Name ______________________

Midterm Examination: Economics 210A
November 7, 2011

Answer Question 1 and any 4 of the other 6 questions. Good luck.

1) Let f be a real-valued concave function whose domain is a convex subset of $\Re^n$. Let g be a function from the reals to the reals and define the composite function $h(x) = g\left(f(x)\right)$.

State whether each of the following claims about the function h is true or false. If true, give a proof, justifying each claim made in your proof. If false, give a counterexample and prove that your counterexample is a counterexample.

A) If g is a strictly increasing function, then h is a concave function.

B) If g is an increasing, concave function, then h is a concave function.

C) If g is a concave function, then h is a concave function.

2) Rocky consumes two goods. He prefers any bundle such that $x_1 > 0$ and $x_2 > 1$ to any bundle for which these two inequalities are not satisfied. His preferences over bundles such that $x_1 > 0$ and $x_2 > 1$ can be represented by the utility function

$$u(x_1, x_2) = \ln(x_1 + 1) + \ln(x_2 - 1)$$

for all (x_1, x_2) .

A) For what price-income combinations does Rocky choose positive amounts of both goods?

B) Find Rocky's Marshallian demand function.

C) Find Rocky's indirect utility function.

3) Rocky, from the previous problem, is one consumer in an economy in which there are n people, $i = 1, \ldots n$. Person i has a utility function of the form

$$u_i(x_1, x_2) = A_i \ln(x_1 + b_i) + \ln(x_2 - 1).$$

What restrictions, if any, do we need to put on the parameters A_i and b_i so that aggregate demands for goods 1 and 2 are determined by prices and the sum of incomes and do not depend on the distribution of income? Relate your answer to the Gorman polar form.

4) A) What restrictions must α_1, α_2, and $f(y)$ satisfy for the following to be a legitimate indirect utility function? Explain.

$$v(p_1, p_2, y) = f(y)p_1^{\alpha_1}p_2^{\alpha_2}$$

B) What restrictions must the functions $w(p_1, p_2)$ and $z(p_1, p_2)$ satisfy for the following to be a legitimate indirect utility function? Explain.

$$v(p_1, p_2, y) = w(p_1, p_2) + z(p_1, p_2)y$$

5) A consumer has utility function

$$u(x_1, x_2) = \left(x_1^{1/2} + 2x_2^{1/2}\right)^2.$$

A) How is the ratio x_1/x_2 in which this consumer consumes the two goods related to the ratio of the prices of goods 1 and 2?

B) What is the elasticity of substitution between the two goods?

C) Find the consumer's Marshallian demands for goods 1 and 2.

D) Find the consumer's indirect utility function.

6) Define the lexicographic preference ordering on $\Re^n$. Which of the following properties does the lexicographic ordering have? Completeness, transitivity, strict monotonicity, strict convexity, continuity. Explain your answers.

7) A consumer buys two goods. The more of a good that the consumer buys the higher the price per unit he must pay. (Resale of the good is impossible-think of haircuts and other services.) The cost of buying x_1 units of 1 is $p_1 x_1^2$ and the cost of buying x_2 units of good 2 is $p_2 x_2^2$. The consumer's income is m. Suppose this consumer's utility function is $u(x_1, x_2) = x_1^a + x_2^a$. For what values of a does this consumer buy positive amounts of both goods? (Hint: You may want to draw a diagram.) Find the "demand functions" $x_i(p_1, p_2, m)$ showing quantities that he will buy given p_1, p_2, and m and "indirect utility function" $v(p_1, p_2, m)$, showing his utility for what he buys. Does Roy's law still apply? Justify your answer.

For Extra Credit: Suppose that there are n goods. A consumer has utility function $u(x)$ and a budget constraint of the form $\sum_i p_i g_i(x_i) = m$, where the g_i's are all strictly increasing functions. Where $x(p, m)$ is the bundle that solves this constrained maximization problem with parameters p and m, let $v(p, m) = u(x(p, m))$ be the corresponding "indirect utility function". Show that you can use a rule that is similar to Roy's Law to recover the demand functions (over the range of interior solutions) from the indirect utility function v.