

Name _____

Final Exam, Econ 210A

Answer as many questions as you can. Enjoy.

1. A poor fellow is an expected utility maximizer with a von Neumann Morgenstern utility function $v(x) = x^{1/2}$. He has a wealth of \$99,000. His shady brother-in-law has given him inside information on the outcome of an upcoming sports event. On the basis of this information, the poor fellow believes that the probability that Team A will defeat Team B is $2/3$, despite the fact that Team B is two-to-one favorite to win among the betting community. It is possible for the poor fellow to bet as much money as he wishes on Team A to win at the prevailing odds. That is, for every dollar that he bets, he will get back \$2 if Team A wins and will get back nothing if Team A does not win.

a. Let x_A be the contingent commodity “wealth if Team A wins” and let x_B be “wealth if Team B wins.” Write a budget equation expressing the combinations of x_A and x_B that the poor fellow can afford by betting some amount of money on Team A.

Draw a graph to show all of these combinations.

b. If this poor fellow bets so as to maximize his expected utility, what will be his wealth if Team A wins?

What will be his wealth if Team B wins?

c. Another poor fellow has a von Neumann Morgenstern utility function $v(x) = -x^{-1}$ and he also has a wealth of \$99,000. He has the same shady brother-in-law and the same inside information on the outcome of an upcoming sports event. This poor fellow also believes that the probability that Team A will defeat Team B is $2/3$, despite the fact that Team B is two-to-one favorite to win among the betting community. It is likewise possible for this poor fellow to make bets such that for every dollar that he bets, he will get back \$2 if Team

A wins and will get back nothing if Team A does not win. If this poor fellow places his bets to maximize his expected utility, what will be his wealth if Team A loses?

What will be his wealth if Team A wins?

2. Consider a set of lotteries in which the prizes are purely monetary and where there are n possible outcomes. Prices of goods are assumed to be the same regardless of the outcome of the lottery and preferences are state independent. A consumer's certainty equivalent for a lottery is the amount of wealth received with certainty that would be exactly indifferent to facing that lottery.

a. Suppose that a consumer is an expected utility maximizer whose preferences on lotteries that have n possible outcomes are represented by the expected utility function

$$\sum_{i=1}^n \pi_i v(x_i)$$

where

$$v(x_i) = \frac{1}{\rho} x_i^\rho$$

and where x_i is the consumer's wealth if event i happens. Write an equation for the certainty equivalent of the lottery $(x_1, \dots, x_n)$.

b. Is the certainty equivalent function homogeneous? If so, of what degree?

c. For what values of ρ is this consumer a risk averter?, risk neutral? a risk lover?

d. Show that a consumer with this utility function has homothetic preferences

over lotteries.

e. Are there any other utility functions v not of the form

$$v(x_i) = \frac{1}{\rho} x_i^\rho$$

such that an expected utility maximizer with this function v has homothetic preferences? If so, give an example. Hint: What do we know about utility for homothetic and separable preferences?

3. a. Define the class of indirect utility functions that are of the Gorman form.

b. What is the connection between the Gorman form and the aggregation of consumers?

c. Show that if preferences are quasi-linear, indirect utility can be represented in the Gorman form.

d. Show that if preferences are identical and homothetic, indirect utility can be represented in the Gorman form.

4. Define each of the following. Use a full grammatical sentence in your definition.

a. concave function

b. convex function

c. quasi-concave function

d. quasi-convex function

e. Prove that a concave function must be quasi-concave.

f. Show that a quasi-concave function is not necessarily a concave function.

5. Where utility functions are of the functional form

$$U(x_1, \dots, x_6) = U^*(f_1(x_1, x_2), f_2(x_3, x_4, x_5), x_6)$$

a. show that the marginal rate of substitution between goods 3 and 4 is independent of the quantities of goods 1 and 2.

b. Construct an example of a utility of this form where the marginal rate of substitution between goods 3 and 6 depends on the quantity of good 1.

6. Let $c(p, y)$ be the cost function representing the cheapest way to produce the single output y when factor prices are given by the vector p .

a. Must the function $c(p, y)$ be homogeneous of degree one in p ? If so, prove it. If not, show that it is not.

b. If you know the function $c(p, y)$, can you find the conditional factor demand $x_i(p, y)$ for factor i ?

c. How are returns to scale of the production function related to the second derivative of $c(p, y)$ with respect to y ?

7. Consider Leontief preferences $u(x_1, x_2) = \min\{\frac{x_1}{\alpha}, \frac{x_2}{1-\alpha}\}$ where $0 < \alpha < 1$. Derive the demand functions, the indirect utility function, the expenditure function, and the Hicksian demand functions for these preferences.

8. A firm has the production function

$$f(x_1, x_2) = (x_1 - 10)^{1/2}(x_2 - 20)^{1/2}.$$

a. Is this function homogeneous of degree one?_____

b. Find the conditional factor demands $x_1(w_1, w_2, y)$, $x_2(w_1, w_2, y)$ and the cost function $c(w_1, w_2, y)$ where w_i is the price of factor i and y is output.

c. Can you provide an interpretation of this function as a description of a technology that involves some setup activity and constant returns to scale once setup is accomplished?

9. Let $u(x_1, x_2)$ be a utility function.

a. What conditions must u satisfy to be a homothetic function?

b. What conditions must u satisfy to be homogeneous of degree $k > 0$?

c. Must a homothetic function be homogeneous of some degree? If so, prove it, if not show by example.

d. Must a homogeneous function that is homogeneous of degree $k > 0$ be homothetic? If so, prove it, if not show by example.

e. If someone has preferences that are represented by a utility function that is homogeneous of degree $k > 0$, can the same preferences be represented by a utility function that is homogeneous of degree 1? If so, prove it. If not show why not.