facilities.
    a. Suppose the UN has two pure strategies: It can either inspect facilities X and Y (both of which are geographically close to each other) or inspect facility Z. Find a Nash equilibrium in mixed strategies.
    b. Suppose the UN can inspect any two facilities, so that it has three pure strategies. The UN can inspect X and Y, X and Z, or Y and Z. Find a Nash equilibrium in mixed strategies.

12. Consider the two-player game below. Find all of the mixed-strategy Nash equilibria.

Player 2

		Slow	Fast
Player 1	Small	2,0	3,8
	Medium	3,7	2,1
	Large	3,4	5,6

13. Consider the two-player game below. Find all of the mixed-strategy Nash equilibria.

Player 2

Player 1		*Left*	*Right*
	Top	1,2	0,2
	Bottom	1,0	3,4

14. Consider the two-player game below. Assume players are allowed to randomize.
   a. Derive players' best-reply functions.
   b. Find all of the mixed-strategy Nash equilibria.

Player 2

Player 1		*x*	*y*
	a	3,3	4,2
	b	6,3	2,6
	c	5,3	3,2

15. Phil, Stu, and Doug are deciding which fraternity to pledge. They all assign a payoff of 5 to pledging Phi Gamma and a payoff of 4 to Delta Chi. The payoff from not pledging either house is 1. Phi Gamma and Delta Chi each have two slots. If all three of them happen to choose the same house, then the house will randomly choose which two are admitted. In that case, each has probability 2/3 of getting in and probability 1/3 of not pledging any house. If they do not all choose the same house, then all are admitted to the house they chose. Find a symmetric Nash equilibrium in mixed strategies.

16. Three retail chains are each deciding whether to locate a store in town A or town B. The profit or payoff that a chain receives depends on the town selected and the number of other chains that put stores in that town; see accompanying table.

Chain's Own Location	Number of Other Chains with Stores in That Town	Chain's Profit
A	0	10
A	1	3
A	2	1
B	0	8
B	1	4
B	2	2

a. Find a symmetric mixed-strategy Nash equilibrium in which chains randomize.
b. Find all mixed-strategy Nash equilibria in which one of the chains puts a store in town A for sure.
c. Find all mixed-strategy Nash equilibria in which one of the chains puts a store in town B for sure.

17. A factory is suspected of hiring illegal immigrants as workers. The authority is deciding whether to conduct an inspection. If the factory has illegal workers and an inspection takes place, the workers will be discovered. The cost of an inspection to the government is 100. The benefit from the inspection is 500 if illegal workers are found, but 0 if none are found. The payoff to the authority from conducting an inspection is the benefit minus the cost, while the payoff from not inspecting is 0. For the factory, the payoff from having illegal workers and not getting caught is 200, from not using illegal workers is 0, and from using illegal workers and getting caught is −300. A factory must decide whether or not to use illegal workers, and the government must decide whether or not to conduct an inspection. Find all mixed-strategy Nash equilibria.

18. For the game below, find all of the mixed-strategy Nash equilibria. The first payoff in a cell is for player 1, the second payoff is for player 2, and the third payoff is for player 3.

Player 3: *Hug*

Player 2

Player 1		*Kiss*	*Slap*
	Cuddle	7,1,5	1,2,4
	Poke	2,2,1	5,3,2

Player 3: *Shove*

Player 2

Player 1		*Kiss*	*Slap*
	Cuddle	5,0,5	3,4,1
	Poke	3,3,3	0,5,4

19. For the Avranches Gap game in Figure 7.10, find the security strategies and security payoffs for General Bradley and General von Kluge.

20. Consider the modified Rock–Paper–Scissors below. Find a symmetric mixed-strategy Nash equilibrium.

Player 2

Player 1		*Rock*	*Paper*	*Scissors*
	Rock	0,0	−2,2	1,−1
	Paper	2,−2	0,0	−1,1
	Scissors	−1,1	1,−1	0,0