Econ 210A Problems, November 9, 2007

1) An economy has M consumers. For $i = 1, \ldots, M$, Consumer i has utility function

$$U_i(x_{1i}, x_{2i}) = x_{1i} + 2a_i\sqrt{x_{2i}}$$

where x_{ji} is the amount of good j that i consumes and where $a_i > 0$.

a) Find Consumer i's demand function $x^i(p, w_i)$for Goods 1 and 2. Be sure to take account of possible corner solutions. For what values of p_1, p_2, and w does Consumer i consume none of Good 1?

b) Show that if "everyone has enough income", all consumers will demand positive amounts of both goods. What do we mean by everyone has enough income in this context?

c) Write an expression for consumer i's indirect utility function. Are these indirect utilities of the Gorman form?

d) In general, aggregate demand is the sum of individual demands which in turn depends on prices and the distribution of income. That is aggregate demand is a function

$$D(p, w_1, \ldots, w_M) = \sum_i x^i(p, w_i).$$

Show that with the utility functions in this problem, there is a range of income distributions such that aggregate demand depends only on prices and total income for all income distributions in this range.

2 When I wrote this problem, I meant to ask the question that you now find as 2b), because it has a parallel structure to Problem 1. But I inadvertently wrote the problem in 2a. In retrospect, Problem 2a is also interesting, so I am going to ask you to do both 2a) and 2b).

2a) An economy has M consumers. For $i = 1, \ldots, M$, Consumer i has utility function

$$U_i(x_{1i}, x_{2i}, x_{3i}) = x_{1i} + 4\sqrt{x_{2i}x_{3i}}$$

where x_{ji} is the amount of good j that i consumes. Answer questions equivalent to parts a-d of Question 1. (Note that the demand "function" is actually a correspondence in this case.)

Hint: Break the problem into two pieces. Given the price vector p, suppose that the consumer spends y on goods 2 and 3. How would he allocate this expenditure between goods 2 and 3. Once you know this, you can write an expression for his utility if he spends y on goods 2 and 3 and consumes x_1 units of good 1. You can write then write his utility and his budget constraint in terms of x_1 and y and solve for the optimal choices of x_1 and y, and hence the optimal choices of x_2 and x_3. Another hint: You will find that preferences in Problem 2a are quasi-concave but not strictly quasi-concave. To see that they are not strictly quasi-concave, note that $U(0, 1, 1) = U(4, 0, 0) = 4$. Now consider the convex combination $1/2(4,0,0)+1/2(0,1,1)=(2,1/2,1/2)$. If preferences are strictly quasi-concave, we would have to have $U(2,1/2,1/2)>4$, but in fact $U(2,1/2,1/2)=4$.At some prices, consumer will buy no Good 1. When you break your problem down to a choice between good 1 x_1 and expenditure y on the other two goods, you will have utility that is linear in x_1 and y. At some prices consumer will buy no Good 2 and no Good 3. At those prices at which he is willing to buy positive amounts of all three goods, there is more than one optimal bundle.

2b) An economy has M consumers. For $i=1,\ldots,M$, Consumer i has utility function

$$U_i(x_{1i},x_{2i},x_{3i})=x_{1i}+4\left(x_{2i}x_{3i}\right)^{1/4}$$

where x_{ji} is the amount of good j that i consumes. Answer questions equivalent to parts a-d of Question 1.

Hint: Break the problem into two pieces. Given the price vector p, suppose that the consumer spends y on goods 2 and 3. How would he allocate this expenditure between goods 2 and 3. Once you know this, you can write an expression for his utility if he spends y on goods 2 and 3 and consumes x_1 units of good 1. You can write then write his utility and his budget constraint in terms of x_1 and y and solve for the optimal choices of x_1 and y, and hence the optimal choices of x_2 and x_3.

3) An economy has M consumers. For $i=1,\ldots,M$, Consumer i has utility function

$$U_i(x_{1i},x_{2i},x_{3i})=x_{1i}+\ln\left(x_{2i}+\ln(1+x_{3i})\right)$$

where x_{ji} is the amount of good j that i consumes. Answer questions equivalent to parts a-d of Question 1. Hint: First see what must be true at interior solutions where all 3 goods are consumed. Then check out the corner solutions.