# Lecture 6 -Law and Economics 

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## Chapter 6

## Law, Economics, and Externalities

## Coase attacks Pigou

In a famous paper, called "The Problem of Social Cost," [1] Ronald Coase attacked the interventionist approach to externalities that Coase identified with the work of A. C. Pigou. Coase argued that Pigou and other economists had lost sight of the fact that one party imposes damages on another, the extent of the damages depends on the action of both parties and efficient allocation of resources may involve adjustments in the behavior of both parties.

According to Coase:
"The traditional approach (to problems where business firms inflict harmful effects on others) has tended to obscure the nature of the choice that has to be made. The question is commonly thought of as one in which $A$ inflicts harm on $B$ and what has to be decided is how should we restrain $A$ ? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to $B$ would inflict harm on $A$. The real question to be decided is: should $A$ be allowed to harm $B$ or should $B$ be allowed to harm $A$ ?" [1] page 2

Coase thought of the establishment of legal liability rules as a specifying initial property rights as a starting point from which private bargaining would determine the ultimate outcome. According to Coase:

It is necessary to know whether the damaging business is liable or not for damage caused since without the establishment of this initial delimitation of rights there can be no market transactions to transfer and recombine them. But the ultimate result (which maximizes the value of production) is independent of the legal system if the pricing system is assumed to work without cost. [1] page 8

Applying Coase's view to the example of Ed and Fiona in chapter ??, the situation where Ed is not liable for the damage caused by his smoking corresponds to assigning initial property rights corresponding to the allocation $X$ in Figure ?? and the situation in which Ed is fully liable for damage caused by his smoking corresponds to the assignment of property rights to the allocation $W_{0}$ in Figure ??. As Coase points out, the legal specification of "property rights" is not necessarily a good prediction of what the final outcome will be. Rather the legal rules determine a starting point from which bargaining can proceed.

## What is the Coase Theorem?-A digression

Every economist has heard of the "Coase Theorem", but those who have read Coase's article will find neither a statement of a theorem nor a proof. There is a series of examples and some very interesting discussion. But the conclusions to be drawn are not stated with the directness that we normally associate with formal economics. The term "Coase Theorem" seems to be due to George Stigler, who explained Coase's ideas in his textbook The Theory of Price. [8], pp 110-114. Stigler claimed that the Coase theorem establishes two results:
a). "under perfect competition, private and social costs will be equal."
b). "the composition of output will not be affected by the manner in which the law assigns liability for damage. "

Stigler offered no proof of these claims but simply presents a numerical example for which the result seems to be true. Later admirers of Coase were more cautious than Stigler and stated versions of the Coase theorem that were qualified by assumptions. For example, according to Richard Zerbe [9],
"The Coase theorem, without violence to common usage, may be expressed as: ${ }^{1}$

[^0]'In a world of perfect competition, perfect information, and zero transactions costs, the allocation of resources will be efficient and invariant with respect to legal rules of liability'. "

There has been a great deal of dispute among economists about what Coase said, what Coase meant, and whether he was right. ${ }^{2}$ One thing that we all learn from teaching classes is that even when we think that our lectures are paragons of clarity, our students may find them puzzling or ambiguous. Coase seems to believe although Pigou infected by bad ideas, he didn't state them clearly enough so that they could be refuted.
"It is strange that a doctrine as faulty as that developed by Pigou should have been so influential, although part of its success has been due to the lack of clarity of exposition. Not being clear, it was never clearly wrong." [5], page 39.

Some of those who have attempted to pin down the claims made by Coase will believe that this fine bit of rhetoric applies at least as well to Coase's "Problem of Social Cost" as it does to Pigou's Theory of Welfare.

## A Modern Interpretation of Coase

Modern mathematical economists have made several efforts to formulate Coase's claims as general propositions that can be proved or disproved under specific assumptions. Significant progress along these lines was made by several authors, including Marchand and Russell, [4], Starrett [7], and Schweizer [6]. I strongly recommend reading Schweizer's discussion, which is particularly clearly-presented and thorough. The presentation that follows is based closely on Schweizer's paper.

We introduce another pair of neighbors, George and Hazel, who are in many ways similar to Ed and Fiona. George, like Ed, imposes a disagreeable externality on his neighbor. In the spirit of Coase's examples, we suppose that the interaction between George and Hazel is a step more subtle than that between Ed and Fiona. Unlike in the case of Ed and Fiona, the damage that George causes to Hazel depends in part on the Hazel's own activity.
following. "The interpretations of Coase's work given here are based on what seems the most useful view of this work, not necessarily on what Coase himself meant. This can be determined only by Coase himself.
${ }^{2}$ A small sample of the large number of papers written in an effort to interpret Coase include papers by Marchand and Russell, [4], Zerbe [9], Gifford and Stone [3], Cooter [2], and Schweizer [6].

George enjoys walking around his backyard in his underwear, singing rugby songs. Hazel enjoys having tea with refined friends in her backyard which abuts George's back yard. The amount of annoyance that Hazel suffers from George's activity is an increasing function of the amount of time that Hazel spends in her back yard as well as of the amount of time that George spends in his backyard, singing in his underwear. We model this story as follows. ${ }^{3}$

George cares about two things-the level of activity $X$ that he performs and his money income $Z_{G}$. Hazel enjoys activity $Y$ and her money income $Z_{H}$, but her enjoyment of $Y$ is diminished by George's performance of $X$. In particular, the damage that George does to Hazel is an increasing function both of $x$, the level of George's activity and of $Y$, the level of Hazel's activity. Both persons have quasilinear utility functions, linear in income. George's utility function is of the form:

$$
\begin{equation*}
U_{G}\left(x, z_{G}\right)=G(x)+z_{G} \tag{6.1}
\end{equation*}
$$

Hazel's utility function is of the form:

$$
\begin{equation*}
U_{H}\left(x, y, z_{G}\right)=H(y)-S(x, y)+z_{H} \tag{6.2}
\end{equation*}
$$

The function $S(x, y)$ represents the amount of damage that George's activity causes to Hazel when George does $x$ units of $X$ ing and Hazel does $y$ units of $Y \mathrm{ing}$.

Assumption 1 The functions $G, H$, and $S$ have the following properties.

- George has a preferred $x^{*}$ such that $G^{\prime}(x)>0$ for $x<x^{*}$ and $G^{\prime}(x)<0$ for $x>x^{*}$. The function $G$ is concave and twice differentiable, so that $G^{\prime \prime}(x) \geq 0$ for all $x \geq 0$.
- For Hazel, there is also a preferred $y^{*}$ such that $H^{\prime}(y)>0$ for $y<y^{*}$ and $H^{\prime}(y)<0$ for $y>y^{*}$. The function $H$ is concave and twice differentiable, so that $H^{\prime \prime}(y) \geq 0$ for all $y \geq 0$.
- The function $S(x, y)$ is an increasing function of both variable $x$ and $y$ and for all $x$ and $y, S_{x y}(x, y)>0$ where $S_{x y}$ is the cross-partial derivative between $x$ and $y$.

[^1]Some day I think that I will expand these notes to apply most of Schweizer's discussion for Hazel and George. On the other hand, the problem set does almost the entire Schweitzer paper for specific utility functions. This together with reading Schweizer might be enough on this subject. I would be happy to have your opinions on this matter.

I will also probably briefly discuss accident law and torts. Diamond and Mirrlees, Calabresi, Shavell, are likely references. Maybe a bit of "value of life" and discussion of the mistake of confounding insurance and punishment.

## Exercises

6.1 George's utility function is

$$
U_{G}(x, y)=48 x-x^{2}+z_{G}
$$

and Hazel's utility function is

$$
U_{H}(x, y)=60 y-y^{2}-x y+z_{H}
$$

where $x$ is the amount of $X$ ing that George does and $y$ is the amount of $Y$ ing that Hazel does and where $z_{G}$ and $z_{H}$ are the amounts of bread that George and Hazel consume, respectively. When George does $x$ units of $X$ ing and Hazel does $y$ units of $Y$ ing, the amount of damage that George does to Hazel is equal to $x y$. George and Hazel each have fixed incomes $W_{G}$ and $W_{H}$. The set of feasible allocations consists of all vectors $\left(x, y, z_{G}, z_{H}\right) \geq 0$ such that $z_{G}+z_{H}=W_{G}+W_{H}$.
a). Find all of the Pareto optimal allocations in which George and Hazel each consume positive amounts of bread. How much $X$ is there and how much $Y$ ?
b). Suppose that George and Hazel are not able to communicate with each other and that there is no government interference either with Xing or with Ying. In consequence, the outcome is a (non-cooperative) Nash equilibrium in which the payoff functions are $U_{G}(x, y)=48 x-x^{2}+z_{G}$ for George and $U_{H}(x, y)=60 y-y^{2}-x y+z_{H}$ for Hazel. In equilibrium, how much $X$ ing is there and how much $Y$ ?
c). Suppose that there is strict legal liability, so that George has to pay Hazel for all damage that he does to her. Suppose also that transaction costs are so high that no deals are struck between George and Hazel. In equilibrium, how much $X$ is there and how much $Y$ ?
6.2 Suppose that George and Hazel of the previous problem are still unable to communicate or make deals between them. The government decides to impose a tax of $t x$ on George where $x$ is the amount of activity $X$ that he does. The government seeks a tax rate that will induce George and Hazel to perform Pareto optimal amounts of activities $X$ and $Y$.
a). If the government gives all of the tax revenue to Isolde, whom you haven't yet met, what tax rate $t$ must it use to induce George and Hazel to perform Pareto optimal amounts of activities $X$ and $Y$.
b). If the government gives all of the tax revenue to Hazel, what tax rate $t$ must it use to induce George and Hazel to perform Pareto optimal amounts of activities $X$ and $Y$.
c). If the government gives half of the tax revenue to Hazel and half to George, what tax rate $t$ must it use to induce George and Hazel to perform Pareto optimal amounts of activities $X$ and $Y$.
6.3 Suppose that the government introduces a market for the right to perform activity $X$. In this market, demand and supply curves are defined as follows: At any price $p, D(p)$ is the amount of $X$ ing that George would choose to do if each unit of $X$ cost him $p$. At price $p, S(p)$ is the amount of $X$ ing that Hazel would want George to do if she is paid $p$ for each unit of Xing that George does. Find the equilibrium price at which $D(p)=S(p)$ and the amount of $X$ ing and of $Y$ ing that would take place when the price of $X$ ing is set at the equilibrium level.
6.4 Suppose that George and Hazel of the previous problems are able to communicate cheaply and easily. Whenever there are possible gains from a deal between them, they will make a deal. The outcome of the deal always turns out to be the Nash bargaining solution (sometimes called the "cooperative" Nash solution). In any situation, they recognize a "threat point", which is the distribution of utility $\left(\bar{U}_{G}, \bar{U}_{H}\right)$ that they would achieve if they make no deal. The result of bargaining is that they find the point on the utility possibility frontier that maximizes the product $\left(U_{G}-\bar{U}_{G}\right)\left(U_{H}-\bar{U}_{H}\right)$ over all possible utility distributions in their utility possibility set. Prove that in doing so, Hazel and George will choose the point on the utility possibility frontier such that $U_{G}-\bar{U}_{G}=U_{H}-\bar{U}_{H}$. Would this be true regardless of the shape of the utility possibility frontier? Explain.
6.5 Suppose that George and Hazel from the previous problems have no transactions costs and always bargain to the Nash bargaining solution.
a). Assuming that there is no government interference, so that the threat point is the noncooperative Nash equilibrium, find the predicted amount of $X$ ing and of $Y$ ing and the predicted distribution of income and of utility between George and Hazel.
b). Assuming that the government imposes strict liability and that the threat point is the noncooperative Nash equilibrium that would obtain under these rules, find the predicted amount of $X$ ing and of $Y$ ing and the predicted distribution of income and of utility between George and Hazel.
6.6 Suppose that George and Hazel from the previous problems have no transactions costs and always bargain to the Nash bargaining solution. The government imposes the tax rate $t$ that induced George to perform a Pareto optimal amount of activity $X$ when George and Hazel were unable to bargain and the money was given to Isolde.
a). If the government gives the tax revenue to Isolde, what levels of activities $X$ and $Y$ will George and Hazel choose?
b). If the government splits the tax revenue between George and Hazel, what levels of activities $X$ and $Y$ will George and Hazel choose?
6.7 Hazel and George have one and only chance to make a deal. The chance works like this. Hazel offers George an all-or-nothing proposition. If George reduces his $X$ ing to a level $x^{*}$, Hazel will give him a lump sum payment of $z^{*}$ dollars. If George rejects this deal, then the outcome will revert to the noncooperative Nash equilibrium in which there is no government interference. After George has either accepted or rejected the deal that he is offered, Hazel can decide on the amount of $Y$ ing that she will do. Hazel knows George's utility function. Assuming that George will accept any deal that is as good or better for him than the Nash equilibrium outcome, what combination of $x^{*}$ and $z^{*}$ should Hazel make in order to maximize her utility?
6.8 This problem is like the previous one except for one thing. Hazel does not know George's utility function. Hazel knows that George's utility function is of the functional form: $U_{G}\left(x, z_{G}\right)=A x-x^{2}+z_{G}$, but all she knows about $A$ is that with probability $1 / 2, A=36$ and with probability $1 / 2, A=60$.
a). Hazel offers to give George a lump sum payment of of $z^{*}$ dollars if he reduces his amount of $X \operatorname{ing}$ to $x^{*}$. What choice of $z^{*}$ and $x^{*}$ would maximize Hazel's expected utility.

Hint: There are only two interesting strategies for Hazel. She might choose $x^{*}$ and $z^{*}$ so that George is sure to accept her offer, whether $A=60$ or $A=36$. Alternatively, she might choose $x^{*}$ and $z^{*}$ so that George will accept her offer if and only if $A=36$. For each of these two strategies there is a best choice of $x^{*}$ and $z^{*}$. Find these two solutions. Then compare Hazel's utility under each solution.
b). Show that for any offer that Hazel makes, there is a probability of at least $1 / 2$ that the amount of $X$ selected by George is not Pareto optimal.

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[^0]:    ${ }^{1}$ Zerbe graciously absolves Coase of responsibility for the Coase theorem by saying the

[^1]:    ${ }^{3}$ There has been some dispute among my students about the activities that constitute $X$ and $Y$. You may for example, prefer to think of activity $X$ as driving a noisy pickup truck with poor brakes and no muffler and activity $Y$ as riding a bicycle on the same road.

