Name ______________________________

Midterm Examination: Economics 210A

October 16, 2015

Question 1.) In their 1972 book, *Economic Theory of Teams*, Jacob Marschak and Roy Radner describe a team theory problem as one in which n players all want to maximize the same objective function. Each player chooses an action x_i from a set X_i and each player gets a payoff equal to $F(x_1, \ldots, x_n)$. There is no "boss" to coordinate their activities. Marschak and Radner define an outcome $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ to be *Person-by-person satisfactory* if each person is taking the action that maximizes F, given what the other players are doing. This means that for all players i:

$$F(\bar{x}_1, \ldots, \bar{x}_i \ldots, \bar{x}_n) \geq F(\bar{x}_1, \ldots, x_i, \ldots, \bar{x}_n)$$

for all $x_i \in X$.

If $\bar{x}_i$ is in the interior of X_i for all i and if $\bar{x}$ is person-by-person satisfactory, what must be true of the first and second order partial derivatives of F?

For now let us consider two-person teams. Assuming that F is twice continuously differentiable, determine whether each of the following three statements is true or false. If it is true, sketch a proof. If it is false, provide a counterexample.

Claim 1) If $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on the set on (X_1, X_2), then outcome $(\bar{x}_1, \bar{x}_2)$ must be person-by-person satisfactory.

Claim 2) If outcome $(\bar{x}_1, \bar{x}_2)$ is person-by-person satisfactory, it must be that $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on (X_1, X_2).

Claim 3) If F is a concave function, then if an outcome is person-by-person satisfactory and x_i is in the interior of X_i for $i = 1, 2$, it must be that $(\bar{x}_1, \bar{x}_2)$ maximizes $F(x_1, x_2)$ on (X_1, X_2).

Question 2.) A consumer has preferences on $X \subset \Re^2_+$ which are defined in the following way. There is a real-valued continuous function u with domain X such that for all x and y in X, $x \succeq y$ if and only if $u(y) - u(x) \leq 1$.

A) Where the relations $\succ$ and $\sim$ are defined from $\succeq$ in the usual ways, describe the relations $x \succ y$ and $x \sim y$ in terms of inequalities involving $u(x)$ and $u(y)$.

B) Determine whether each of the following three statements is true or false. If it is false show a counterexample. If it is true, prove it.

Claim 1) The relation $\succeq$ is transitive.

Claim 2) The relation $\succ$ is transitive.

Claim 3) The relation $\succeq$ is continous.

C) Suppose that $\succeq$ is described as above, where $u(x_1, x_2) = x_1 x_2$. Let B be the set of things the consumer can afford with income 10 and where the price of good 1 is 1 and the price of good 2 is 2. Is $c(B)$ non-empty? Sketch the set $c(B)$. Describe this set, using two inequalities.