Practice Problems on Gradients and Directional Derivatives

1) Consider the function $f(x,y) = \ln(\sqrt{x^2+y^2})$. Find its gradient at the point $(x,y) = (1,-1)$. At this point, what is the directional derivative of f in the direction $(3/5, 4/5)$?

2) Pikabo Street is skiing on a Colorado mountain. Colorado is a large square state with a surface of many high bumps. Surveyers have mapped out the state into "sections" which are 1 mile by 1 mile squares. The "origin" for each section is its Southwest corner. Each point within a section is assigned a number (x,y) where x feet is the number of feet that this point lies East from the origin and y is the number of feet that it lies North of the origin is denoted by (x,y). The altitude of the earth's surface at the point (x,y) is given by the formula $z = f(x,y)$. In the region of interest to us, f is a continuously differentiable function.

Suppose that $f(x,y) = (100x + 2xy - 3y^2)/100$. Pikabo is currently at the point $(x,y) = (100,100)$. What is the gradient of the surface at this point? What is the slope of her skis if she faces East? What is the slope of her skis if she faces North?

Pikabo is pointing her skis in the direction $(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$. What is the slope of her skis in the direction she has them pointed? Suppose that Pikabo points her skis in the direction that would lead to the steepest descent. What direction is that and what is the slope of her skis in this direction? Suppose that Pikabo wants to point her skis in a direction such that her skis are level. In what direction(s) should she point them?

3) (a four dimensional problem) Captain Econ finds himself flying in a faraway planetary system. The temperature in a (3-dimensional) rectangular coordinate system in his neighborhood is

$$T(x,y,z) = 2x + yz.$$

If he is currently at the point $(1,1,1)$ and is flying in the direction $(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}})$, what is the rate of change of temperature per unit of distance that he travels? In what direction should he fly to maximize the rate of increase of temperature per unit of distance that he flies. In what direction should he fly to maximize the rate of decrease of temperature per unit of distance that he flies. In what direction(s) can he fly if he wants to make the derivative of his temperature equal to zero?