

# Public Goods, Perfect Competition, and Underproduction

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Recently, the classical position that free markets will underprovide public goods has come under attack. Critics have argued that, if exclusion is costless, the proposition is false. Underlying this argument, however, is the assumption that preference maps of consumers are known. In this paper, this assumption is relaxed. It is shown that a system of atomistic competition will lead to suboptimal levels of public goods, and that those public goods which are produced will be inefficiently allocated. Furthermore, owing to different intensities of use, units of public goods will command different prices. However, for any particular unit of public good, price will be the same for all individuals.

## I. Introduction

The question of how a market economy, left to its own devices, will allocate public goods has received considerable attention in recent years. With a few notable exceptions, the literature has focused upon that subset of public goods for which exclusion is not possible, or, if possible, prohibitively costly. This includes the case of pure public goods, such as defense, and impure public goods, as represented by production and consumption externalities. The consensus of such studies is that private markets will systematically underprovide for collective goods.<sup>1</sup>

Recently several investigators (Demsetz 1970; Thompson 1968) have explored the consequences of market provision of public goods for which exclusion is costless. It is surprising that each author arrives at the conclusion that competitive markets will not underprovide public goods. Indeed, Thompson (1968) arrives at the conclusion that overprovision will emerge. Both authors also conclude that competitive equilibrium will

An earlier version of this paper was read to the March 1973 meetings of the Public Choice Society. I am grateful to Wallace Oates and James Buchanan for useful suggestions.

<sup>1</sup> See, for example, Mishan (1971), Musgrave (1959), and Samuelson (1954). For an exception to this statement, see Buchanan and Kafoglis (1963).

[*Journal of Political Economy*, 1974, vol. 82, no. 5]

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be characterized by price differentials among individuals for the public good.

It is not the purpose of this paper to offer a critical review of the merits of the aforementioned studies.<sup>2</sup> Instead, our aim is to relax one of the key assumptions made by both authors—that preferences of individual consumers are known to sellers of the public good.<sup>3</sup> Since such an assumption is not generally made with respect to private goods and has, to date, been the root of difficulties confronting policy makers with respect to public goods, it seems interesting and important to relax it. Such a procedure, we feel, will lead to a more adequate framework for dealing with the question of the competitive supply of public goods for which exclusion is not prohibitively costly.

The results of our analysis are similar in several important respects to those already published. Needless to say, they also differ in important respects. As in most previous studies, but in direct contradiction to at least two—Owen (1969) and Ekelund and Hulett (1973)—competition is shown to lead to a determinant, nonzero output of the public good. Furthermore, price differentials exist in equilibrium. However, unlike the case in existing studies, prices here differ among units of the public good and not among individuals. Competitive equilibrium is shown to depart from Pareto optimality for two reasons: (1) certain individuals may be excluded from consuming a portion of the output of public goods; (2) the level of production of public goods is too small. Thus, we are able to demonstrate that, in the absence of firms' knowledge concerning the preferences of specific individuals, the orthodox underprovision doctrine carries over to the case where exclusion is costless.

## II. The Model

Consider an economy composed of  $N$  individuals and two goods,  $A$  and  $B$ . Let good  $A$  represent a private good and good  $B$  a public good. Good  $B$  has the property that each of the  $N$  individuals in the economy can consume its total production,<sup>4</sup> although any individual can be costlessly excluded. We shall assume that production in the economy is characterized by constant costs—that is, the production-possibility curve is linear.<sup>5</sup>

<sup>2</sup> For a critique of Thompson's paper, see Ganguly (1969), Owen (1969), and Rodgers (1969); see also Thompson's reply (1969).

<sup>3</sup> Thompson (1968) assumes that the preference map of each consumer is known to both sellers and other consumers. Demsetz (1970), on the other hand, makes the weaker assumption that individual demand schedules for the public good are known to sellers.

<sup>4</sup> For concreteness, we can think of the public good as the production of television tapes, among which individuals are indifferent; transmission costs are taken to be zero, and scrambling devices can be costlessly installed on receivers. This example is given by Demsetz (1970).

<sup>5</sup> This assumption is made for expositional purposes only. We need only require that the appropriate convexity conditions hold.

We shall take the price of good  $A$  to be fixed at one so that all values can be expressible in terms of units of  $A$ . Entry into the production and sale of both goods is assumed to be free, and we assume that the market for  $A$  is perfectly competitive in the usual sense of the term. Finally, each individual,  $i$ , is assumed to have a strictly quasi-concave utility function,  $u^i$ , defined over the two goods, that is,  $u^i = u^i(A^i, B^i)$ , where  $A^i$  and  $B^i$  correspond to the  $i$ th individual's consumption of  $A$  and  $B$ , respectively.

To begin with, we shall assume that, in the market for  $B$ , each individual behaves parametrically in terms of price. Hence, we can define the inverse-demand function

$$p^i = f^i(B^i) \quad (i = 1, \dots, N) \quad (1)$$

for each individual. To arrive at (1), each individual maximizes his utility subject to his budget restraint. Since the consumption of  $B$  is nonrival, we obtain the market demand,  $R$ , by summing the individual demand functions vertically:

$$R = \sum_{i=1}^N P_i = \sum_{i=1}^N f^i(B). \quad (2)$$

Equation (2) corresponds to the marginal valuation of the community for any quantity  $B$  of the public good.

Turning to the supply side, we assume that firms have no knowledge of the preferences of specific individuals. Since marginal cost of extending consumption to another individual is zero, each firm is assumed to induce every individual to consume its entire output; firms will enter the industry as long as unit price as given by (2) exceeds the constant cost of production, which we represent by  $c$ . Hence, the supply function for the economy is

$$R = c. \quad (3)$$

Combining (2) and (3), we obtain the equilibrium condition for the economy:

$$\sum_{i=1}^N f^i(B) = c. \quad (4)$$

Together, equations (1) and (4) define the equilibrium values of  $B$  and the  $p^i$ . It can be easily shown that this solution is Pareto-optimal.<sup>6</sup>

One of the interesting features of the model just described is that in equilibrium each individual pays a different price for the public good and consumes the same quantity. This is precisely the reason why the solution described by (1) and (4) is likely not to be achieved. As Samuelson (1969) so aptly phrased it, the above only represents a "pseudo-equilibrium."

It will be recalled that the solution defined by (1) and (4) was based

<sup>6</sup> Readers will recognize the similarity between the present model and the Lindahl theory of voluntary exchange; see Johansen (1965, chap. 6).

upon the premise that consumers behaved parametrically with respect to price and that all firms attempt to sell their entire output to all individuals. It is easy to see, however, that if the number of individuals is large, such parametric price behavior of consumers is not based on rationality. Inspection of (1) and (4) reveals that it is in the interest of each individual to reduce his price offer. Supply decisions are based upon  $\sum P^i$ , of which any  $P^i$  can be assumed to constitute a negligible fraction. Thus, the rational individual will behave parametrically with respect to quantity and not with respect to price. In other words, each individual thinks of himself as a monopsonist who is faced with a perfectly inelastic supply schedule. If preferences are not known, rational consumers will find it in their own interest to misrepresent their preferences by making price offers substantially below those corresponding to (1).<sup>7</sup> Indeed, in the limit, each consumer will express no demand whatever for the collective good, irrespective of the price. Each individual is caught in the well-known "prisoner's dilemma." The result will be little or no private production of the public good, which is exactly what orthodox underprovision theory predicts.<sup>8</sup>

It was perhaps in response to this dilemma that Demsetz (1970) and Thompson (1968) introduce the assumption that preferences are known. For it might seem that the individual buyer is able to exploit his monopsonistic position only by misstating his preferences. As long as perfect competition is assumed on the supply side, however, it is not clear that the knowledge of preferences will have any impact on the outcome. It is also not clear that a model built upon such a premise can be described as "competitive." Whatever the case, however, the assumption is not necessary in order to develop a model that produces a nonzero equilibrium output of the public good. This will be shown in what follows.

### III. An Alternative Approach

Thus far, our discussion of the competitive production of a public good has been based upon the premise that a particular unit of public good

<sup>7</sup> See, however, the recent experiments of Bohm (1972), which indicated that individuals may voluntarily reveal their preferences for public goods even if cost shares are dependent upon their response. However, these experiments were carried out in a small-number setting, and thus may be of limited relevance to the problem at hand.

<sup>8</sup> Similar arguments concerning the implausibility of the model have been set forth by, among others, Buchanan (1968), Musgrave (1959), Olson (1965), and Samuelson (1954, 1958, 1969). It has also been described by Owen (1969) in the present context. Ekelund and Hulett (1973) arrive at a different conclusion regarding the outcome. They argue that a process of price cutting would lead to a single supplier of the public good. Freedom of entry would limit profits of the remaining firm to normal levels. However, I find their argument unconvincing. It is based upon the allegation that a firm whose revenues and costs initially satisfy eq. (4) can profit by luring customers away from other firms. Since firms are assumed to be selling their entire output to all individuals, this clearly cannot be the case.

can command a different price for different individuals. The motivation for this assumption of a public good to an additional person is zero. Thus, profit-maximizing firms, taking unit prices as given, will attempt to induce each consumer to purchase their entire output. Since individual tastes and income vary, the latter can be accomplished only by charging different persons a different price. In other words, in order to clear the  $N$  markets for the public good,  $N$  independent prices are generally required. Thus, price differentials among consumers are a requirement for competitive equilibrium.

On the other hand, the requirement that each person purchase total industry output leads to the situation where each person is a monopsonist in the market for public goods. From the point of view of the individual, this monopsony power is so strong that it leads him to attempt to consume public goods free of charge. Such behavior would appear to preclude the production of public goods in a competitive framework. Thus, from the point of view of establishing a viable competitive equilibrium for public goods, little is gained by assuming price discrimination among individuals. In the analysis to follow, we shall attempt to develop a competitive equilibrium under the assumption that the price for the services provided by a particular unit of the public good is identical for all persons. In this case, a consumer cannot gain by misrepresenting his true preferences because from his point of view such activities will have a negligible effect upon the price he pays. Price uniformity across individuals for a given unit will clearly be the case if resale of consumption rights is permitted. It will also prevail because we assume that firms are unable to distinguish individual demand curves. The only knowledge firms have concerning individual tastes is whether or not a particular individual chooses to purchase the services of their own output. This is clearly an extreme assumption since, for many services, costs of determining something about a particular consumer's demand function may not be prohibitive. For example, occupation or housing characteristics may be correlated with demand elasticity. Nevertheless, we feel the polar case of consumer indistinguishability is of considerable interest. It is the assumption most frequently employed in the analysis of private goods. Furthermore, the preference-revelation problem is generally regarded as the most serious obstacle to the efficient provision of public goods. By making an extremely weak assumption about the knowledge of consumer preferences, we are able to isolate the role of exclusion in providing such information. Finally, existing studies have been based upon an equally strong, but opposite, assumption about knowledge of consumer tastes. Our results can thus be used to define the range of possible outcomes. It is likely that intermediate cases will carry the flavor of both extremes.

Despite the fact that the price of a given unit of public good will be the same for all individuals, it does not follow that the prices of different units

will be identical.<sup>9</sup> Indeed, it will be shown that, in general, such price differentials tend to emerge from the competitive process. In the following discussion, it will be convenient to think of each firm as producing a single unit of collective good. Given the assumption of constant costs of production, this assumption is innocuous.

Consider, first, the supply side of this economy. It is clear that a particular unit of public good will be produced if and only if revenues cover costs; that is,

$$P \geq c/n, \quad (5)$$

where  $n$  denotes the number of individuals consuming the unit ( $0 < n \leq N$ ), and  $P$  represents the price paid by each individual consuming the good. Because of our assumption of free entry, it must be the case that the equality in (5) prevails. Thus, we have

$$P = c/n. \quad (n = 1, 2, 3, \dots, N). \quad (6)$$

This relation states that all units achieving an intensity of use  $n$  must be sold at the same unit price  $P_n$ , and the latter must just cover average costs  $c/n$ . Thus, to the extent that units of collective goods are used at different intensity levels, their prices will differ. This case is in direct contrast with earlier models, where each firm is assumed to receive the same unit price and all individuals necessarily consume all units.

Turning to the demand side, it is clear that individuals will be subjectively indifferent between units of public goods having differing intensities of use. Therefore, they will purchase those units of  $B$  that are cheapest. The lowest price they will face will be  $c/n$  and the highest price they may face is  $c$ . In general, they may face as many as  $N$  different prices with different quantities of public goods available at each price. Let the function  $x_n = h(P_n)$  represent the number of units available at price  $P_n$ . The function  $h$  is a step function since, in equilibrium, the  $P$ s can only assume the discrete values defined by (6). The problem facing each consumer can be described as follows:

$$\begin{aligned} & \max u^i(A^i, B^i), \\ & \text{subject to: } \sum_{j=1}^N v_j^i = B^i, \\ & \quad \sum_{j=1}^N P_j v_j^i + A^i = \bar{A}^i, \\ & \quad 0 \leq v_j^i \leq x_j = h(P_j), \quad j = 1, \dots, N \\ & \quad A^i \geq 0, \end{aligned}$$

<sup>9</sup> Throughout the paper, the term price refers to that amount paid by a consumer to enjoy the services of a particular unit of public good. This should be distinguished from that amount received by the firm per unit of public good,  $\sum p^i$ .

where  $v_j^i$  represents the quantity collective good available at price  $P_j$  which is consumed by the  $i$ th individual. This is a well-defined non-linear programming problem. The Kuhn-Tucker conditions generate the following relations:

$$P_j = MRS^i, \quad 0 < v_j^i < x_j \tag{7}$$

$$P_j \geq MRS^i, \quad v_j^i = 0 \tag{8}$$

$$P_j \leq MRS^i, \quad v_j^i = x_j$$

$$MRS^i = \frac{\delta u^i}{\delta B^i} \bigg/ \frac{\delta u^i}{\delta A^i}, \tag{9}$$

where  $MRS^i$  is the marginal rate of substitution for the  $i$ th individual. Inspection of these conditions quickly reveals that (7) can hold for no more than one  $P_j$ , and if it holds, the strict inequality must hold in (8) and (9). In other words, the consumer will consume the maximum available at a given price before consuming units at a higher price. Combining (7), (8), and (9) yields the condition

$$P_j \leq MRS^i \leq P_{j-1}, \tag{10}$$

where  $P_j$  is the highest price at which the individual trades. Together with the consumer's budget constraint, (10) defines the  $i$ th person's demand  $\hat{B}^i$ .

Consider the ordered vector  $\hat{B} = (\hat{B}^1, \hat{B}^2, \dots, \hat{B}^N)$ , where the superscripts  $i$  are chosen such that the  $\hat{B}^i$  are arranged in descending order. The conditions for equilibrium are:

$$\begin{aligned} \hat{B}^N &= x_N, \\ \hat{B}^{N-1} - \hat{B}^N &= x_{N-1}, \\ &\vdots \\ \hat{B}^{N-k-1} - \hat{B}^{N-k} &= x_{N-k-1}, \\ &\vdots \\ \hat{B}^1 - \hat{B}^2 &= x_1. \end{aligned} \tag{11}$$

The task then, is to find a step function  $h(P_n)$  which will satisfy (11). A sketch of the proof that such a function exists is as follows: Set  $x_N$  equal to  $\hat{B}^i(\min)$  which is arrived at by solving the consumer's problem when the price is fixed at  $c/N$ ; set  $x_{N-1}$  equal to  $\hat{B}^i(\min)$  ( $i \neq N$ ) when the price of the first  $x_N$  units is  $c/N$  and the price of all others is  $c/N - 1$ ; etc.

Several characteristics of this solution are worth noting. First, the number of different prices for which nonzero quantities are traded is equal to the number of distinct consumer demands ( $\hat{B}^i$ ). If all individuals are identical in tastes and resources, each unit of public good will be sold

at  $P_N = c/N$ .<sup>10</sup> If, on the other hand, each individual's tastes and real incomes are unique, trade may occur at as many as  $N$  different prices.<sup>11</sup> In general, therefore, some form of price discrimination will emerge from the process of competition. Such price discrimination takes the form of different prices for different units of collective good. Units of public good consumed by relatively few individuals will command a relatively higher price. Despite the fact that resale of consumption rights is permitted, arbitrage among consumers cannot eliminate these price differentials. This owes to the fact that lower-priced units are already consumed by those who purchase the higher-priced units and a given unit cannot be consumed more than once by a particular consumer.

Second, units sold at a price exceeding  $c/N$  will be characterized by "excess capacity"; that is, certain additional consumers could be accommodated at zero cost. There is, however, no incentive for the firm to lower its price to attract these consumers. For in order to do so, it would suffer a revenue loss on its existing clientele of an amount exactly equal to the revenue received from the new customers. Thus, the solution described by (6) and (11) is a stable equilibrium.

Third is the unusual industry structure of firms in the public goods sector. Firms are neither price takers nor price setters in the usual sense. Instead, they face a schedule of prices and associated sales that they take as given. While they may choose which point on the schedule to operate, they are indifferent among alternative strategies. Freedom of entry insures that the actions taken by an individual firm have no influence on the price-sales schedule they face. It is such parametric behavior that gives the present model its strong competitive flavor.

Fourth is the key role played by the assumption that firms do not know the preferences of particular individuals. Recall, a firm selling to  $n$  customers,  $n < N$ , would find it profitable to attract new customers if it could price discriminate between its existing and new clientele. Clearly, arbitrage would rule out this possibility. However, even if the firm were able to prevent arbitrage, it is unable to distinguish among its customers with respect to demand price. Hence, if it were to give one individual a price break, the same treatment would have to be extended to all of its customers—thereby negating the benefit of the additional customer. It is important to note that the model does not imply that consumer preferences are not revealed in the marketplace. Indeed, such revelation is a necessary condition for the existence of private markets. What is assumed in the present paper is that preference orderings of specific individuals are not known by firms—an assumption certainly not required by the standard competitive model.

<sup>10</sup> In this case our conclusions are the same as Demsetz's (1970).

<sup>11</sup> That each individual differs in tastes and real income is a necessary, but not sufficient, condition for the emergence of  $N$  different prices.



The nature of the equilibrium and the roles played by key assumptions can be clarified by appeal to a particular example, television tapes. Let us assume that television tapes are perfectly substitutable for one another and that no consumer wishes to view the same tape twice. Furthermore, tapes are assumed to be produced at constant cost. According to the solution described in (11), then, the price for viewing tapes will vary among the different tapes, but will be the same for all viewers of a given tape. Furthermore, utilization rates will also vary among tapes—the more expensive being the less fully utilized, and vice versa.

Since entry is free and arbitrage possible, how can the price differences among tapes be sustained? Will not competition and the fact that marginal cost is zero for nonfully utilized tapes force price to its lowest possible level,  $c/N$ ? The answer to these questions is no, as we shall demonstrate.

Freedom of entry means that firms are willing to supply any quantity of tapes as long as they can cover their production costs— $c$ . This means that a firm is indifferent to supplying a tape to  $n$  users ( $1 \leq n \leq N$ ) or  $n + k$  users ( $1 \leq n + k \leq N$ ) if the prices it can receive are  $c/n$  and  $c/(n + k)$ , respectively. If any tape, carrying price  $c/n$ , is sold to more than  $n$  customers, the resulting profit will attract competitors, forcing the price down to a level consistent with zero profit. In other words, the elasticity of supply for tapes used by  $n$  different people is infinite at  $c/n$ . What remains, then, is to show that different tapes will have different numbers of subscribers. Consider first the price  $c/N$ . This is the lowest achievable price because the cost of the tape is spread over the largest possible number of consumers. At this price, consumers, unless identical in tastes and income, will want to consume at different levels. Let  $x_0$  correspond to the smallest individual demand at price  $c/N$ . Then freedom of entry will assure that  $x_0$  tapes will be produced and that each tape will be sold to all households.

What about those people who wanted to view more than  $x_0$  tapes at price  $c/N$ ? Suppose we consider the person with the second-smallest demand at price  $c/N$ , say  $x_1$  tapes. Under what conditions will the  $(x_1 - x_0)$  tapes be produced? Clearly, we must require that the costs be covered. If there are  $(N - 1)$  people who have demands at least as great as  $x_1$ , at price  $c/N$ , the lowest possible price for the  $(x_1 - x_0)$  goods is  $c/(N - 1)$ . Higher prices will be beaten down by free entry; lower prices will be unprofitable.

So far, we have  $x_0$  tapes being sold to all consumers at price  $c/N$  and  $(x_1 - x_0)$  being sold to all but one consumer at price  $c/(N - 1)$ . Can such a price differential be sustained? Consider the supply side. Firms selling to all individuals make no more of a profit than those selling to  $(N - 1)$  individuals. If a firm that presently sells to all individuals raises its price above  $c/N$ , it will lose his entire clientele to a new entrant. By similar reasoning, a firm selling to  $(N - 1)$  people cannot increase its

price above  $c/(N - 1)$ , nor will it lower its price to below  $c/(N - 1)$ . To offset the reduction in price, such a firm would have to attract an additional customer. By definition, however, the necessary price would be  $c/N$ . If the firm could charge the lower price to only the one individual, it would find this profitable. However, arbitrage among the consumers of a given tape rules out this possibility. More important, however, is the fact that each of the original  $(N - 1)$  customers could demand the same treatment, and freedom of entry would ensure these demands would be met. Hence, to attract an additional consumer, firms selling to only  $(N - 1)$  individuals would have to lower their price for all individuals to  $c/N$ —negating the benefit of the additional customer.

Finally, we can ask whether arbitrage among consumers can eliminate any price differentials between units. Since consumers will purchase the services of lower-price tapes before higher-priced units, an individual who watches higher-priced tapes is consuming all lower-priced tapes. Hence, individuals who buy at low prices cannot sell to those buying the higher-priced tapes because the latter are already viewing the low-priced tapes. Thus, arbitrage among tapes of different prices is not possible.

#### IV. Optimality Properties

It is obvious that, in general, the solution given by (11) does not satisfy the requirements of a Pareto-optimum allocation. The mere fact that some individuals are excluded from consuming part of the output of collective goods is sufficient to preclude Pareto optimality. The extent of the deadweight loss will depend upon the degree of diversity of consumer tastes and resources. Only in the limiting case of identical individuals does this source of inefficiency (as well as any other) disappear. That the use of average-cost prices to allocate collective goods will result in inefficiency is a well-known proposition and dates as far back as Dupuit's analysis of the uncongested bridge.

What can be said concerning the optimality of the level of public good production implied by (6) and (11)? In order to answer this question, it might appear that we would have to define the optimal level of collective goods output and compare it with the level which satisfies (6) and (11). In general, however, the Pareto-optimal level of public goods production is not unique but depends upon the initial distribution of income and the way in which the costs of the collective good are allocated. It is quite possible, therefore, that our answer to the question posed above will be ambiguous. This is not the sense, however, in which the terms under- or overproduction have been used by economists. Consider, for example, an often-used explanation of the inefficiency involved in the case of monopoly. Suppose all but one market satisfy the conditions of Pareto optimality; in this case, marginal cost corresponds to true opportunity costs. Since price exceeds marginal cost and since the former is an indicator of

marginal personal benefit, it is usually alleged that monopoly results in the underproduction of the good in question (and by implication the overproduction of the other goods, taken together). The reasoning is that the consumer can be made better off and the firm no worse off if an additional unit of good is produced and made available to the consumer at a price equal to marginal cost. Absent from this argument is any explanation of how such a change is brought about. Furthermore, it ignores any income effects which result from the increase in the consumer's and/or monopolist's welfare. Despite these drawbacks, the argument has strong intuitive appeal and is often invoked in welfare discussions of monopoly.

Similar reasoning is employed by Thompson (1968) in his attempt to refute the traditional underproduction hypothesis. To establish his result he simply demonstrated that the sum of individual's marginal valuations was less than the marginal cost of production. Thus, by producing one less unit of public good it is possible to increase the welfare of some subset of society without reducing the welfare of the remaining individuals. It is in this spirit that overproduction has been used.

Carrying this logic forward to the case at hand, it is easy to show that the sum of individuals' marginal valuations exceeds marginal cost of production; hence, underproduction can be said to exist. However, it is not necessarily legitimate to employ the marginal rate of substitution defined by (10) for this purpose. Consider those individuals whose consumption of  $B$  is less than the total amount produced. Their  $MRS$ s reflect the fact that they are denied access to existing units. Clearly such marginal valuations are inappropriate for purposes of deciding whether an additional unit of output should be produced. The relevant magnitude is the marginal valuation that would exist if they consumed total industry output.<sup>12</sup> These marginal valuations, in turn, depend upon the prices that we assume they pay for the extension of their consumption. In order to avoid the complications associated with income-effect feedbacks, we assume that the additional consumption is provided free of charge. As we shall demonstrate below, this particular assumption does not affect the qualitative nature of the conclusions to be derived. Let  $K$  individuals ( $0 \leq K \leq N$ ) comprise the class of consumers who, in equilibrium, are denied access to some part of the output of public goods. Let  $V^K$  be the sum of marginal valuations of this group, once individuals are allowed to extend, free of charge, their consumption to total output. For each of the  $(N - K)$  individuals who do consume total output, we have by (10)

$$\frac{c}{N - K} \leq MRS^i, \quad i = (N - K, N - K + 1, \dots, 1) \quad (12)$$

<sup>12</sup> This assumes that the marginal utility of the public good is nonnegative. Otherwise, extension of consumption should continue only as long as marginal utility is positive.

Summing (12) over the  $(N - K)$  individuals, we have

$$c \leq \sum_{i=1}^{N-K} MRS^i. \quad (13)$$

Now if  $V^K > 0$ , underproduction is seen to exist. That is, if the excluded group is willing to pay any positive sum for an additional unit of output, once they are allowed to consume total output, the traditional proposition holds. This appears to be a very weak condition, especially if we assume the marginal utility of the collective good to be nonnegative. In general, the presumption is that  $V^K > 0$ , irrespective of how the extension of public good consumption to the  $K$  individuals is financed. Of course, no such conclusion can be drawn if  $K = 0$ , the case where all individuals are identical. In this limiting case, (13) will hold as an equality. Our conclusion, therefore, is that underproduction is the general outcome of the competitive production of public goods. In no event will over production be the case.

## V. Conclusion

This paper has explored the consequences of private provision of public goods when exclusion costs are zero, unit production costs are non-decreasing, entry into production and sale is free, and there is a lack of knowledge of specific individual preferences by firms. It was shown that a determinate nonzero level of production would result. However, unlike provision of private goods, the market outcome for public goods is unsatisfactory in two major respects: (1) some individuals are excluded from consuming all units of the public good; (2) the level of public good output is suboptimal. Hence, the orthodox position that private markets will underprovide for public goods for which exclusion is prohibitively costly can be extended to those for which exclusion costs are zero.

Before concluding, we must face the criticism that real-world examples of public goods produced in competitive markets are difficult to find. Thus, why study such cases? Several considerations can be brought to bear on this issue. (1) While it is true that under present arrangements examples are difficult to come by, we need not take such arrangements as given. Thus, one could imagine market provision of broadcasting and outdoor recreation facilities such as parks, police, and fire protection. Before rejecting private provision of such excludable services as an alternative, we must know some of its characteristics. (2) There is increasing pressure on state and local governments to consider user finance for many of their activities. The present analysis sheds light on the outcome of a decentralized self-supporting finance arrangement. (3) It is of inherent theoretical interest. It is interesting to know whether the problems posed by public goods stem from characteristics other than their

inability to be produced in competitive markets. The present paper demonstrates that the lack of competitive markets for public goods is not solely responsible for deviations from optimality.

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