The provision point mechanism with reward money*

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Abstract

We modify the provision point mechanism by introducing reward money, which is distributed among the contributors in proportion to their contributions only when the provision point is not reached. In equilibrium, the provision point is always reached as competition for reward money and preference for the public good induce sufficient contributions. In environments without aggregate uncertainty, the mechanism not only ensures allocative efficiency but also distributional. At a specific level of reward money, there is a unique equilibrium, where all consumers contribute the same proportion of their private valuations. The advantages of the mechanism are also demonstrated for collective action problems.

Keywords: Public goods; private provision; provision point mechanism; aggregative game; distributional efficiency; collective action problem *JEL codes*: C72, D82, H41.

1 Introduction

In this paper, we propose a new mechanism for the provision of threshold public goods. It is an extension of the provision point mechanism with refunds by an additional clause. The clause specifies a sum of reward money to be distributed among the contributors in proportion to their contributions if the sum of contributions is below the provision point.

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Hence, in the event of insufficient contributions each contributor gets his contribution refunded and, additionally, a share of the promised reward money. But the distribution of reward money never occurs in equilibrium. When provision is sufficiently desirable by agents, competition for reward money and preference for the public good induce contributions up to the level where the provision point is reached. Importantly, this mechanism not only resolves the free-riding problem but can also implement the public good in the unique Nash equilibrium.

In equilibria, obtained under the proposed mechanism, every consumer receives a payoff from the public good at least as high as that from the share of the reward money assigned to him if he deviates. Therefore, the effect of the introduction of reward money is a reduction of the set of individually rational strategies that can be supported in equilibrium. A higher level of reward money implies a smaller set of equilibrium strategies. In environments without aggregate uncertainty about the total value of the public good, the mechanism can uniquely implement the project with reward money set at the net value of the public good. In environments with aggregate uncertainty, the mechanism also results in the only equilibrium outcome of provision without dispensing reward money as long as the public good is sufficiently efficient, but otherwise there is no equilibrium.

The aforementioned unique implementation achieved with the reward money set at the net value of the public good has a special feature. Every consumer contributes the same proportion of his valuation, where the proportion is equal to the ratio of the cost of the public good and its total value. Therefore, the mechanism ensures not only allocative but also distributional efficiency. Taken from a different perspective, the mechanism effectively levies a Lindahl tax and can be expressed as a demand to pay a proportional tax on the private valuation for the public good. The reward money ensures that consumers have the right incentives to reveal their privately known valuations truthfully.

The problem of underprovision and, relatedly, of free riding arises when externalities are not internalized. Reward money can be viewed as a device to compensate consumers for externalities they create. For the same reason, we argue that the suggested mechanism can prove useful in other situations where the problem that externalities are not internalized arises. Specifically, we demonstrate this on a collective action problem, where participation in a project is individually rational only when a critical mass of participants is reached. Reward money effectively eliminates undesirable equilibria leaving only the efficient one, which, by design, does not lead to the distribution of reward money. Moreover, the mechanism implements the efficient outcome in weakly dominant strategies. In the case of negative externalities, e.g., the problem of the commons, the mechanism fails to achieve the efficient outcome without distributing reward money. The reason is that the efficient outcome is not individually rational with negative externalities unlike in the case with positive externalities.

Generally, the idea behind our mechanism relates to the augmented revelation principle of Mookherjee and Reichelstein (1990) (also see Ma et al. (1988)). They show that the revelation principle augmented with specially designed transfer payments eliminates the undesirable equilibria produced by the direct mechanism. At the same time, as in our mechanism, transfer payments are never paid in equilibrium. Taken from this more general perspective, our mechanism when equivalently reformulated as a direct mechanism can be seen as a practically applicable example of the augmented revelation principle.

The literature on the private provision of public goods and, specifically, on the provision point mechanism is immense to be discussed in any greater detail here. The provision point mechanism with refunds has a practical appeal as it is simple for implementation. It was successfully applied by Benjamin Franklin in the 18th century and is presently used in online donation platforms. This mechanism is formally introduced and analyzed in Bagnoli and Lipman (1989) (see Palfrey and Rosenthal (1984) for a discrete version). They show that under complete information it uniquely implements the efficient outcome in undominated perfect equilibrium, but it certainly gives rise to a multiplicity of Nash equilibria including inefficient ones. Experimental studies reveal that this mechanism implements the public good in about 50 percent of cases (Isaac et al. (1989), Cadsby and Maynes (1999), Marks and Croson (1999)) but the problem of free riding is sizable (see Ledyard (1995) and Chen (2008) for reviews). In the field, the implementation rate is much lower (Rose et al. (2002)). With the introduction of seed money, i.e., significant first-move donations, the efficiency of the provision point mechanism improves (List and Lucking-Reiley (2002)), but a multiplicity of equilibria, including free riding, is still a problem (Andreoni (1998)). Attempts are made to improve the performance of the provision point mechanism by introducing different rebate rules of contributions exceeding the provision point such as proportional rebate, winner-takes-all, etc. For experimental evidence, see Marks and Croson (1998), Rondeau et al. (1999), Spencer et al. (2009), who show improvements in allocative efficiency, but there are concerns regarding distributional efficiency. All this calls for further effort on improving the provision point mechanism.

The idea to reward the contributors in the event of insufficient contributions is not new. Tabarrok (1998) applies it to the problem where agents have a binary choice of making or not making a pre-determined contribution toward the public good which is then provided conditional on a sufficient number of contributors. He proposes an "assurance" contract that implements the public good project in dominant strategies.[1] The contract specifies a reward that each contributor receives in case the number of contributors misses the target needed for implementation. With such a reward, like in the present paper, the mechanism designer can effectively and at no cost eliminate inefficient outcomes. The present paper extends this idea by allowing agents to contribute any amount toward the public good and also applies it to other problems with positive externalities. But the main advantage of our mechanism with continuous contributions lies in its superior properties of distributional efficiency, as it can induce agents to make contributions in proportion to their valuation for the public good. For a similar reason, our mechanism can also be applied in environments with very uneven distributions of private valuations, where fixed contributions may be restrictive in raising sufficient funds.

Generally, this paper belongs to the strand of literature on public good games with rewards to contributors. Falkinger (1996) proposes a mechanism that rewards contributors with above-average contributions. Morgan (2000) studies the mechanism that induces contributions with the help of lotteries. Goeree et al. (2005) demonstrate the advantages

[1] I thank Ted Bergstrom for bringing my attention to this work.

of the all-pay auction design in soliciting contributions. For experimental evidence on the performance of these mechanisms, see Falkinger et al. (2000), Morgan and Sefton (2000), Lange et al. (2007), and Corazzini (2010), who all report improved allocative efficiency. However, distributional efficiency may be failed. In the case of the lottery mechanism, it can happen (and it is empirically supported, see Kearney (2005)) that it is poorer people who end up financing the public good, i.e., lotteries are regressive. Morgan (2000) points out that adverse distributional effects may override allocative gains, leaving this problem open. The same applies to the mechanisms of Falkinger (1996) and Goeree et al. (2005). Lastly, the mechanisms that reward contributors, discussed above, lead to the distribution of promised rewards in equilibrium, which is not the case with the mechanism proposed in the current paper.

The remainder of the paper is organized as follows. After introducing a set-up in Section 2, we study the performance of the mechanism in two different environments: (i) without aggregate uncertainty (Section 3) and (ii) with aggregate uncertainty (Section 4). Section 5 deals with an application of the proposed mechanism to a collective action problem. The last section concludes the study.

2 Set-up

There is an economy that consists of a set $N = \{1, ..., n\}$ of consumers with quasi-linear utility functions

$$U_i = w_i + v_i h(C). \tag{1}$$

In (1), w_i denotes the wealth of consumer i in the numeraire good, and $v_i h(C)$ denotes his utility from the public good in the amount of C provided. The public good cannot be provided in amounts below a threshold of $C_{\min} > 0$. The function $h(C)$ is strictly increasing and concave, $h'(.) > 0$ and $h''(.) < 0$. Privately known valuation v_i takes values in a compact set $\mathcal{V} \subset \mathbb{R}^+$; a vector of valuations $\mathbf{v} = (v_1, ..., v_n)$ is drawn from a distribution $F(.)$ with the support $\mathcal{V}^n$. The public good is provided by transforming the numeraire good into C on a one-for-one basis. We assume that the marginal utility of the public good, $v_i h'(C)$, is smaller than 1 for every i so that no consumer finds it individually beneficial to increase the amount of the public good already provided. Throughout the paper, wealth constraints are assumed to be non-binding for all consumers.

In the economy, there is a public authority that seeks to implement the public good project of a given size C. The authority has a capacity to raise a budget of at most B, which is, however, insufficient to provide the public good in any amount, i.e., $B < C_{\min}$. To raise the required funds C, the authority turns to the public with the following mechanism. Consumers are asked to make voluntary contributions toward the public good. Let $\mathbf{g} = (g_1, ..., g_n) \in \mathbb{R}_+^n$ denote a profile of their contributions and G the sum of contributions. If $G \geq C$, the public good is financed out of the contributions collected, with the excess amount $G - C$ wasted (assumed for the ease of exposition). If $G < C$, the public good is not provided, the contributions are refunded, and the authority distributes an ex-ante promised level of reward money R among the consumers in proportion to their contributions.

For brevity, we label the mechanism by its amount of reward money, R. Until further notice, we shall ignore the constraint that $R \leq B$ and assume that the authority can credibly promise any level of reward money R. For ease of exposition, we normalize $h(C) = 1$ so that v_i denotes consumer i's willingness to pay for the public good C. The payoff to consumer i under a mechanism R is given by

$$\pi_i(g_i, G) = \begin{cases} \mathcal{I}(G \geq C)\left[v_i - g_i\right] + \mathcal{I}(G < C)\left[\frac{g_i}{G}R\right] & \text{if} \quad G > 0 \\ 0 & \text{if} \quad G = 0, \end{cases} \tag{2}$$

where $\mathcal{I}(.)$ is an index function.

The game induced by a mechanism R is aggregative: A consumer's payoff depends on own contribution and the sum of all contributions. We apply techniques developed to study aggregative games which simplify our analysis greatly.[2] In the next section, we analyze the mechanism in environments without uncertainty about the total value of the public good. Specifically, letting $\overline{V}(\mathbf{v})$ denote the sum of the components of the vector $\mathbf{v}$,

[2] See Jensen (2010), Martimort and Stole (2011), Martimort and Stole (2012). For an application toward public good games, see Cornes and Hartley (2007).

we impose on the support of $F(.)$ a property such that $\overline{V}(\mathbf{v})$ is the same for each $\mathbf{v}$. Later, we study the problem with aggregate uncertainty, in doing which we also generalize our results obtained for the case without aggregate uncertainty.

3 No Aggregate Uncertainty

Here, we study the situation when the total value of the public good project can be estimated without knowing the exact valuations of consumers. When the number of consumers is large then, by the law of large numbers, the total value of the public good can be approximated by $\overline{V}(\mathbf{v}^e)$, where $\mathbf{v}^e$ is the expected vector of valuations under the distribution $F(.)$. But even without reversion to the law of large numbers, one can think of situations where the total value of the project is known. For instance, the total value of, e.g., a park, can be inferred from expected price changes of the nearby property.[3] The rest of the incomplete-information framework is retained.

Assumption 1 $\overline{V}(\mathbf{v}) = V$ *for each* $\mathbf{v}$ *and is publicly known.*

We assume that consumers choose contributions (without randomizing) that constitute a Nash equilibrium of the game induced by a mechanism R. Letting G_{-i} denote the sum of all contributions of consumers other than i, we define

Definition 1 *A profile of contributions* $\mathbf{g}$ *is a Nash equilibrium if* g_i *maximizes* π_i *given* G_{-i} *for each* $i \in N$.

Next, given Assumption 1, we characterize the set of pure-strategy Nash equilibria, denoted by $\Gamma(R)$, under a mechanism $R > 0$. In the proof, we rely on the aggregative structure of the game, which allows us to study it as a two-player game between an individual consumer and the "aggregate" of other consumers.

Proposition 1 *If* $0 < R \leq V - C$, *then* $\Gamma(R) = \left\{\mathbf{g} : \forall i, g_i \leq \frac{C}{R+C} v_i, \sum_j g_j = C\right\}$. *If* $R > V - C$, *then* $\Gamma(R) = \{\emptyset\}$. *If* $R' > R$, *then* $\Gamma(R') \subseteq \Gamma(R)$.

[3]The assumption of the known sum of private characteristics is not uncommon in economic modeling. E.g., Bergstrom and Varian (1985) present a general result; Marks and Croson (1999) discuss it in relationship to the performance of the provision point mechanism.

Proof. When $R > 0$, there is no equilibrium $\mathbf{g}$ such that $\Sigma_j g_j < C$ as any consumer could obtain a larger share of the reward money R by marginally increasing his contribution. Similarly, $\Sigma_j g_j > C$ cannot hold in equilibrium as any consumer could gain in utility by marginally decreasing his contribution. Thus, the equilibrium candidates need to have $\Sigma_j g_j = C$. For every $i \in N$, let $\widetilde{G}_{-i}$ denote the total amount consumer i believes to be contributed by others. Consumer i contributes $g_i = \max\{0, C - \widetilde{G}_{-i}\}$ to have the public good provided if

$$\widetilde{G}_{-i} \geq C - \frac{C}{R+C} v_i. \tag{3}$$

This condition follows from the individual rationality condition $v_i - g_i \geq \frac{g_i}{C} R$, where the right-hand side is the upper-bound utility of the consumer when he contributes marginally less than needed to have the public good provided. The largest individually rational contribution leading to the provision of the public good is given for every i by

$$g_i \leq \frac{C}{R+C} v_i. \tag{4}$$

Summing up (4) we see that the public good can be provided in equilibrium only if $C + R \leq V$. Next, we check the consistency of beliefs in (3). From (4) we see that it is rational to expect $\widetilde{G}_{-i} \leq \frac{C}{R+C} \Sigma_{j \neq i} v_j = \frac{C}{R+C}(V - v_i)$. It immediately follows that beliefs in (3) are consistent also if $C + R \leq V$. Thus, if $R \leq V - C$, then $\Gamma(R) = \left\{ \mathbf{g} : \forall i, g_i \leq \frac{C}{R+C} v_i, \sum_j g_j = C \right\}$. But if $R > V - C$ then $\Gamma(R) = \{\emptyset\}$ because the largest amount raised in the individually rational way is lower than C. The last part of the proposition follows from the observation that if (4) holds for R' it also holds for $R < R'$. But the reverse is not true. Hence, $\Gamma(R') \subseteq \Gamma(R)$. ■

With a promise to reward the contributors in the event the provision point is not reached, the mechanism actually induces a sufficient amount of contributions for the public good to be provided. The reason is that when $R \leq V - C$ there is always a consumer willing to increase his contribution to have either a larger share of the reward money or the public good provided. From a different perspective, in the game induced by the mechanism the consumers need to decide which "prize" to divide, the reward money

R or the net utility of the public good $V - C$, and they choose whichever is bigger. Therefore, if $V - C \geq R$, then they choose the public good and otherwise the reward money. In the latter scenario, there is no equilibrium because the set of contributions that sum to less than R is not compact. (With a discrete contribution space, $R > V - C$ would result in the equilibrium outcome $\Sigma_j g_j = C - \delta$, where δ is the smallest currency unit.)

In equilibrium, each consumer needs to obtain a utility level from the public good at least as high as that obtained from the share of the reward money the consumer is entitled to if he deviates. A higher level of reward money implies a more profitable deviation and, thus, a higher level of utility for each consumer in equilibrium, which reduces the set of equilibria. But if the promised amount is too generous, it makes consumers seek utility from the reward money rather than from the public good. Interestingly, when reward money is set at the net value, $R = V - C$, the mechanism implements the public good in the unique equilibrium.[4] This equilibrium has a special feature that all consumers contribute the same proportion of their private valuations, $g_i = \frac{C}{V} v_i$. Hence, the ratio C/V can be interpreted as a voluntary Lindahl tax, levied on consumers' private valuations for the public good. Thus, the mechanism with $R = V - C$ achieves distributional efficiency.

Discussion

Even though reward money is never distributed in equilibrium, the capacity of raising it needs to be credible. There are several possible sources of reward money, the simplest of which is seed money generated from individual donors. If the source of reward money is the budget of the authority, then it has to be that $R \leq B$, restricting the set of feasible mechanisms. And if unique implementation is a desirable property, then this constraint on reward money can be binding for public good projects of large size. Under this circumstance, the public authority would have to either reduce the amount of the public

[4] Formally, the uniqueness result can be explained by the fact that at the equilibrium point of provision the payoff function π of each consumer is continuous and concave in own contribution. At the same time, we have a multiplicity of equilibria with $R < V - C$ because of the discontinuity of payoff functions π_i at the point of provision, but these functions are upper semicontinuous. But if $R > V - C$, then at the point of provision the upper semi-continuity of at least one payoff function π_i is violated, which is behind the non-existence of equilibria.

good sought in order to preserve uniqueness or proceed without unique implementation.

If the authority raises its budget through taxes imposed on consumers, then there is also a question when the promised reward money needs to be raised: before the announcement of the fund-raising campaign or after it. It is important because the timing of taxation can have an effect on consumer payoffs in (2). To illustrate our argument, suppose that the authority can levy a lump-sum tax $\tau = R/N$ from every consumer. Because of quasi-linear preferences, ex-ante taxation has no effect on consumer preferences for the public good. Therefore, Proposition 1 continues to hold in its entirety.[5]

Ex-post taxation, however, implies a change in consumer payoffs in (2) as the second term becomes $\mathcal{I}(G < C)\left[\frac{g_i}{G}R - \tau\right]$. Analogously to (4), the individually rational contribution has to satisfy

$$g_i \leq \frac{C}{R+C}(v_i + \tau). \tag{5}$$

Thus, the upper bound on individually rational contributions increases with ex-post taxation. The reason is that, when the provision point is reached, the consumer also avoids paying the tax, making his gain from the public good $v_i + \tau$ rather than v_i. But for the same reason, however, with ex-post taxation we can obtain equilibria with contributions larger than valuations. Consumers with $v_i < \frac{C}{N}$ may pledge contributions above their valuations to increase the likelihood of reaching the provision point done to avoid the tax. Finally, with ex-post taxation the existence of equilibria is independent of the condition $R \leq V - C$ as potential gains from reward money are diminished by taxes. The outcome of the unique equilibrium is, accordingly, not preserved. Despite these disadvantages, ex-post taxation has an advantage that it is reverted to only in the non-equilibrium event of the distribution of reward money, whereas with ex-ante taxation the cost of raising taxes needs to be incurred immediately.

It is also worthwhile to discuss the negative side of the mechanism, which is the "bad" non-equilibrium outcome, when the promised reward money needs to be distributed. It hardly has any impact on the social welfare (none, in fact, with quasi-linear preferences).

[5] If the authority sets the provision point at $C - R$ rather than at C, supplying the remaining funds from tax revenues when the provision point is reached, then we have the public good provided whenever $V \geq C$ (i.e., independently of R) and, correspondingly, a multiplicity of equilibria.

On the individual level, the "bad" outcome is not, however, without an element of justice. Unlike in the typical scenario of the private provision of public goods, the mechanism with reward money leaves free-riders worse off than contributors, who then can be thought of as receiving a "compensation" for the public good being not provided in proportion to their revealed preference for it.

4 Aggregate Uncertainty

In the previous section, the mechanism with reward money is analyzed under the assumption of no aggregate uncertainty, which we relax here. Without the structure imposed by Assumption 1 the simple characterization of equilibria given in Proposition 1 may no longer hold. The consistency of beliefs about the sum of contributions collected by others can be infringed if the condition $V \geq R + C$ is not met. Certainly, if for every possible vector of valuations $\mathbf{v}$ we still have that $\overline{V}(\mathbf{v}) \geq R + C$, then Proposition 1 continues to hold. As we show next, however, if in the support there is a vector $\mathbf{v}$ such that $\overline{V}(\mathbf{v}) < R + C$, then there is no equilibrium.

Formally, let each consumer i choose a strategy $\overline{g}_i : \mathcal{V} \to [0, C]$, which is a mapping from own valuation $v_i \in \mathcal{V}$ to contributions. Denote by $\overline{G}(\mathbf{v})$ the resultant sum of contributions at vector $\mathbf{v}$ when a strategy profile $\overline{\mathbf{g}}$ is played. Consumer i's expected payoff is given by

$$\overline{\pi}_i(\overline{g}_i, \overline{G}) = \int \mathcal{I}(\overline{G}(\mathbf{v}) \geq C)\,[v_i - \overline{g}_i(v_i)] + \mathcal{I}(\overline{G}(\mathbf{v}) < C)\left[\frac{\overline{g}_i(v_i)}{\overline{G}(\mathbf{v})}R\right] dF(\mathbf{v}) \tag{6}$$

Let $\overline{\mathbf{g}}^*$ be an equilibrium profile. In this equilibrium (as in any other), by the principle of aggregate concurrence[6] all consumers must agree on the choice of the aggregate $\overline{G}^*(\mathbf{v})$. If we replace $\overline{g}_i(v_i)$ in (6) with $\overline{G}(\mathbf{v}) - \overline{G}^*(\mathbf{v}) + \overline{g}_i^*(v_i)$, then for each i the following needs to hold

$$\overline{G}^* \in \arg\max_{\overline{G} \geq \overline{G}^*_{-i}} \overline{\pi}_i(\overline{G} - \overline{G}^* + \overline{g}_i^*, \overline{G}), \tag{7}$$

[6] See Martimort and Stole (2011) and Martimort and Stole (2012), on which the following analysis is based.

where $\overline{G}^*_{-i}(\mathbf{v})$ is the sum of equilibrium contributions of consumers other than i.

We can also observe that the game is aggregate-invariant: The total welfare of consumers with uniform weights is invariant to the composition of individual contributions as long as their sum is the same. This observation simplifies the search of the equilibrium aggregate $\overline{G}^*$. Define the total welfare by

$$\begin{aligned}
\Lambda(\overline{G}, \overline{G}^*) &= \frac{1}{n} \sum_{i \in N} \overline{\pi}_i(\overline{G} - \overline{G}^* + \overline{g}_i^*, \overline{G}) \\
&= \frac{1}{n} \int \mathcal{I}(\overline{G}(\mathbf{v}) \geq C) \left[\overline{V}(\mathbf{v}) - (n\overline{G}(\mathbf{v}) - (n-1)\overline{G}^*(\mathbf{v})) \right] + \\
\mathcal{I}(\overline{G}(\mathbf{v}) &< C) \left[\frac{n\overline{G}(\mathbf{v}) - (n-1)\overline{G}^*(\mathbf{v})}{\overline{G}(\mathbf{v})} R \right] dF(\mathbf{v}). \qquad (8)
\end{aligned}$$

Then, for each i we have in equilibrium

$$\overline{G}^* \in \arg \max_{\overline{G} \geq \overline{G}^*_{-i}} \Lambda(\overline{G}, \overline{G}^*). \qquad (9)$$

Proposition 2 *The game induced by a mechanism R has no equilibrium if $\overline{V}(\mathbf{v}) < R + C$ for some $\mathbf{v}$.*

Proof. First, we show that $\overline{G}^*(\mathbf{v}) = C$ for all $\mathbf{v}$ such that $\overline{V}(\mathbf{v}) \geq R + C$. If for some $\mathbf{v}$ we have $\overline{G}^*(\mathbf{v}) > C$, then a consumer with a non-zero contribution can increase his welfare and, in fact, the total welfare by choosing $\overline{G}(\mathbf{v}) = \overline{G}^*(\mathbf{v}) - \varepsilon > C$. Thus, the maximum amount contributed in equilibrium must be C. Now, consider a vector $\mathbf{v}$ such that $\overline{G}^*(\mathbf{v}) = C$. It has to be that at $\mathbf{v}$ every consumer i finds that the sum $\overline{G}^*(\mathbf{v}) = C$ leads to a higher welfare than $\overline{G}(\mathbf{v}) = C - \varepsilon$ for any $\varepsilon > 0$. Using (8), this implies

$$\overline{V}(\mathbf{v}) - C \geq \frac{C - n\varepsilon}{C - \varepsilon} R. \qquad (10)$$

As the right-hand side is at most R for any $\varepsilon > 0$, we get that in equilibrium $\overline{G}^*(\mathbf{v}) = C$ only if $\overline{V}(\mathbf{v}) \geq R + C$. Analogously, we can establish that if $\overline{V}(\mathbf{v}) < R + C$ then $\overline{G}^*(\mathbf{v}) = C$ cannot hold as it is consumer welfare improving to have a lower $\overline{G}(\mathbf{v})$. But because the integrand of (8) is not continuous from the left (and not upper semicontinuous) at $\mathbf{v}$ with

$R > \overline{V}(\mathbf{v}) - C$, there is no such $\overline{G}^*(\mathbf{v})$ that consumers could agree upon. ■

The proof of Proposition 2 offers an alternative proof of Proposition 1 and it formalizes the earlier claim that Proposition 1 continues to hold if $\overline{V}(\mathbf{v}) \geq R + C$ for each $\mathbf{v}$. The intuition behind the result of Proposition 2 is straightforward. In their attempt to maximize own welfare, the consumers want to have the highest total payoff for each realization of $\mathbf{v}$. For low realizations such that $R > \overline{V}(\mathbf{v}) - C$, the consumers rather divide the reward money, which leads to the no equilibrium outcome. As a simple example illustrating the proposition, consider a setting with $C = 50$, $n = 2$, where $v_1 = 40$ but v_2 takes a value of either 0 or 40 with equal probability. Suppose the authority applies the mechanism with $R = 20$. In equilibrium, we should have $\overline{g}_1^*(40) + \overline{g}_2^*(40) = 50$. But the optimal contribution $\overline{g}_2^*(0)$ needs to be as close as possible to but less than $50 - \overline{g}_1^*(40)$, which is not possible given a continuous contribution space.

In environments with aggregate uncertainty, welfare implications from the proposed mechanism are harder to assess as the provision of the public good is not always in equilibrium. The expected welfare depends on the likelihood that the project is sufficiently efficient. But as before, in comparison to the standard provision point mechanism with refunds, a significant advantage of the mechanism with reward money is the elimination of the zero-contribution equilibrium.[7]

Individual contributing behavior

Next, we study individual contributing behavior by looking at the best-response contribution of consumer i with valuation v_i, denoted by $\overline{g}_i^{BR}(R; v_i)$, given a mechanism R and any strategy profile of others $\overline{\mathbf{g}}_{-i}$ (not necessarily equilibrium). Specifically, interest lies with the comparative statics of the best response with respect to R. As only the aggregate contribution of others matters for consumer i in deciding on own contribution, we let $\mathcal{G}(.)$ be the distribution of the sum G_{-i} of other contributions resulting from $\overline{\mathbf{g}}_{-i}$ and assume the differentiable probability density function $\gamma(.) > 0$ and a compact support.

[7] In an experimental study, Marks and Croson (1999) show that with uncertainty the efficiency rate of the provision point mechanism remains similar to that obtained for the case without uncertainty, which is about 50 percent.

The best response of consumer i maximizes his expected payoff and is given by

$$\overline{g}_i^{BR}(R; v_i) = \arg\max_{\overline{g}}(1 - \mathcal{G}(C - \overline{g}))(v_i - \overline{g}) + \int_0^{C-\overline{g}} \gamma(G_{-i}) \frac{\overline{g}}{G_{-i} + \overline{g}} R dG_{-i}. \qquad (11)$$

Above, $1 - \mathcal{G}(C - \overline{g})$ stands for the probability that the provision point is reached when the consumer contributes $\overline{g}$, and the second term stands for the expected share of the reward money when the provision point is not reached. The expected payoff on the right-hand side of (11), defined as a function $\widetilde{\pi}_i(\overline{g}, R; v_i)$, is not super- or sub-modular in $\overline{g}$ and R in a general case, which implies a non-trivial comparative statics of $\overline{g}_i^{BR}(R; v_i)$ with respect to R. Intuitively, an increase in R gives rise to two countervailing effects. First, an increase in contribution raises the probability of the provision point being reached and, accordingly, lowers the probability of obtaining a share of the reward money. Second, an increase in contribution raises the expected share of the reward money. Assuming the validity of the first-order approach, i.e., $\partial^2\widetilde{\pi}_i/\partial\overline{g}^2 < 0$, we can establish the following result.

Proposition 3 *Assume that the elasticity of density* $\gamma(.)$ *is less than* 1 *and* $\overline{g}_i^{BR}(\min(\mathcal{V}), R) = 0$. *There is a threshold valuation* $\overline{v}_i \in \mathcal{V}$ *such that for* $R' > R$ *we have*

$$\overline{g}_i^{BR}(v_i, R') \lesseqqgtr \overline{g}_i^{BR}(v_i, R) \text{ if } v_i \gtreqqless \overline{v}_i. \qquad (12)$$

Proof. By the implicit function theorem and suppressing the arguments, we have

$$\frac{d\overline{g}_i^{BR}}{dR} = \frac{\partial^2\widetilde{\pi}_i/\partial\overline{g}\partial R}{-\partial^2\widetilde{\pi}_i/\partial\overline{g}^2}. \qquad (13)$$

The sign of this expression coincides with that of the numerator. The cross derivative

$$\frac{\partial^2\widetilde{\pi}_i}{\partial\overline{g}\partial R} = -\gamma(C - \overline{g}_i^{BR})\frac{\overline{g}_i^{BR}}{C} + \int_0^{G-\overline{g}_i^{BR}} \gamma(G_{-i}) \frac{G_{-i}}{(G_{-i} + \overline{g}_i^{BR})^2} dG_{-i} \qquad (14)$$

is strictly decreasing in $\overline{g}_i^{BR}$ because of the assumption about the elasticity of $\gamma(.)$. Thus, this derivative can only cross zero from the left. If it never crosses zero, then we have

$\partial^2 \widetilde{\pi}_i / \partial \overline{g} \partial R > 0$ (by continuity and the fact that it is positive at $\overline{g}_i^{BR}(\min(\mathcal{V}), R) = 0$), given which the proposition trivially holds with $\overline{v}_i = \max(\mathcal{V})$. Now, denote the point where the derivative crosses zero by g^0. Observing that the best response $\overline{g}_i^{BR}(R; v_i)$ is increasing in v_i, define $\overline{v}_i = \max(v_i \in \mathcal{V} : \overline{g}_i^{BR}(R; v_i) \leq g^0)$. Then, if $v_i \leq \overline{v}_i$ we have $\partial^2 \widetilde{\pi}_i / \partial \overline{g} \partial R \geq 0$, and *vice versa*, which proves the proposition. ■

In words, Proposition 3 says that low-valuation consumers are likely to increase and high-valuation consumers – to decrease their contributions when more reward money is offered, keeping the behavior of others constant (or assuming that the aggregate effect of changes of their behavior is of a second order, e.g., the elasticity of density $\gamma(.)$ remains to be less than 1). Consumers with higher valuations and, accordingly, higher contributions are entitled to larger shares of the reward money. Thus, an increase in reward money makes them more seek utility from the reward money, which they attempt by diminishing the probability of provision with lower contributions. This effect is weaker for consumers with low contributions, who rather increase their shares of the reward money with higher contributions. Off the equilibrium path, it is hard to predict the aggregate effect, but it has to be zero in equilibrium.

5 Collective Action Problem

In the private provision of public goods, the purpose of reward money can also be viewed as a way to compensate consumers for externalities they create. The problem that externalities are not internalized arises in many different situations hindering the achievement of socially optimal outcomes. Next, we apply our mechanism to one such situation, specifically, a collective action problem, where participation in a project is individually rational only when a critical mass of participants is reached (see Myatt and Wallace (2009) for a recent discussion on collective action problems).

Imagine a government that plans to populate a new area with a capacity of at most M settlers. The individual cost of settling in this area is fixed at c, whereas the benefit, denoted by $v(m)$, depends on the total number of settlers, m. Let the benefit be increasing in m implying positive externalities from settlement. Assume that it is individually rational to settle in the area only if there are at least $\underline{m}-1$ other settlers, where $\underline{m} = \min\{m : v(m) \geq c\}$ and let $1 < \underline{m} < M$. There is a large population of people who simultaneously decide whether to file an application for a settlement. Applications are contractually binding and in case more than M applications are filed, M of them are randomly selected. As $v(1) < c$, we can have two equilibrium outcomes (i) "bad" – nobody settles and (ii) "good" – there are M settlers.

A much applied way to eliminate the "bad" equilibrium in similar problems is via subsidies. In our example, the government can offer $\underline{m}-1$ subsidies of size s, which are randomly distributed if there are more than $\underline{m}-1$ applications. However, for the subsidy scheme to eliminate completely the "bad" equilibrium it has to be that $s = c - v(1)$ with the total budget of $(\underline{m}-1)(c-v(1))$ required. Now suppose that the government applies a mechanism with reward money. The government announces that if the number of settlers is smaller than $\underline{m}$ then the settlers equally share a pre-specified endowment of R; if $m \geq \underline{m}$ then no money from the government is distributed. Obviously, for R sufficiently large the only equilibrium outcome is when there are M settlers. The threshold $\underline{R}$ such that with $R > \underline{R}$ there is only "good" equilibrium is determined by

$$\underline{R} = \max_{m \leq \underline{m}-1} m(c - v(m)). \tag{15}$$

To see this, if $R \geq c - v(1)$ then there will be at least one settler, if $R \geq \max\{c - v(1), 2(c-v(2))\}$ at least two, if $R \geq \max\{c-v(1), 2(c-v(2)), 3(c-v(3))\}$ at least three, and so on until we establish (15).

Unlike in the case of subsidies, no money is distributed in equilibrium under the mechanism with reward money. Furthermore, as $\underline{R} < (\underline{m}-1)(c-v(1))$ the budget at stake is lower than that with subsidies. The mechanism with reward money compensates settlers for the externalities they create as long as the critical mass is not attained (after which externalities play no important role). Moreover, it is straightforward to see that the promise of reward money $R > \underline{R}$ has an implication that filing an application is a weakly dominant strategy.

An interesting question is whether similar effects can be achieved when the mechanism is applied to problems with negative externalities, e.g., the problem of the commons. The answer, however, is no. The mechanism with reward money is designed in such a way that the events of distribution of reward money and of achievement of the social optimum are exclusive. In the case with positive externalities, the social optimum is also individually optimal, i.e., is a Nash equilibrium, but it is not in the case with negative externalities. Therefore, it is impossible to achieve the social optimum without the distribution of reward money.

6 Conclusion

In this paper, we propose a modification of the provision point mechanism with refunds that leads to significant improvements in performance. The modification is reward money introduced to benefit contributors in case the provision point is not reached. In environments without aggregate uncertainty the proposed mechanism leads to the unique efficient outcome without dispensing the reward money. Moreover, the mechanism with the reward money set at the net value of the public good implements the efficient outcome uniquely with every consumer contributing the same proportion of his private valuation. Thus, the proposed mechanism not only achieves allocative efficiency but also distributional. In environments with aggregate uncertainty, the unique equilibrium outcome is also the provision of the public good as long as the project is sufficiently efficient. We also apply the mechanism to a collective action problem and show that it can implement the efficient outcome in weakly dominant strategies. Lastly, this mechanism remains fairly simple for the purpose of practical applications.

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