Name ____________________

First Midterm, Graduate Microeconomic Theory, October 2014

Question 1

Suppose that the n functions $f_1(x), \ldots, f_n(x)$ are all concave functions from $\Re^n_+$ to $\Re$. Define the function $h(x)$ from $\Re^n_+$ to $\Re$ where

$$h(x) = g\left(f_1(x) + \ldots + f_n(x)\right)$$

with g being an increasing function from $\Re$ to $\Re$. Answer each of the following questions by either offering a proof or showing a counterexample.

a) Must the function $h(x)$ be concave?

b) Must the function $h(x)$ be quasi-concave?

c) Suppose that the functions f_i are quasi-concave, but not necessarily concave, must the function $h(x)$ be quasi-concave?

Question 2

a) Find the Marshallian demand correspondence and the indirect utility function for a consumer with utility function

$$U(x_1, x_2) = x_1 + 2\sqrt{x_2}.$$

(Be sure to account for possible non-interior solutions.)

b) Find the Marshallian demand correspondence and the indirect utility function for a consumer with utility function

$$U(x_1, x_2) = \sqrt{(x_1 + a)x_2}$$

where $a \geq 0$. (Be sure to account for possible non-interior solutions.)

c) Find the Marshallian demand correspondence and the indirect utility function for a consumer with utility function

$$U(x_1, x_2) = \sqrt{x_1 x_2} + 2\sqrt{x_3}.$$

(Hint: One way to tackle this problem is to break it into two pieces. Suppose that at price vector p, the consumer buys x_3 units of good 3. Then he will have $m - p_3 x_3$ left to spend on goods 1 and 2. Use the indirect utility function for someone with utility $\sqrt{x_1 x_2}$ to find the best utility he can achieve if he buys x_3. Then find the best x_3 that he can afford at these prices.) Holding prices constant, how do changes in income affect this consumer's demand for good 3?

Question 3 Suppose that $X = \Re^3_+$ and we define weak preference by $x \succeq y$ if for at least two of the three components, x gives as much of the commodity as y. That is, if $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ then $x \succeq y$ if $x_i \geq y_i$ for two or three of $i = 1, 2, 3$.

a) Is $\succeq$ transitive? If so, prove it. If not, show a counterexample.

b) Define strict preference from these weak preferences by the rule: $x \succ y$ if $x \succeq y$ and not $y \succeq x$. Show that $\succ$ defined in this way is equivalent to the following alternative: $x \succ y$ if x gives strictly more than y in at least two components.

c) Is $\succ$ as defined in part b) transitive? Is $\succ$ negatively transitive? (Prove your answers or provide counterexamples)