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Lecture 9

Excludable Public Goods

The standard definition of public goods requires that consumption of these goods is *non-rival* in the sense that more than one person can simultaneously "consume" the good at the same time, without diminishing the other's enjoyment of that good. Even if a good is non-rival, it may be technically possible to exclude people from access to it. Cable television is a good example of such a good. If a good is non-rival, then excluding anyone from access to the entire existing stock is wasteful in a fairly obvious sense. On the other hand, as we have seen, it is not always easy to find efficient ways to determine people's preferences for public goods if they are supplied publicly at a price of zero. Thus it is interesting both from a positive and from a normative viewpoint to consider markets in which prices are charged for access to excludable public goods.

The Oakland Model of Competitive Supply

The best model of "competitive" supply of excludable public goods that I know of is that of William Oakland, published in the 1974 *Journal of Political Economy*. Oakland assumes that an excludable public good can be produced at constant marginal cost. The "competitive" part comes from his assumption that there is free entry into the industry in the sense that any firm can produce a perfect substitute for the units of the public good that are produced by others and can do so at the same constant marginal cost. The assumptions of constant marginal cost and free entry imply that in equilibrium, all firms in the industry will be making zero profits.

The idea of Oakland's competitive equilibrium is illustrated by the following fable.

A fable

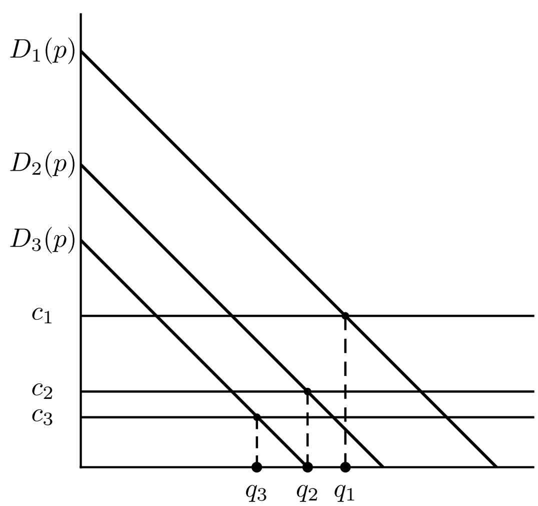
In the small country of Couch Potato, people love to watch soap operas on cable television and eat hot dogs. They consume nothing else. Their currency is pegged to the hot dog, so that the price of a hot dog is 1. Television programming is financed entirely by paid subscriptions from viewers. The cost of producing soap operas is c per minute of program. There is free entry into the soap opera production industry and because of this, soap opera producers make zero profits in equilibrium.

Couch Potato has three kinds of people, which we will call Types 1, 2, and 3. There are 1000 people of each type. People of type i have quasi-linear utility functions of the form

$$U_i(x_i, y_i) = x_i + A_i y_i - \frac{y_i^2}{2} \tag{9.1}$$

where x_i is the number of units of other $A_1 > A_2 > A_3$. With this utility function, we see that if a type i could buy access to soap operas at the price p per minute, she would purchase $q = A_i - p$ minutes of programming. The total amount demanded by type is at price p is therefore $D_i(p) = 1000(A_i - p)$ Figure 9.1 shows demand curves for each of the three types.

Figure 9.1: Soap Opera in Couch Potato



If all 3000 consumers purchase access to a soap opera, then the supplier could recover its costs by charging each subscriber $c/3000$ per minute. In

Figure 9.1, we have drawn a horizontal line at $c_3 = c/3000$. At this price, the type 3s, who have the lowest demand curve would want to purchase q_3 units, while the type 1s and type 2s would want to purchase more than that. But in order to be able to sell minutes of soap opera at a price of $c/3000$, suppliers must be able to sell to access for these minutes to all three types. Thus in equilibrium, suppliers will supply only q_3 units at price $c/3000$.

Suppose that some suppliers offer to sell additional units of soap opera at a price of $c_2 = c/2000$ in hopes that these units will attract demand from the type 1s and 2s. We see from Figure 9.1 that if they could buy only at price c_2 then the type 2s would want to buy q_2 units. In fact, type 2s can buy q_1 units at the lower price c_3. Since we have assumed quasi-linear utility, there are no income effects on demand and so the type 2s would buy the q_3 units that are available at price c_3 and would buy the remaining $q_2 - q_3$ units at the higher price c_2. Thus in equilibrium, suppliers would supply $q_2 - q_1$ units at price $c_2 = c/2000$.

There remains a possibility for producers to produce additional minutes of soap opera which would be consumed only by the highest demanders. These would have to be sold at a price of $c_1 = c/1000$. From Figure 1, we see that at this price, type 1s would demand q_3 units in total. Since they can purchase a total of q_2 units at prices lower than c_1, the number of units they will buy at price c_1 is given by $q_1 - q_2$ in Figure 9.1

This example is a special case of Oakland's model. Oakland describes equilibrium for a model in which there are many types, whose demand curves may possibly cross each other, and where there may be income effects on demand.

Characteristics of Oakland Equilibrium

Oakland asks us to notice a number of unusual features of equilibrium in this market. These include the following:

1. Despite the fact that all firms produce perfect substitutes for each others' output, there is no "law of one price". Some units are sold at higher prices than others.

2. In equilibrium there is "excess capacity" in the sense that some of the units produced are not made available to some consumers, even though they have positive value for these units and even though the marginal cost of extending access to these consumers is zero.

3. Firms are neither price takers nor price setters. They do not offer to meet all demand at a specified price, nor do they choose a quantity and sell it at what the market will bear. Instead they offer specific price-quantity combinations.

4. Firms know nothing about the demands of individual consumers and are unable to practice price discrimination between different consumers.

5. In equilibrium, despite the different prices paid for different units, there are no possibilities for profitable arbitrage even if the goods are freely transferable.

6. The equilibrium outcome has both underconsumption and underproduction of the public good relative to full Pareto efficiency.

Some Thoughts on the Oakland Model

(In progress) All of the examples of excludable public goods that I can think of are ones in which separate units are not perfect substitutes for each other, at least in the eyes of interested consumers. This includes television programs, novels, movies, music performances, and academic journals. When one reads two novels, one is not interested in reading two separate books with identical text. Same goes for the other products.

Copyright laws prevent one seller from marketing a perfect copy of the other producer's product. In fact, people who will pay a positive amount one copy are likely to be unwilling to pay anything for a second identical copy. Readers of detective stories will want to read different books and possibly different authors. Music lovers will want CD's of different pieces of music, though perhaps of the same type. In some applications, for example academic journals, I suspect that different journals in the same field are complements rather than substitutes.

These issues deserve more modelling.

Monopoly models

The Brennan-Walsh Model

This model was introduced by Geoffrey Brennan and Cliff Walsh in the American Economic Review (1981).

A public good is produced by a monopolist who is able to exclude potential users from consuming any unit unless they pay the "price" that the monopolist charges for that unit. The monopolist is unable to tell his consumers apart and is not able to price discriminate. The monopolist can choose both a price to charge and a quantity to produce. Consumers of type z have demand functions for the public good that take the form $q(z,p)$ where p is the price that demanders must pay for the public good. We assume that demand functions are continuous in q and z and that curves of different types of people never "cross each other." That is, for all p, if $z' > z$, then $q(z',p) > q(z,p)$. There are a total of N consumers. Let us index consumers in such a way that the fraction z of all consumers are of type less than or equal to z. Thus z ranges over the interval $[0,1]$.

Let us assume that the monopolist has a fixed cost C_0 and constant marginal cost c, so that his total costs of producing Q units of the public good is $C(Q) = C_0 + cQ$. Since the public good is "non-rival", each unit of the public good could be sold to every consumer in the economy. If the monopolist produces Q units and sells at a price p, then consumers of type z where $q(z,p) < Q$ will purchase $q(z,p)$ units, while consumers of type z where $q(z,p) > Q$ will only be able to purchase Q units.

The monopolist will never choose to produce a price-quantity pair (p,Q) such that $q(z,p) < Q$ for all $z \in [0,1]$, nor would it choose (p,Q) such that $q(z,p) > Q$ for all $z \in [0,1]$. (In the former case, he could increase his profits by reducing Q, in the latter case by increasing p.) Therefore for any profit-maximizing choice, $q(s,p) = Q$ for some type $s \in [0,1]$. It follows that the monopolist's decision can be described as a choice of a price p, and a marginal consumer type s, such that the quantity supplied is $Q = q(s,p)$.

Define $R(z,p) = Npq(z,p)$. The monopolist's profits can be written as a function

$$\Pi(s,p) = \int_0^s R(z,p)dz + (1-s)R(s,p) - C(q(s,p)). \tag{9.2}$$

Let $q_1(z,p)$ and $q_2(z,p)$ denote the partial derivatives of q(z,p) with respect to its first and second arguments and let $R_2(z,p)$ be the partial derivative of $R(z,p)$ with respect to price. The first order conditions for profit maximization are found by differentiating with respect to p and s respectively. Setting the derivative of Equation 9.2 equal to 0, we have:

$$\int_0^s R_2(z,p)dz + ((1-s)R_2(s,p) - c))\, q_2(s,p) = 0 \tag{9.3}$$

Setting the derivative of 9.2 with respect to s equal to zero and simplifying, we find

$$(1-s)R_1(s,p) - cq_1(s,p) = [(1-s)Np - c]q_1(s,p) = 0. \tag{9.4}$$

From Equation 9.4, we find that for a profit-maximizing monopolist, price and marginal cost are related by the simple markup formula

$$pN = \frac{c}{(1-s)}. \tag{9.5}$$

This relation should not be surprising. The gain to a monopolist who is charging price p from producing an extra unit of public good is that he will be able to sell the extra unit to the fraction $1-s$ of the population at price p. Thus the marginal revenue from producing an extra unit (while holding price constant) is $pN(1-s)$. He will therefore gain from producing an extra unit so long as $pN(1-s) > c$.

Equation 9.3 can be simplified by substituting from Equation 9.5. We have

$$\int_0^s R_2(z,p)dz + (1-s)[R_2(s,p) - Npq_2(s,p)] = 0 \tag{9.6}$$

But $R_2(s,p) = Nq(s,p) + Npq_2(s,p)$. Therefore, Equation 9.6 simplifies to

$$-\int_0^s R_2(z,p)dz = (1-s)Nq(s,p). \tag{9.7}$$

The profit maximizing conditions are therefore equivalent to the two relatively simple Equations, 9.5 and 9.7. What can we make of these conditions? Let us monkey with some special functional forms, get some explicit solutions and try to interpret the result. It may be helpful to express things in terms of elasticities and then take a look at the special case of constant elasticity. Applying the standard definition of price elasticity to the demand of a type z, we have

$$\xi(z,p) = \frac{p}{q(z,p)}q_2(z,p). \tag{9.8}$$

Then

$$R_2(z,p) = q(z,p) + pq_2(z,p) = q(z,p)\left(1 + \xi(z,p)\right) \tag{9.9}$$

Therefore Equation 9.7 is equivalent to

$$-\int_0^s (1 + \xi(z,p))q(z,p)dz = (1-s)q(s,p). \tag{9.10}$$

Consider the special case where $q(z,p) = f(z)p^{-\xi}$. Then there is a constant price elasticity elasticity of demand $\xi(z,p) = \xi$ for all types z. Equation 9.10 simplifies to

$$-(1+\xi)\int_0^s q(z,p)dz = (1-s)q(s,p). \tag{9.11}$$

Dividing both sides of this equation by $p^{-\xi}$, we have

$$-(1+\xi)\int_0^s f(z)dz = (1-s)f(s).. \tag{9.12}$$

We can go further toward finding a solution if we assume that the function $f(z)$ takes a convenient functional form. Let us assume that $f(z) = \alpha z^\beta$ for constants, $\alpha > 0$ and $\beta > 0$. Then Equation 9.12 is $\alpha(\xi-1)s^{\beta+1}/(\beta+1) = \alpha s^\beta(1-s)$. This simplifies to $(\xi-1)/(\beta+1) = (1-s)/s$, which has a unique solution, $s = (\beta+1)/(\xi+\beta)$.

Having found a solution $\bar{s}$ to Equation 9.12, we can return to Equation 9.4 to find the profit maximizing price, $\bar{p}$. According to Equation 9.5, we have $\bar{p} = c/N(1-\bar{s})$. The monopolist will produce $\bar{Q} = f(\bar{s})\bar{p}^{-\xi}$.

What comparative statics results can we find for this model? It would be good to have some applications in mind so that we can decide what are the interesting questions to investigate. We could compare this solution with various alternative solutions, like oligopoly, competition, regulated monopoly and so on.

Perhaps one should try some other simple special functional forms for demand.

Exercises

9.1

A city has 2 types of people, and 1000 people of each type. There is one private good and one public good. Let X_i denote the amount of private consumption consumed by citizen i and let Y denote the amount of public good available in the city. All type 1's have the utility function $U(X_i, Y) = X_iY$, type 2's have the utility function $U(X_i, Y) = X_iY^2$. The price of private goods is \$1 per unit. Type 1's have an income of \$10,000 and Type 2's have an income of $\$15,000$. Public goods can be made from private goods with constant returns to scale. It takes 30 units of the private good to make one unit of the public good. The following questions relate to alternative arrangements for provision of public goods in this city.

a). Calculate the Lindahl equilibrium prices and quantities for this city.

b). Suppose that the public good is excludable and marketed competitively as in the Oakland (1974) model. In the Oakland competitive equilibrium with free entry for firms, how many units will be consumed by the type 1's? the type 2's? What will be the total number of units produced? What will the competitive prices be? How many units of the public good will the low price seller sell? How much will the high price seller sell.