Chapter 12

Benefit-Cost Analysis

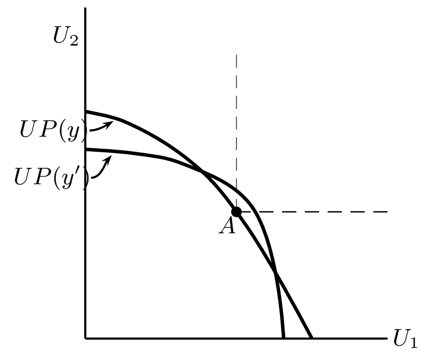
Utility Possibilities and Potential Pareto Improvement

Without explicit instructions about how to compare one person's benefits with the losses of another, we can not expect benefit-cost analysis to tell us whether a public project should or should not be adopted. The best we can hope for from benefit-cost analysis is to learn whether a project is *potentially* Pareto improving.

Consider a community with two selfish people, one private good and m public goods. An allocation is determined by the quantities of private good for persons 1 and 2 and the vector y of public goods that is available. The two people are endowed with a total of W units of private good, which is to be allocated between consumption for person 1, consumption for person 2, and inputs for the production of public good. Suppose that the amount of private goods needed to produce the vector y of public goods is $C(y)$. Then if the supply of public goods is y, the total amount of private goods to be divided between 1 and 2 is $W - C(y)$. Given the vector y of public goods, each feasible distribution of private goods determines a distribution of utilities between persons 1 and 2. We define the graph of such utility distributions to be the *y-contingent utility possibility frontier*. One such curve is shown as $UP(y)$ in Figure 12.1.

Suppose that a public project will increase the amounts of public goods from y to y' at a cost of some reduction in total private consumption. With the project in place, there is a new utility possibility frontier $UP(y')$. In Figure 12.1, neither of the two curves, $UP(y)$ and $UP(y')$ lies entirely beneath the other. Thus there is no unambiguous way to determine which is the better outcome. Some utility distributions are attainable only if the project is implemented and others are attainable only if it is not.

Figure 12.1: Utility Possibilities and benefit-cost



Let us suppose that initially the amount of public goods is y and that the distribution of private goods corresponds to the utility allocation marked A in the figure. We see that the curve $UP(y')$ includes points that are above and to the right of A. This implies that it is possible to change the supply of public goods from y to y' and still have enough private goods left over so that both individuals can be made better off than they were at A. When this is the case, we say that the project is potentially Pareto improving.

These ideas extend naturally to the case of more than two consumers. Where x_i is private consumption of consumer i, the set of all feasible allocations is $\{(x_1, \ldots, x_n, y) | \sum x_i + C(y) \leq W\}$, where $C(y)$ is the cost in terms of private goods of producing the vector y of public goods. The y-contingent utility possibility frontier in n dimensions is then defined in the obvious way.

Definition 1 *Suppose that the initial allocation of private and public goods is $A = (x_1, \ldots, x_n, y)$. A change in the amount of public goods from y to y' is potentially Pareto improving if there exists a feasible allocation $A' = (x'_1, \ldots, x'_n, y')$ such that A' is Pareto superior to A.*

A Benefit-cost Test

Where the initial allocation is $A = (x_1, \ldots, x_n, y)$, we define an individual's willingness-to-pay for changing the vector of public goods from y to y' as the quantity of private goods that she would be willing to sacrifice in return for this change. This number could be either positive or negative, depending on whether i prefers y' to y or *vice versa*.

Definition 2 *If the initial allocation is $(x, y) = (x_1, \ldots, x_n, y)$, then individual i's willingness to pay for changing the amount of public goods to y' is w_i where w_i solves the equation $U_i(x_i - w_i, y') = U_i(x_i, y)$.*

For an economy with selfish individuals, a simple benefit-cost test determines whether a change is potentially Pareto improving.

Theorem 1 *If individuals are selfish and the initial allocation is (x, y) where $\sum_i x_i + C(y) = W$, then a change in the amount of public goods from y to y' is potentially Pareto improving if and only if the sum of individual willingnesses-to-pay for the change exceeds the difference in cost $C(y') - C(y)$.*

A detailed proof of Theorem 1 is found in the Appendix. But the idea behind the proof is quite simple. If the sum of willingnesses-to-pay exceeds total cost, then it is possible to change the amount of public goods from y to y' and pay for this change by collecting an amount from each individual that is smaller than her willingness to pay. Doing so constitutes a Pareto improvement. Conversely, if the change is potentially Pareto improving, there must be a way to distribute the costs of the change so that nobody is worse off after the project is implemented and cost shares are assigned. By definition, this implies that each individual's share of the cost is smaller than his willingness-to-pay for the project. Since the cost shares add to the total cost of the project it follows that the project would pass the benefit-cost test.

A Calculus-Based Necessary Condition

Theorem 1 is very general in the sense that it does not depend on preferences being convex or continuous and it applies whether the change being considered is large or infinitesimal. But the weakness of this theorem is that it only gives us a way to evaluate projects one at at time. Applying the benefit-cost criterion in this theorem would requires a separate survey of consumers to determine the merits of every possible public project.

If we are willing to assume that preferences are smooth and convex, then calculus-based methods allow us to make more sweeping judgments about the direction of potential Pareto improvements, which depend simply on a comparison of marginal costs and marginal willingnesses to pay. The first order necessary condition for efficient provision of public goods was elucidated by Samuelson [7] and is known as the Samuelson condition. This condition requires that at an interior Pareto optimum, for each public good, the sum of all consumers' marginal rates of substitution between that public good and the private goods is equal to the marginal cost of that public good.

While the Samuelson condition can be used to determine whether an existing allocation is Pareto optimal, this is not exactly the result that is needed for benefit-cost analysis. The task of benefit-cost studies is to determine whether specific *changes* in the amount of public goods could be financed in a way that the outcome is Pareto improving. In a "convex environment," it turns out that a simple extension of Samuelson's result allows one to determine whether an increase or decrease in the amount of public goods is potentially Pareto improving.

Definition 3 *At a feasible allocation (x, y), we say that an increase in the amount of public good j passes (fails) the Samuelson test if at the allocation (x, y) the sum of marginal rates of substitution between public good j and the private good is greater than (less than) the marginal cost $C_j(y)$ of public good j.*

Under appropriate convexity assumptions, the Samuelson test gives us simple necessary conditions for an increase or for a decrease in the amount of a public good to be potentially Pareto improving. A proof of the following result is found in the Appendix.

Theorem 2 *If preferences of each individual are selfish and convex and if the cost function $C(y)$ is convex in y, then a necessary condition for an increase in the amount of public goods to be potentially Pareto improving is that an increase passes the Samuelson test. A necessary condition for a decrease in the amount of public goods to be potentially Pareto improving is that an increase fails the Samuelson test.*

Benefit-Cost as a First Step

An attractive feature of benefit-cost analysis is that it seems to allow analysts to make policy recommendations about specific public expenditures without taking a stand on questions of income distribution. This independence of distributional considerations is achieved by focussing on *potential* rather than actual Pareto improvements. Efforts to construct a useful benefit cost analysis in the absence of distributional judgments has a long history in economics. In 1939, Kaldor [5] and Hicks [4] articulated the view that has come to be known as the *New Welfare Economics*, and which centers on the "compensation principle." The compensation principle states that an institutional change constitutes an improvement if it is possible for the gainers to compensate the losers for their losses and still be better off after the change, *whether or not the redistribution actually takes place.*[1]

The modern consensus is that the case for distribution-independent project evaluation is much weaker than was originally hoped. Samuelson [6] demonstrated that if the utility possibility sets corresponding to alternative policies are not nested, then the social orderings implied by these criteria are highly unsatisfactory. Chipman and Moore point out that in economies with several private goods and no public goods, nesting of the utility possibility sets requires essentially that preferences be identical and homothetic, or with some further qualifications, of the Gorman polar form. Bergstrom and Cornes [1] show that for an economy with public goods, utility possibility sets corresponding to amounts of public goods will be nested only under special circumstances that are formally dual to the Gorman polar form.

It is noteworthy that Hicks' founding manifesto of the New Welfare Economics [4], does not appear to advocate acceptance of reforms that pass the compensation test if compensation is not actually paid. Hicks states that

> "The main practical advantage of our line of approach is that it fixes attention on compensation. Every simple economic reform inflicts a loss on some people... Yet when such reforms have been carried through in historical fact, the advance has usually been made amid the clash of opposing interests, so that compensation has not been given, and economic progress has accumulated a roll of victims, sufficient to give all sound policy a bad name." [4], p. 711

Much of the controversy surrounding the use of the criterion of "potential Pareto improvement" seems to be avoidable if we think of benefit-cost analysis as only a first step in a project evaluation. For example, if an increase in the amount of a public good passes the marginal Samuelson test, then we know that *some* increase in the amount of the public good is potentially Pareto improving. Of course this does not mean that every possible way of implementing the project is Pareto improving. All we know is that there would be some way of increasing the amount of this public good and dividing costs so that everyone benefits. The next step in evaluation of the project is to consider alternative ways of financing this project and estimating who will then be the winners and the losers. Incentive problems will

[1] An elegant and enlightening intellectual history of the compensation principle and the New Welfare Economics is found in Chipman [3]. Chipman and Moore [2] present a rigorous treatment of these issues, using modern techniques.

normally prevent policy makers from knowing exactly how much each individual values the project and so estimates of the distribution of winners and losers will be statistical and not exact. It is usually not reasonable to expect that literally everyone will be better off after the change, but may be possible to use available information to ensure that a very large fraction of the population benefits from the policy and that very few individuals are significantly harmed.

There is an interesting asymmetry in the results of a marginal benefit-cost tests. If an increase in the amount of a public good *fails* the test, then it must be that there is *no* way to divide the costs of the project in such a way to achieve a Pareto improvement. The project can reasonably be described as "special interest legislation". To make a case in favor of a project that fails the test, one would need to argue that implementing this project and paying for it with a specified tax scheme is likely to achieve redistributive goals that for some reason could not be more efficiently achieved through redistribution of private goods.

Appendix

Proof of Theorem 1

Let (x, y) be the initial allocation and let w_i be i's willingness to pay for the change from y to y'. Suppose that the sum of willingnesses to pay exceeds $C(y') - C(y)$. Then there exists $\epsilon > 0$ such that $\sum_i (w_i - \epsilon) > C(y') - C(y)$. For each i, let $x'_i = x_i - (w_i - \epsilon)$. From the definition of w_i, it follows that for each i, $U_i(x'_i, y') > U_i(x, y)$. Now

$$\sum_i x'_i = \sum_i x_i - \sum_i (w_i - \epsilon) < \sum_i x_i - \big(C(y') - C(y)\big) \qquad (12.1)$$

Rearranging terms in 12.1, we have

$$\sum_i x'_i + C(y') < \sum_i x_i + C(y) \le W \qquad (12.2)$$

Expression 12.2 implies that that the allocation (x', y') is feasible. Therefore if the sum of willingnesses to pay for the movement from y to y' exceeds $C(y') - C(y)$, then the change from y to y' is potentially Pareto improving.

Conversely, suppose that the change from y to y' is potentially Pareto improving. Then there exists a feasible allocation (x', y') such that $U_i(x'_i, y') \ge U_i(x_i, y)$ for all i with strict inequality for some i. From the definition of

i's willingness to pay, w_i, it follows that $x_i - x'_i \leq w_i$ for all i with strict inequality for some i Therefore

$$\sum_i w_i > \sum_i (x_i - x'_i) \tag{12.3}$$

Since $\sum x_i + C(y) = W$ and since feasibility of (x', y') implies that $\sum x'_i + C(y') \leq W$, it must be that

$$\sum_i (x_i - x'_i) \geq C(y') - C(y) \tag{12.4}$$

Then from Expressions 12.3 and 12.4 it follows that $\sum w_i > C(y') - C(y)$.

Bibliography

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