Chapter 4

4.1 There are three Nash equilibria: (b, x), (a, z), and (d, z).

4.2 The payoff matrix is

		Maria			
		8 am	9 am	10 am	11 am
Juan	8 am	−3,−3	7,3	7,4	7,5
	9 am	3,7	−2, −2	8,4	8,5
	10 am	4,7	4,8	−1,−1	9,5
	11 am	5,7	5,8	5,9	0,0

If Juan believes Maria will show up at 8 am then he'll plan to show up at 11, but if Juan shows up at 11 then Maria will want to show up at 10, not 8. Hence, 8 am for Maria is not part of a Nash equilibrium. If Juan believes Maria will show up at 9 then he'll show up at 8, but if Juan shows up at 8 then Maria will want to show up at 11, not 9. Hence, 9 am for Maria is not part of a Nash equilibrium. If Juan believes Maria will show up at 10 then he'll show up at 9, but if Juan shows up at 9 then Maria will want to show up at 8, not 9. Hence, 10 am for Maria is not part of a Nash equilibrium. Finally, if Juan believes Maria will show up at 11 am then he'll show up at 10, but if Juan shows up at 10 then Maria will want to show up at 9, not 11. Hence, 11am for Maria is not part of a Nash equilibrium. Thus, there is no Nash equilibrium.

4.3 To be a Prisoners' Dilemma, one strategy needs to strictly dominate the other strategy and, in addition, players must have a lower payoff from both choosing the dominant strategy than if both chose the other strategy. These conditions are only satisfied when $x > 5$ and $y < 3$. As a coordination game has players rank strategy profiles in the same way, we just need $x = y$. For example, if $x = y < 3$ then both players have a ranking of (alpha, alpha) > (beta, beta) > (beta, alpha) = (alpha, beta).

4.4 The best replies for each player are circled. A Nash equilibrium is a pair of strategies for which each player's strategy is a best reply. Thus, there are three Nash equilibria: (a, w), (a, z), and (c, y).

		Player 2			
		w	x	y	z
Player 1	a	4,3	1,1	0,3	3,3
	b	2,4	3,1	2,2	3,1
	c	0,6	4,4	3,7	1,2
	d	2,3	3,2	1,0	0,0

4.5 There are two Nash equilibria: (c, y, I) and (a, x, II).

4.6 First note that for A (B) to prevail, both shareholders 2 and 3 must vote for A (B). If one does not then the most the option can receive is 65% which is below the 70% threshold. Thus, shareholder 1 is never pivotal which means all of her strategies yield the same payoff (given the strategies of the other two shareholders) and thus all of shareholder 1's strategies are not weakly dominated. Next note that shareholder

3 can ensure that C prevails, which is 3's most preferred option, by voting for C as then A or B will receive at most 60%. Furthermore, 3 voting for A (B) is weakly dominated by voting for C because option A (B) would prevail if 2 votes for A (B) and that is inferior to C for shareholder 3. We conclude that, for shareholder 3, voting for C weakly dominates voting for A and voting for B. Turning to shareholder 2, let us show that voting for B weakly dominates voting for A and for C. If 3 voted for C then all three strategies for 2 result in C being the outcome. If 3 voted for B then B would prevail by 2 voting for B (which is the most preferred outcome for 2), while if 2 voted for A or C then C would prevail. If 3 voted for A then C would prevail when 2 votes for B or C, while if 2 voted for A then A would prevail which is worse than C for 2. Thus, for shareholder 2, voting for B does better than voting for A or C when 3 votes for B; voting for B does the same as voting for A or C when 3 votes for C; and voting for B does better than voting for A and just as well as voting for C when 3 votes for A. Hence, for shareholder 2, voting for B weakly dominates voting for A and for C. Summing up, a strategy profile is a Nash equilibrium in strategies that are not weakly dominated when: 1 votes for A, B, or C; 2 votes for B; and 3 votes for C.

4.7 The best replies are circled in the payoff matrix below. There are 19 Nash equilibria. 15 of them have player 1 winning for sure; one has players 2 and 3 each having a 50% chance of winning; one has player 2 winning for sure; and one has player 3 winning for sure.

Player 3: *Positive*

Player 2

Player 1	*Positive*	*Negative - 1*	*Negative - 3*
Positive	1,0,0	1,0,0	1,0,0
Negative - 2	1,0,0	½,0,½	1,0,0
Negative - 3	1,0,0	1,0,0	1,0,0

Player 3: *Negative against 1*

Player 2

Player 1	*Positive*	*Negative - 1*	*Negative - 3*
Positive	1,0,0	0,½,½	1,0,0
Negative - 2	1,0,0	0,0,1	1,0,0
Negative - 3	½,½,0	0,1,0	1,0,0

Player 3: *Negative against 2*

Player 2

Player 1	*Positive*	*Negative - 1*	*Negative - 3*
Positive	1,0,0	1,0,0	1,0,0
Negative - 2	1,0,0	1,0,0	1,0,0
Negative - 3	1,0,0	1,0,0	1,0,0

4.8 In round 1 of the IDSDS, no strategies are strictly dominated for players 1 and 2, while *up* strictly dominates *down* for player 3. In round 2, *bottom* strictly dominates *top* for player 1, while no strategy is strictly dominated for players 2 and 3. In round 3, no strategies are strictly dominated. Thus, a strategy profile survive the IDSDS if it has player 1 choose *middle* or *bottom*, player 2 choose *left*, *center* or *right*, and player 3 choose *up*. This is a total of 6 strategy profiles. In contrast, there are only two Nash equilibrium strategy profiles: (*middle*, *right*, *up*) and (*bottom*, *center*, *up*).

4.9 The table below shows the evolution of their beliefs and actions. Thelma and Louise settle on a convention of driving on the left.

Encounter Number	Thelma's Belief That Louise Will Choose Left	Louise's Belief That Thelma Will Choose Left	Thelma's Optimal Strategy	Louise's Optimal Strategy
18	$8/17 \simeq .47$	$14/17 \simeq .82$	Right	Left
19	$9/18 \simeq .50$	$14/18 \simeq .78$	Right	Left
20	$10/19 \simeq .53$	$14/19 \simeq .74$	Right	Left
21	$11/20 \simeq .55$	$14/20 \simeq .70$	Right	Left
22	$12/21 \simeq .57$	$14/21 \simeq .67$	Right	Left
23	$13/22 \simeq .59$	$14/22 \simeq .64$	Right	Left
24	$14/23 \simeq .61$	$14/23 \simeq .61$	Left	Left
	$15/24 \simeq .63$	$14/24 \simeq .63$	Left	Left
.				
∞	$\simeq 1$	$\simeq 1$	Left	Left

Chapter 5

5.1 Let (x, y, z) represent a strategy profile in which x women wear Lilly, y wear Goth, and z wear vintage. With seven women, let's start with (0, 1, 6). This is not an equilibrium, as a woman wearing vintage has a payoff of zero (since women wearing Goth are in a smaller clique), but she could have a payoff of 1 by wearing either Goth or Lilly. With (0, 2, 5), a woman wearing vintage can improve by wearing Lilly or Goth; in either case, she'll be in the smallest group. With (0, 3, 4), a woman wearing vintage can raise her payoff from 0 to 1 by wearing Lilly. (Note, though, that wearing Goth will not help her social standing.) (1, 1, 5) is a Nash equilibrium, as a woman wearing Lilly and a woman wearing Goth each have a payoff of 1 and thus cannot do better, while a woman wearing vintage will still not be in the smallest group even if she were to switch to Lilly or Goth, since women only receive a payoff of 1 if the size of the clique of which she is a member is no larger than any other clique. (1, 3, 3) is not a Nash equilibrium, as a woman wearing Goth or vintage can instead wear Lilly and do better. Finally, (2, 2, 3) is a Nash equilibrium. We then conclude that it is a Nash equilibrium for there to be (1) one woman wearing Lilly and one wearing Goth, and (2) two wearing Lilly and two wearing Goth. There are other comparable equilibria with the labels just switched around. That is, in general, the Nash equilibria have (1) one woman choosing one type of clothing, a second woman choosing a second type of clothing, and the other women choosing the third type of clothing; or (2) two women choosing one type of clothing, two women choosing a second type of clothing, and the other women choosing the third type of clothing.