Name ______________________________

Midterm Examination: Economics 210A

October 2011

The exam has 6 questions. Answer as many as you can. Good luck.

1) A) Must every quasi-concave function must be concave? If so, prove it. If not, provide a counterexample. (In all answers where you provide a counterexample, you must show that your example is really a counterexample.)

B) Must every concave function be quasi-concave? If so, prove it. If not, provide a counterexample.

2) Let F and G be real-valued concave functions with the same domain, A. Define the function H so that for all $x \in A$, $H(x) = F(x) + G(x)$. Is H a concave function? If so, prove it. If not, provide a counterexample.

3 Let F and G be real-valued concave functions with the same domain, A. Define the function H so that for all $x \in A$, $H(x) = F(x)G(x)$.
A) Is H a concave function? If so, prove it. If not, provide a counterexample.

B) Is H a quasi-concave function? If so, prove it. if not, provide a counterexample.

4) A consumer has preferences represented by the utility function

$$U(x_1, x_2) = x_1 + x_2 + 2x_2^{1/2}.$$

Good 1 is the numeraire and has price 1. The price of good 2 is p_2 and the consumer's income is m.

A) Find this consumer's Marshallian demands for goods 1 and 2 as a function of p_2 and m. Be careful to account for corner solutions if there are any.

B) Use your solution to Part A and the relevant homogeneity property of Marshallian demand to find this consumer's demands for goods 1 and 2 for arbitrary non-negative prices p_1, p_2, and income m. (Simplify your expressions for answers as much as possible.)

C) Find this consumer's Hicksian demand functions $h_1(p_1, p_2, u)$ and $h_2(p_1, p_2, u)$. Be careful to account for corner solutions if there are any.

D) Find this consumer's expenditure function $e(p_1, p_2, u)$.

E) Verify that Shephard's lemma applies in this case.

5) A consumer has utility function

$$u(x_1, x_2) = \min\{v_1(x_1, x_2), v_2(x_1, x_2)\}$$

where v_1 and v_2 are both quasi-concave functions Is u quasi-concave? If so, prove it. If not, provide a counterexample.

6) A sculpture is placed on top of a horizontal grid. The height of the sculpture above the point on the grid with coordinates (x_1, x_2) is $x_1^2 - 3x_1x_2 + x_2^2$. An ant is crawling on the surface of the sculpture.
A) If the ant is initially at the point directly above the point $(2, 1)$ on the grid, what is the directional derivative of the height of the ant if it is crawling on the surface of the sculpture in the direction $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ as measured on the grid?

B) In what direction on the surface should the ant crawl if it wants to climb most steeply? (Hint: Directions should be described by a vector whose length is 1 unit.)

C)In what direction should it crawl if it wants to descend most steeply?

D) If the ant is moving along the surface of the statue in the direction of steepest climb at the rate of one unit per second, at what rate is its height above the ground increasing?