Marriage Markets and Bargaining Between Spouses

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Introduction

Proposing marriage, an eager suitor might promise a lifetime of devoted service to the whims of his beloved. But a sensible young woman, even if she hasn't studied game theory, is likely to be skeptical. She is more likely to base her expectations about marriage on what she knows of the way her mother and other married female acquaintances have fared, than on her suitor's flattering, but unenforceable promises.

At the time of a marriage, property transfers could be arranged between the family of the bride and the family of the groom, but it is simply not possible to write a premarital contract that legally binds the new couple to detailed courses of action over the course of their marriage. Most of the important decisions that they make will have to be resolved as they arise, after marriage. A theory of courtship and mating that deals satisfactorily with each individual's prospects in a marriage, must include postmarital bargaining between spouses. Conversely, since a person's bargaining power within a marriage may depend on the threat of exercising the "outside options" of divorcing and reentering the marriage market, a satisfactory theory of bargaining within marriage must include a theory of courtship and mating.

Cooperative Nash Bargaining Solutions

The pioneering work on models of household bargaining was done by Marilyn Manser and Murray Brown (1980) and Marjorie McElroy and Mary Horney (1981), who proposed to model household decision making with the Nash cooperative bargaining model. In these papers, a marriage is modelled as a static bilateral monopoly. A married couple can either remain married or they can divorce and live singly. There is a convex utility possibility set S containing all utility distributions (U_1, U_2) that could possibly be achieved if they remain married. The utility of person i if he or she divorces and lives singly is given by V_i. It is assumed that there are potential gains to marriage, which means that the there are utility distributions (U_1, U_2) in S that strictly dominate the utility distribution (V_1, V_2). These papers propose that the outcome in a marriage will be the symmetric Nash bargaining solution where the "threat point" is dissolution of the marriage with both persons choosing to live singly. According to the Nash bargaining theory, the outcome in this household will be the utility distribution (U_1^*, U_2^*) that maximizes $(U_1 - V_1)(U_2 - V_2)$ on the utility possibility set S.[1] In this theory the outcome in a marriage is completely determined by the utility possibility set and by the position of the threat point, (V_1, V_2). This theory has the interesting prediction that social changes that affect the utility of being single will affect the distribution of utility within the household and hence may change household spending patterns, even if they have no effect on the budget of the household, while changes in the apparent distribution of earned income within the household will have no effect on the distribution of utility in the household if they do not change the threat point from being single.

Shelly Lundberg and Robert Pollak (1991) propose an alternative Nash bargaining model. They suggest that for many marriages the relevant threat point for the Nash bargaining solution should be not divorce, but an "uncooperative marriage" in which spouses would revert a "division of labor based on socially recognized and sanctioned gender roles." Lundberg and Pollak suggest that with their model, if government child-allowances are paid to mothers rather than to fathers in two-parent households, this threat point will shift in the mothers' favor. Accordingly, the outcomes of cooperative bargaining within households are likely to be more favorable to women. By contrast, in the divorce-threat model, changing who gets the welfare payments when the couple is together will have no effect on the distribution of utilities if there is no change in who gets these payments in the event of a divorce.

[1] This expression is sometimes known as the Nash product. John Nash (1950) proposed a set of axioms for resolution of static two-person bargaining games such that the only outcomes that satisfy the axioms maximize the Nash product on the utility possibility set.

Should the threat point be divorce as suggested by Mansur, Brown, McElroy and Horney? Should it be an uncooperative marriage as suggested by Lundberg and Pollak? Will the threat point depend on whether either party can end the marriage or whether mutual consent or a court decree is required to end the marriage? Nash's axioms for the cooperative bargaining solution give us no direct guidance about the appropriate threat points for bargaining in a marriage. Recent work on the noncooperative foundations of bargaining theory not only offers a more convincing foundation for the Nash bargaining solution, but also yields useful insight into the appropriate choice of threat points.

Ariel Rubinstein (1982) developed an extensive-form, multi-period bargaining game for two agents in which a cake is to be partitioned only after the players reach agreement. Players alternate in proposing how to divide the cake with one time period elapsing between each offer. Each agent i is impatient, discounting future utility by a factor $\delta_i < 1$, so that the utility to player i of receiving w units of cake in period t is $w\delta_i^t$. Rubinstein proved that in the limit as the time between proposals becomes small, the only subgame perfect equilibrium is for the cake to be divided in the first period with player i's share of the cake being $\alpha_i = \delta_i/(\delta_1 + \delta_2)$. More generally, if agent i's utility from receiving w_i units of cake in period t is $u_i(w_i)\delta_i^t$ where u_i is a concave function, then the only perfect equilibrium is the allocation that maximizes the "generalized Nash product", $u_1^{\alpha_1} u_2^{\alpha_2}$ on the utility possibility set $\{(u_1(w), u_2(1-w))\,|0 \le w \le 1\}$. In case the two agents have equal discount rates, this outcome is the same as the symmetric Nash equilibrium where the threat point is $(0,0)$.

Ken Binmore (1985) extended the Rubinstein model to the case where each of the bargaining agents has access to an "outside option". Binmore's model is like the Rubinstein model, except that each agent i has the option of breaking off negotiations at any time and receiving a payoff of m_i units of cake, in which case the other player receives no cake. Given that the outcome in the game without outside options is the same as the Nash cooperative equilibrium with threat point $(0,0)$, one might conjecture that the effect of the outside options would be to move the threat point to (m_1, m_2). (If negative values of m_i

are considered, this conjecture might be amended to $(\max\{0, m_1\}, \max\{0, m_2\})$. Binmore shows that this is not the answer. The only subgame perfect equilibrium for the game with outside options is an agreement in the first period on the utility distribution (u_1, u_2) that maximizes the Nash product $u_1^{\alpha_1} u_2^{\alpha_2}$ on the utility possibility set $\{u_1(w), u_2(1-w) | 0 \leq w \leq 1\}$ *subject to the constraint that* $u_i \geq m_i$ *for each* i. In general, this solution is not the same as maximizing $(u_1 - m_1)^{\alpha_1}(u_2 - m_2)^{\alpha_2}$ on the utility possibility set, which would be the outcome of shifting the threat point to (m_1, m_2).[2]

Noncooperative Bargaining Theory and Marriage

To many persons with marital experience, it seems unlikely that couples generally resolve disagreements about ordinary household matters by negotiating under the pressure of divorce threats. If one spouse proposes a resolution to a household dispute and the other does not agree, the expected outcome is not a divorce. More likely, there would be harsh words and burnt toast until the next offer is made. If the couple were to persist forever in inflicting small punishments upon each other, it might well be that the outcome would be *worse* for one or both of them than a divorce. But divorce imposes large irrevocable costs on both parties, while a bargaining impasse need last only as long as the time between a rejected offer and acceptance of a counteroffer.

The Rubinstein-Binmore model, as applied to marriage lends formal support to these speculations. This model leads to the conclusion that so long as the gains from marriage are divided in such a way that both parties are better off being married than being divorced, a threat of divorce is not credible. Instead, the relevant threat is the threat of delayed agreement and burnt toast, followed by a counterproposal. Here we will explain the workings of the Rubinstein-Binmore model as applied to a highly simplified model of a household.

Consider a married couple who expect to live forever in a stationary environment.

[2] Binmore, Shaked, and Sutton (1989) tested this theory with a laboratory experiment in which subjects played a Rubinstein bargaining game with outside options. Behavior in this game was better predicted by the Binmore model than by the competing model in which the outside option is the threat point.

Assume that each spouse discounts future utility by the same per-period discount factor δ and that in every time period, the utility possibility frontier is the simplex $\{(u_h, u_w)|u_h + u_w = 1\}$, where u_h and u_w are the utilities of husband and wife respectively. Each spouse has an intertemporal utility function of the form $\sum_{t=0}^{\infty} u_t \delta^t$. In any period where they remain married, but do not reach agreement, the husband will get a utility of b_h and the wife will get a utility of b_w, where $b_h + b_w < 1$. If either person asks for a divorce, they will divorce and the husband will get a utility of m_h forever and the wife will get a utility of m_w forever, where $m_h + m_w < 1$. [3]

The spouses alternate in making offers of feasible utility distributions. For concreteness, let us suppose that the wife gets to make the first offer[4] and that she proposes a utility distribution $(u_h, u_w) > (m_h, m_w)$. The husband could either accept the offer, refuse the offer and make a counteroffer, or refuse the offer and ask for a divorce. If the husband accepts the offer, then the distribution of utility in the household will (u_h, u_w) and will remain (u_h, u_w) in every subsequent period unless in some future period the husband changes his mind and decides to reject his wife's outstanding offer of (u_h, u_w). Since this is a stationary model, if the husband accepts the offer in the first period, he will continue to accept it in all subsequent periods. If the husband refuses the offer and asks for a divorce, he will get a utility flow of $m_h < u_h$ in all future periods. Therefore, if the only way to refuse an offer were to ask for a divorce, the wife could extract all of the gains from marriage by offering the husband a utility that is just equal to his utility from being divorced.[5] But the husband has the additional alternative of refusing the wife's

[3] A more realistic model would allow the possibility that divorced persons can remarry with some probability at some interval of time after divorcing. While it would be worthwhile to develop the model in this direction, it appears that the qualitative conclusions would be little different from the model sketched here

[4] If the husband makes the first offer, the same discussion applies with the words *husband* and *wife* reversed.

[5] We follow the convention in the principal-agent literature, by assuming that if the agent is offered a deal in which he is just indifferent between two options, he will take the one that the principal wants him to take. This saves mathematical clutter that would arise if we had the principal offer the agent a tiny bit more for taking the desired option.

offer and making a counteroffer in the next period. In equilibrium, it must be the case that the husband can not do better by refusing the offer and waiting for his own turn to make a counteroffer. Since the wife will want to make the smallest offer that the husband will accept, it must be that in equilibrium, the wife offers terms that leave the husband indifferent between accepting immediately and making a counteroffer. If the divorce threat is not credible for either spouse, this process has a unique equilibrium in which the wife gets b_w plus the fraction $\frac{1}{1+\delta}$ of the total gain $1 - b_h - b_w$ from agreement and the husband gets b_h plus the fraction $\frac{\delta}{1+\delta}$ of the gains $1 - b_h - b_w$..[6] Thus if the wife gets to make the first offer, the equilibrium is

$$(\bar{u}_h, \bar{u}_w) = \left(b_h + \delta \frac{(1 - b_h - b_w)}{1 + \delta}, b_w + \frac{(1 - b_h - b_w)}{1 + \delta} \right).$$

If $\bar{u}_h > m_h$ and $\bar{u}_w > m_w$, then the divorce threat is not credible for either spouse and the solution will be $(\bar{u}_h, \bar{u}_w)$. If $\bar{u}_i < m_i$, then the divorce threat will be relevant for person i, and as Binmore observes, the only equilibrium outcome is one in which person i gets utility m_i and his partner gets utility $1 - m_i$.

If the time between offer and counteroffer is small, then the discount rate for waiting one period is close to 0, so that δ is close to 1. In the limit as δ approaches 1, if divorce threat is not relevant, then the gains from cooperative rather than noncooperative marriage will be divided nearly equally. Thus in the limit as the time between offer and counteroffer becomes small, the equilibrium approaches one of the following three cases.

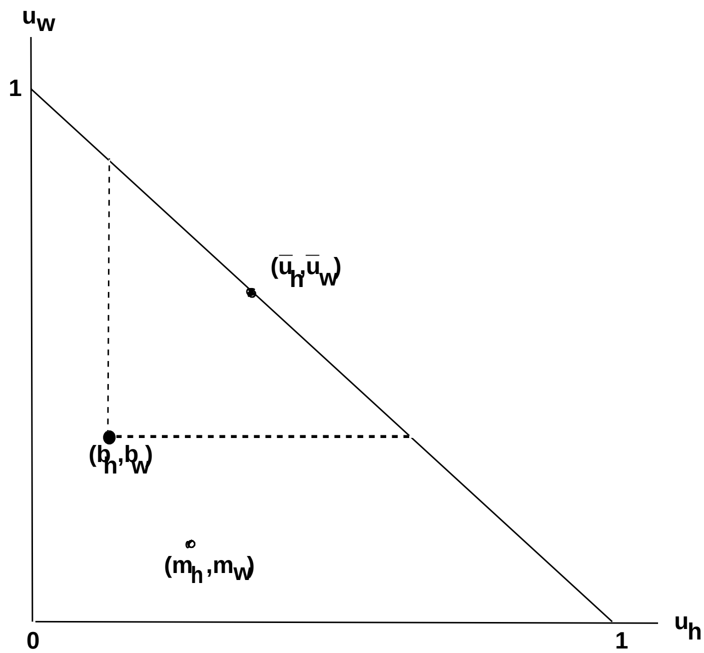
Case (i) Divorce threats are not credible. If $b_h + (1 - b_h - b_w)/2 > m_h$ and $b_w + (1 - b_h - b_w)/2 > m_w$, then the outcome is $(\bar{u}_h, \bar{u}_w) = b_h + (1 - b_h - b_w)/2, b_w + (1 - b_h - b_w)/2$. The geometry of Case (i) is illustrated in Figure 1. The point $(\bar{u}_h, \bar{u}_w)$ is the point on the simplex that splits the gains above (b_h, b_w) equally. In the example shown here, noncooperative marriage for a single period is worse for the husband (and better for the wife) than being divorced for a single period, but the bargained equilibrium $(\bar{u}_h, \bar{u}_w)$ is better for both spouses than divorce. It is not difficult to see that it would be possible

[6] In the Appendix, we present a simple algebraic proof of this proposition. (This proof is not new. A similar argument can be found in Binmore (19897)

to construct examples that fall into Case (i) where a single period of noncooperative marriage is worse for both spouses (or better for both spouses) than a single period of divorce, but where the equilibrium from the noncooperative threat point is better for both spouses than divorce.

Figure 1--Household Bargaining Equilibrium

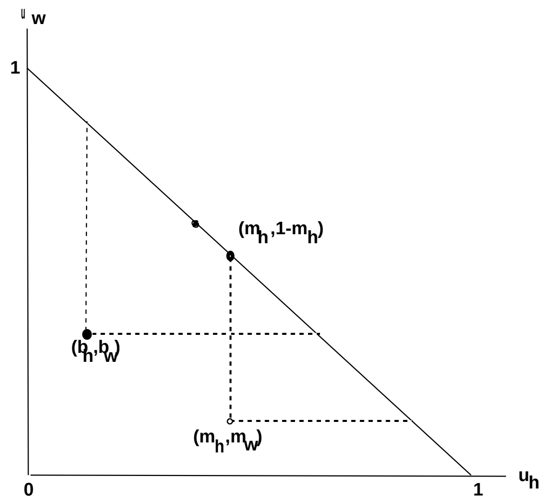
Case (i), Divorce threat is not binding



Case (ii) Divorce threat is credible for the husband, but not for the wife. This happens if $b_h + (1 - b_h - b_w)/2 < m_h$.In this case the solution is $u_h = m_h$ and $u_w = 1 - m_h > m_w$. This case is illustrated in Figure 2. In Case (ii), not only is noncooperative marriage worse for the husband than divorce, but the equilibrium found taking noncooperative equilibrium as a threat point is worse for the husband than divorce. In this case, equilibrium is the outcome where the husband is indifferent between divorce and marriage and the wife has utility $1 - m_h$.

Figure 2--Household Bargaining Equilibrium

Case (ii), Divorce threat binds on u_h



Case (iii) Divorce threat is credible for the wife, but not for the husband. This happens if $b_w + (1 - b_h - b_w)/2 < m_w$. In this case the solution is $u_w = m_w$ and $u_h = 1 - m_w > m_h$.

The first case corresponds to the Lundberg-Pollak cooperative solution where the threat point is not divorce, but a noncooperative marriage. In the other two cases, the divorce threat is relevant, but notice that the outcome is never the outcome predicted by the Mansur-Brown and McElroy-Horney models. In an equilibrium where both persons are better than they would be if divorced, equilibrium is calculated as if the threat point were eternal burnt toast rather than divorce. Small changes in the utility of being divorced would have no effect on the outcome of household bargaining. In the only cases where the divorce threat is relevant, the gains from marriage are not split equally as in the divorce-threat bargaining models. In this case, one partner enjoys all of the surplus and the other is indifferent between being divorced and being single.

To some observers, this model's stately minuet of offer and counteroffer may seem not to reflect the realities of domestic conflict. But Rubinstein's canonical bargaining model can be much relaxed in the direction of realism without altering the main results. Binmore (1985) shows that qualitatively similar results obtain when the length of time between offers and the person whose turn it is to make the next offer are randomly determined after every refusal. It is also a straightforward matter to add a constant probability of death for each partner without seriously changing the model. On the other hand, stationarity of the model seems to be necessary for Rubinstein's beautifully simple result. This stationarity is lacking in a model where children grow up and leave the family and where the probability of death increases with age. It would be useful to know more about the robustness of the Rubinstein results to more realistic models of the family. For the time being, Rubinstein's model and its extensions seem to be "the only game in town" as far as giving us a theoretical basis for distinguishing among plausible alternative bargaining theories of the household.

Marriage Markets for Bargaining Spouses

A satisfactory theory of bargaining between spouses should be embedded in a theory of marriage markets. In this discussion, in order to illustrate issues that arise when marriage markets are combined with bargaining between spouses, we use a much simpler model than is normally dealt with in the marriage market literature. In particular, we make the barbaric assumption that every pair of possible spouses faces the same utility possibility frontier if they marry as every other pair.[7] The only difference between individuals is the utility that they could achieve by remaining single.

Assume that the utility possibility frontier for every married couple is the unit simplex and that there is a continuum of persons of each sex. Let $F_h(u)$ be the number of males in the population for whom the utility of being single is less than u and let $F_w(u)$ be the

[7] The theory of mating and matching, which is thoroughly surveyed by Al Roth and Marilda Satomayor (1990), incorporates models in which different individuals could have arbitrarily different rankings over members of the opposite sex as possible partners. While the theory sketched here should be enriched to incorporate this feature, it seems apparent that the qualitative results found here would extend to such models.

number of females in the population for whom the utility of being single is less than u. Assume that these distribution functions are strictly increasing and continuous, and that $F_h(0) = 0$, $F_h(1) > 0$, $F_w(0) = 0$ and $F_w(1) > 0$.

Let us first think about the marriage market that would arise if it were possible before marriage to determine the distribution of utility within marriages by a binding contract. Then there would be a unique equilibrium utility distribution $(u_h^*, 1 - u_h^*)$ such that the number of males who are willing to marry and get utility u_h^* equals the number of females who are willing to marry and get $u_w^* = 1 - u_h^*$. When the utility distribution between husbands and wives is (u_h, u_w), the supply of men wanting to marry is $F(u_h)$ and the supply of women wanting to marry is $F(u_w)$. The unique equilibrium utility distribution (u_h^*, u_w^*) is found by solving the equation $F_h(u_h^*) - F_w(1 - u_h^*) = 0$.[8]

Suppose, on the other hand, that neither party to a marriage can credibly promise a utility distribution within marriage. Instead the utility distribution within marriages is determined by the model of non-cooperative bargaining that we have just discussed. Suppose that the utility distribution for any couple during a period where they have not reached agreement is (b_h, b_m) and that the time between offer and counteroffer is very short. Then, as predicted in our model of non-cooperative bargaining, the distribution of utility in all marriages will be (approximately)

$$(\bar{u}_h, \bar{u}_w) = \left(b_h + \frac{(1 - b_h - b_w)}{2}, b_w + \frac{(1 - b_h - b_w)}{2} \right).$$

Given this utility distribution within marriages, the number of males who wish to marry will be $F_h(\bar{u}_h)$ and the number of females who wish to marry will be $F_w(\bar{u}_w)$. It is interesting to notice that there is no reason to expect that $F_h(\bar{u}_h) = F_w(\bar{u}_w)$. Therefore, there will in general be either more men seeking wives than women seeking husbands or *vice versa*. The inability to make prior commitments to utility distributions within marriage has the same kind of effect as price inflexibility in a commodity market. If, for example, the equilibrium bargained utility distribution within marriages is such as to leave

[8] Existence follows from the assumption of continuity and the assumption that $F_h(1) - F_w(0) > 0$ and $F_h(0) - F_w(1) < 0$. The assumption that F_h and F_w are strictly increasing functions implies that $F_h(u) - F_w(1 - u)$ is a strictly decreasing function of u. Therefore equilibrium must be unique.

an excess demand for wives, then all women who wish to marry under the current terms of marriage will be able to do so, but some men who want to marry will not find wives. Such a man would be willing to offer more favorable terms for a wife than the current equilibrium utility. If he could make such promises credible, then he would be able to induce some woman who currently prefer remaining single to marry him, but she realizes that once married, they will be playing a bargaining game in which the inevitable result is the equilibrium utility enjoyed by all other married women.

The two best-known theories of marriage assignments are the theory of stable marriage algorithms, developed by David Gale and Lloyd Shapley and the linear programming assignment model which was introduced to economics by Martin Beckmann and Tjalling Koopmans and applied to marriage markets by Gary Becker (1981). Both of these models are more general than the example considered here in that they allow for differences in preference rankings over possible marriage partners. In the Gale-Shapley theory no "side-payments" are allowed and there are no possibilities for negotiation about the terms of marriage.[9] The assignment problem assumes transferable utility, allowing binding premarital agreements on any possible distribution of utility for any possible married couple. The model of bargaining with non-cooperative marriage as the threat point could be applied to the more general environment assumed in these models. In such a model, for any possible marriage there is a unique distribution of utility that will be determined by the utility possibility frontier, the time-discount rates of each party and the distribution of utility that will prevail if they remain married but do not reach agreement. Therefore, the appropriate model would be like the original Gale-Shapley in that each person assigns a fixed utility to each possible marriage partner and that utility can not be altered by proposing different terms of marriage.

[9] Crawford and Knoer (1981) show how the Gale-Shapley algorithm can be extended to allow side payments.

Appendix–The algebra of noncooperative equilibrium

Let u_1^h be the equilibrium utility for the husband if he gets to make the first offer and let u_2^h be his equilibrium utility if the wife gets to make the first offer. Let u_1^w be the equilibrium utility for the wife if she gets to make the first offer and let u_2^w be her equilibrium utility if the husband gets to make the first offer. Let b_h and b_w be the utilities that the husband and wife respectively would get in any period where they do not reach agreement. Let $b_h + b_w < 1$ and let the utility possibility frontier for each period be $\{(u_1, u_2) \geq 0 | u_1 + u_2 = 1\}$. Let us suppose that there if the wife makes the first offer, the equilibrium payoffs will be $\bar{u}_1^w$ for the wife and $\bar{u}_2^h$ for the husband and if the husband makes the first offer, the equilibrium payoffs will be $\bar{u}_1^h$ for the husband and $\bar{u}_2^w$ for the wife.

In the first period, if the husband accepts the offer of $\bar{u}_2^h$, then since the problem is stationary, he will continue to accept $\bar{u}_2^h$ in all subsequent periods. Therefore his utility will be $\sum_0^\infty \bar{u}_2^h \delta^t$. If he rejected her offer, he would receive b_h in the first period and in the next period it would be his turn to make the offer. Then he would demand $\bar{u}_1^h$ and offer his wife $\bar{u}_2^w$ and she would accept the offer and continue to accept $\bar{u}_2^w$ in all subsequent periods. The husband's utility if he follows this strategy would be $b_h + \sum_{t=1}^\infty \bar{u}_1^h \delta^t$. In equilibrium, the husband must be just indifferent between accepting his wife's initial offer and waiting one period to make a counteroffer. This will be the case if $\sum_0^\infty \bar{u}_2^h \delta^t = b_h + \sum_{t=1}^\infty \bar{u}_1^h \delta^t$, or equivalently if

$$\bar{u}_2^h - b_h = \frac{\delta}{1-\delta}(u_1^h - u_2^h). \tag{1}$$

Similarly, it must be that if $\bar{u}_1^w$ and $\bar{u}_2^w$ are equilibrium strategies for the wife, then she will be indifferent between accepting u_2^w if it is her husband's turn to make an offer and refusing his offer and countering with a demand of $\bar{u}_1^w$ in the next period. This leads by an exactly parallel argument to the equation

$$\bar{u}_2^w - b_w = \frac{\delta}{1-\delta}(u_1^w - u_2^w). \tag{2}$$

The feasibility constraints for offers are:

$$\bar{u}_1^w + \bar{u}_2^h = 1 \tag{3}$$

$$\bar{u}_1^h + \bar{u}_2^w = 1 \tag{4}$$

When we solve the linear equations 1-4 for the variables $\bar{u}_1^w$, $\bar{u}_2^w$, $\bar{u}_1^h$, and $\bar{u}_2^h$, we find that the solutions are:

$$\bar{u}_1^w = b_w + \frac{1}{1+\delta}(1 - b_h - b_w),$$
$$\bar{u}_2^w = b_w + \frac{\delta}{1+\delta}(1 - b_h - b_w),$$
$$\bar{u}_1^h = b_h + \frac{1}{1+\delta}(1 - b_h - b_w)$$

and

$$\bar{u}_2^h = b_h + \frac{\delta}{1+\delta}(1 - b_h - b_w).$$

This is the result claimed in the text.

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