Name

## Second Midterm Examination: Economics 210A November, 2014

This exam has five pages and three questions-each with multiple parts. Answer as many as you can. Good luck.

Question 1) Arthur has utility function

$$
U\left(x_{1}, x_{2}\right)=\left(x_{1}^{1 / 2}+x_{2}^{1 / 2}\right)^{2 k}
$$

Betsy has utility function

$$
U\left(x_{1}, x_{2}\right)=\left(x_{1} x_{2}\right)^{k}
$$

where $k>0$.
Part A)
(i) What is Arthur's elasticity of substitution between goods 1 and 2?
ii) What is Betsy's elasticity of substitution between goods 1 and 2?

Part B)
(i)Find Arthur's expenditure function.
ii) Find Betsy's expenditure function.

Part C)
(i) Find Arthur's indirect utility function.
(ii) Find Betsy's indirect utility function.

Question 2) Velma, the strict vegetarian, consumes only four goods. Goods 1 and 2 are fruits and Goods 3 and 4 vegetables. Her utility function can be written as

$$
U\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=f\left(x_{1}, x_{2}\right)+2^{3 / 2} v\left(x_{3}, x_{4}\right)
$$

where

$$
f\left(x_{1}, x_{2}\right)=\left(x_{1}^{1 / 2}+x_{2}^{1 / 2}\right)^{2}
$$

and

$$
v\left(x_{3}, x_{4}\right)=x_{3}^{1 / 4} x_{4}^{1 / 4}
$$

Part A) Where $p_{i}$ is the price of good $i$ for $i=1, \ldots, 4$, and where Velma has income $M$, how much money should she spend on fruit and how much should she spend on vegetables? State your answers as functions of the variables $M, p_{1}, p_{2}, p_{3}$, and $p_{4}$. (Hint: Maybe your answers to Question, Part C, will be helpful for this.)

Part B) If prices and income are such that Velma consumes both fruit and vegetables, how do small changes in $M$ affect her consumption of vegetables? Explain.

Question 3) In an economy with a large number of people, there are three commodities. Each person $i$ has preferences representable by a utility function of the form

$$
U_{i}\left(x_{1}, x_{2}, x_{3}\right)=\left(x_{1}+a_{i}\right)^{1 / 4}\left(x_{2}+b_{i}\right)^{1 / 4}\left(x_{3}+c_{i}\right)^{1 / 2}
$$

where the $a_{i}$ 's and $b_{i}$ 's and $c_{i}$ 's are all positive numbers, but may be different for different people.

Part A) For price-income combinations where person $i$ buys all three goods, find $i$ 's Marshallian demand function for each of the three goods.

Part B) Find conditions on prices and income that guarantee that $i$ 's Marshallian demands for each of the three goods is positive.

Part C) In this economy, at prices $p$ if all consumers have income sufficient to consume positive amounts of each good, show that aggregate demand depends only on total income and not on the distribution of income.

Part D) Show that in this economy, indirect utility is of the Gorman polar form.

