Consumer Credit, Unemployment, and Aggregate Labor Market Dynamics

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Abstract

This paper studies the dynamic aggregate relationship between unemployment and consumer credit. First, using micro-level data I identify a consistent negative effect of unemployment on both a household’s use of and access to consumer credit. Upon job loss, households increase applications for credit, get denied more frequently, and experience significant reductions in both debt outstanding and average monthly charges. I interpret these effects as an increase in credit constraints for the unemployed and examine how this relationship impacts macroeconomic variables over the business cycle. To do so, I extend the canonical Mortensen and Pissarides (1994) model of unemployment to include a goods market with search and financial frictions. Households have limited commitment in repaying debts and face endogenous borrowing constraints that are disciplined by the ability of lenders to enforce financial contracts. The model predicts that job loss is followed by a contraction in borrowing constraints. In the aggregate, this channel leads to a strategic complementarity between (un)employment and firm hiring incentives as a higher fraction of unemployed consumers decreases the expected revenue from a labor match. I calibrate the model to match the estimated fall in credit upon job loss and examine how these individual unemployment-credit shocks affect aggregate business cycles. I find that productivity shocks do a poor job of generating the co-movement of credit and unemployment we observe in the data. However, I find that aggregate financial shocks contribute significantly to the observed dynamics of both real and financial variables.

JEL Classification: D53, E24, E32, E44

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1 Introduction

Between the late 1970s and 2007, the U.S. experienced a rapid increase in households’ use of debt to finance consumption. At its peak, consumer credit accounted for nearly a third of personal consumption expenditures.¹ This trend was abruptly reversed during the 2007-2009 Great Recession, which featured a large contraction in the consumer credit market, coinciding with a dramatic decline in consumption spending and historically high unemployment.² A growing body of research suggests that consumer debt is an important channel through which shocks to households get amplified leading to large and persistent responses in consumption. Empirically, this literature finds considerable cross-sectional evidence that regions of the U.S. that had the largest declines in household borrowing during the Great Recession also experienced the largest declines in consumption and employment (Mian and Sufi (2010), Midrigan and Philippon (2011), Mian et al. (2013), Mian and Sufi (forthcoming)). Recent theoretical work has examined the role of shocks to aggregate credit and liquidity constraints as a mechanism that prevents households from smoothing consumption.³

In this paper, I empirically and theoretically examine the effects of credit constraints for a salient group during downturns, job losers. Credit constraints are likely to be relevant for this group as these households value access to credit the most. Additionally, employment status and income are key criteria used by lenders in evaluating the credit-worthiness of borrowers (Crossley and Low, 2012). First, using household-level data from the 2007-2009 panel of the Survey of Consumer Finances, I identify a consistent negative effect of entering into unemployment on both a household’s use of and access to consumer credit. I find that upon job loss, households increase demand for credit, get denied more frequently, and experience significant reductions in both debt outstanding and average monthly charges compared to households that maintained employment between 2007 and 2009. This effect is particularly pronounced for borrowing on credit cards. While I cannot directly observe if the decline in credit for these households translated into a fall in consumption, I do find that there is a significant positive effect of unemployment on the likelihood of having zero liquid assets. This suggests that these households are limited in replacing their lost income by dis-saving.⁴

¹Throughout this paper, I will use the definition of consumer credit from the Federal Reserve Boards Z.1 Flow of Funds Accounts. Consumer credit is mostly comprised of credit cards, auto loans and student debt.
²The fall in consumption during the Great Recession has been extensively documented in the literature. For instance, see de Nardi et al. (2012) and Petev and Pistaferri (2012).
³These include Midrigan and Philippon (2011), Hall (2011), Guerrieri and Lorenzoni (2011), among others. Additionally, there is considerable evidence that households are credit or liquidity constrained. Early work by Zeldes (1989) finds that households in the Panel Study of Income Dynamics with low liquid assets are indeed those households in which the test of the permanent income hypothesis fails. Others include Jappelli et al. (1998), Japelli (1990), Gross and Souleles (2002), and Argarwal et al. (2007). See Jappelli and Pistaferri (2010) for a review of this literature.
⁴I use a broad measure of liquid assets and consider balances in checking, savings, and CD accounts as well as any
I interpret the effect of the fall in consumer debt as an increase in credit constraints for the unemployed and examine if this effect can explain the aggregate co-movement of debt, employment and consumption over the business cycle. To do so, I develop a model that features search in labor and goods markets in which credit is constrained by the ability of lenders to enforce financial contracts. The starting point is the canonical model of equilibrium unemployment by Mortensen and Pissarides (1994), hereafter MP, in which both firms and workers must go through frictional hiring before production can occur. However, in this framework firms are considered to sell their output seamlessly in a competitive, frictionless environment and there is no inherent role for credit in the goods market. I describe a household’s need for liquidity (through credit) by incorporating search and matching frictions in the goods market in the style of Diamond (1990), Shi (1996), and Lagos and Wright (2005). A firm matched with a worker in the labor market produces intermediate output that it can either sell in a decentralized, frictional goods market or in a competitive, frictionless market as in MP. These two markets open sequentially and I assume that households have quasi-linear preferences over consumption in the decentralized and competitive market. This feature, combined with the fact that all labor income (wages and unemployment benefits) is received after the decentralized market closes, generates a need for credit on the side of households.5

The key friction of the model is that households lack commitment to repay debt. The amount of borrowing within the period depends on the ability of lenders to enforce debt contracts. I assume enforcement constraints are a function of both aggregate credit market conditions, similar to those used in the literature on firm financial constraints (i.e. Jermann and Quadrini (2012) and Monacelli et al. (2011)), as well as idiosyncratic household income. Similar to MP, a worker that enters into unemployment experiences a fall in their income. This fall causes the enforcement constraint to become tighter which leads to a fall in borrowing. In the model, firm revenues depend on the extent to which households are credit constrained. A fall in borrowing in the event of a job loss, decreases the demand for the output of a labor match. In equilibrium, this causes a lower number of firms to post vacancies and an increase in unemployment. Household credit constraints generate strategic complementarities, which if strong enough will lead to multiple equilibria as in Kaplan and Menzio (2014) or Bethune et al. (2015).

Finally, I calibrate the model to match the fall in credit for the unemployed between 2007-2009 and examine the importance of this channel in amplifying the response of macroeconomic variables to productivity and aggregate financial shocks. In order to discipline the extent of household financial shocks, I use an approach outlined Jermann and Quadrini (2012) with regards to firm credit. Using data on consumer credit and household income from the Flow of Funds, I construct treasury bills. See Section 2.2.

5This motivation for liquidity can also be found in studies of bank runs a la’ Diamond and Dybvig (1983).
a time series for household financial shocks using the model’s enforcement constraint under the assumption that it is always binding. This methodology is analogous to the standard approach of identifying productivity shocks using Solow residuals from the production function. I compare the response of unemployment and other labor market variables in the model to shocks to labor productivity and financial conditions. As first pointed out in Shimer (2005), and more recently in Hall (2014), productivity shocks in the context of the MP model do a poor job of generating sufficient movement in labor market variables. Similarly, I find that the credit effect of unemployment does not improve the performance of the model in this dimension. However, I do find that aggregate financial shocks contribute significantly to the observed dynamics of the labor market. Financial episodes are particularly pronounced in the Great Recession and the recession of the early 1980s.

This paper is closely related to the literature on financial frictions and unemployment. Wasmer and Weil (2004), Monacelli et al. (2011), Petrosky-Nadeau and Wasmer (2013), and Petrosky-Nadeau (2014) consider how financial frictions facing firms affect hiring and unemployment in the context of the MP framework. I differ in that my focus is on credit to households and financial frictions arise as a consequence of limited commitment and enforcement constraints, whereas in these papers financial frictions are in the form of search frictions. On the household side, recent empirical and theoretical work (including Mian and Sufi (2010), Keys (2010), Mian et al. (2013), Hsu et al. (2014), Haltenhof et al. (2014), Gropp et al. (2014), Athreya et al. (forthcoming), Mian and Sufi (forthcoming), among others) stresses the importance of the household debt channel in accounting for movements in the labor market, particularly during the Great Recession. This paper is the first two connect how credit constraints depend on an individual’s job status and show that these constraints have implications on the labor market during business cycles.

In terms of empirical evidence of credit constraints among the unemployed, this paper is closest to work by Sullivan (2008) and Crossley and Low (2012). Sullivan (2008) finds that low-asset households, or those in the bottom decile of the asset distribution, do not borrow from unsecured credit markets in response to job loss. Using Canadian data, Crossley and Low (2012) find that a quarter of recent job losers could not borrow to increase consumption. I further this work by showing similar patterns for the U.S. and by quantifying the aggregate effects of the credit-unemployment channel. Finally, this paper is complementary to recent work by Herkenhoff (2013) that considers the impact of consumer credit access on unemployment. In his framework, greater access to unsecured credit among the unemployed decreases the consumption decline upon job loss, increases reservation wages, and leads to longer and deeper labor market recoveries. The current paper differs in that I consider the reverse, the impact of unemployment on consumer credit access.

Sullivan (2008) finds that households in the second and third deciles do replace lost income, however they only do so by 11.5 to 13.4 percent.
I show that an income shock, in the form of a job loss, is also a significant, negative credit shock which, in the aggregate, also leads to longer and deeper labor market recoveries.

This paper proceeds as follows. Section 2 analyzes the relationship between consumer credit use and unemployment both in the aggregate and at the micro-level. Section 3 outlines the model. Section 4 gives a theoretical characterization of the equilibrium. Section 5 discusses the calibration, which includes identifying household financial shocks and shows the results of the quantitative experiments. Finally, Section 6 concludes.

2 Evidence on Household Credit and Unemployment in the Data

In this section, I use aggregate and household-level data to analyze the relationship between consumer credit use, unemployment, vacancies, and output over the business cycle. First, using aggregate time series I establish a strong business cycle correlation between consumer credit and macroeconomic variables. The co-movement is most pronounced during the Great Recession and the recessions in the early 1980s. Next, using household-level data I identify a consistent negative unemployment effect on both the access to and use of consumer credit. When a household loses their job, it is more difficult to obtain credit precisely the time when the demand for credit should increase.

2.1 Aggregate Data on Consumer Credit, Firm Entry, and Unemployment

Figures 8 and 9 plot the dynamics of consumer credit outstanding (dashed red line) in relation to key macroeconomic aggregates (solid blue lines). Each series has been logged and de-trended using a Hodrick-Prescott filter with a smoothing parameter of 100,000.\footnote{This parameter is commonly used in the macroeconomic labor literature, for instance see Shimer (2005). In addition to allowing comparison to this literature, there are two other reasons to depart from the standard smoothing parameter used in the real business cycle literature of 1,600. First, as pointed out in Bethune et al. (2012), the standard HP filter is very sensitive to movements at the ends of data series and has a tendency to underestimate the trend component during large contractions. Since I am interested in the dynamics of aggregate time series following the Great Recession, this is problematic. One option is to use a band-pass filter as suggested in Christiano and Fitzgerald (2003). The band-pass filter allows the user to pull out a particular frequency component of a data series, such as the business cycle, and because it is a one-sided filter it has less error at the end points. Another option is to use a simple linear trend. The method chosen in this paper is somewhere in the middle of these two approaches. Secondly, as Reinhart and Rogoff (2009) point out, the duration of financial crises are considerably longer than the period considered in most business cycle studies. This suggests the need to analyze key financial and macroeconomic variables over a longer frequency.} The time series of consumer credit comes from the Flow of Funds Accounts. Consumer credit includes both revolving accounts, such as credit cards, as well as non-revolving accounts, such as automotive and education loans.\footnote{See Appendix 7.5 for a more detailed description of the data sources.}
It does not include loans secured by real estate, such as home equity lines of credit. In Table 1, I report the contemporaneous correlation between consumer credit and standard macroeconomic variables. For reference, I also report the standard deviation of the variable of interest, along with its relative standard deviation to consumer credit.

Consumer credit is pro-cyclical; it tends to decrease relative to trend during recessions and increase during expansions. The magnitude of the fall in credit during the Great Recession was not unique. Similar declines occurred during the recessions of the early 1980s and the time period surrounding the 1991 recession. We see that consumer credit is also highly correlated with movements across labor aggregates. Unemployment and consumer credit have a negative correlation of 61.7%. The co-movement of these two variables is the particularly pronounced during the Great Recession and the recessions in the early 1980s. Additionally, declines in the demand for labor, measured as changes in the amount of job vacancies, and employment also coincide with those in credit. Consumer credit has a 51% correlation with the rate of vacancy creation and a 64.6% correlation with the employment to population ratio.

Additionally, consistent with the fall in labor demand and increase in unemployment, I find evidence of a decline in the demand for firms' output. For instance, the declines in retail trade sales almost identically track the declines in consumer credit. The contemporaneous correlation between sales and credit is 64.3%. Household consumption expenditures, while as a whole are not as volatile as consumer credit, experience the same turning points. Focusing on consumer durable purchases, the two series follow nearly identical deviations from trend, with a correlation of 77.1% and a relative standard deviation of 0.92.

The high correlation between consumer credit, labor, and macroeconomic aggregates illustrates the need for a better understanding of the relationship between household finances, unemployment, and real activity. In the next section, I provide household level evidence that, at least during the Great Recession, consumer credit was more difficult to obtain, limits on credit cards fell, and households decreased borrowing. I finish this section by showing that these patterns are considerably more pronounced if a household lost their job.

2.2 Micro Evidence on Consumer Credit and Unemployment

This section presents evidence on the relationship between consumer credit and unemployment at the household level. The goal is to identify the effect of unemployment on a household’s use of and access to consumer credit. To do so I use data from the Federal Reserve Board’s Survey

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9The notable exception is the recession in 2001 following the stock market bubble when consumer credit accelerated relative to trend. The acceleration of debt during this time period is also evident in other forms of borrowing, namely mortgages.
of Consumer Finances (SCF). The SCF is the preeminent data source used to study household finances in the United States. Regarding consumer credit, it contains detailed information about levels of debt outstanding, credit limits, whether a household applied for credit, and whether they were denied. While the SCF is typically a triennial cross-sectional survey, respondents from the 2007 survey were re-interviewed in 2009, creating a two-period panel data set which allows me to observe changes in both employment and credit at a household level.

Sample Selection and Experimental Design  In order to examine how unemployment affects a household’s access to credit, I use a difference-in-difference approach to compare changes in credit for households that entered into unemployment over the 2007 to 2009 period versus those that remained employed. This time period corresponds to the largest recession in the U.S. since the Great Depression. The aggregate unemployment rate increased from 4.6% to 9.3%. The rate of monthly job layoffs increased from 1.3% of total employment to 1.7%. This equates to an additional five-hundred-thousand jobs lost per month due to involuntary reasons.\footnote{Data from the Job Openings and Labor Turnover Survey (JOLTS). JOLTS defines layoffs and discharges as separations initiated by the employer. These include layoffs with no intent to rehire, formal layoffs lasting or expected to last more than 7 days, discharges resulting from mergers, downsizing or closings, firings or other discharges for cause, terminations of permanent or short-term employees, and terminations of seasonal employees.} The timing of the survey allows me to identify both the effect of the Great Recession on all households’ credit access and use as well as the differential effect from entering into unemployment.

The SCF contains observations at a household level. Since the goal of the experiment is to identify the effect of a change in an individual’s labor market state, I first restrict the sample to single households defined as those that reported not having a spouse or partner as well as not sharing finances with any other person.\footnote{The SCF defines a household as a primary economic unit (PEU). The PEU is defined as the core individual or core couple in a household plus any minor children or other financially interdependent individuals with the core individual or couple. See Bricker et al. (2011) for more details on the design of the 2007-2009 panel survey.} Additionally, I only consider households that stayed single throughout the survey. Doing so mitigates the effects of any changes in debt or earnings due to changes in the number of earners in a household and also makes classifying observations into employment states easier.

I use a broad definition of employment and classify households as employed if they reported that they were either currently working, accepted a job and waiting to start, or were on sabbatical or extended leave and expected to go back to work.\footnote{The survey question asked households about their job status at the date of the interview.} My treatment group consists of households that were employed at the 2007 survey date but reported being unemployed at the 2009 survey date. I compare this group to those households that reported being employed in both the 2007 and 2009 surveys and also reported not having any unemployment spell in the year previous to the 2009
survey date. This limits the potential for respondents that were unemployed between 2007 and 2009, but found work close to the 2009 survey date. Finally, I restrict the analysis to households in which the head is between 20 and 70 years of age. This results in a sample of 3,820 households.

**Consumer Credit and Household Labor, 2007-2009** First, in Table 2, I describe the change in credit and labor market variables for the sample as a whole during the initial two years of the Great Recession. The mean of the variables of interest are reported for 2007 and 2009 in columns (1) and (2), respectively. Column (3) reports the difference in means between the two time periods. In regard to certain variables, the experiences of the sample between 2007 and 2009 coincide with the patterns we observe in the aggregate, discussed in Section 2.1. For instance, I find significant reductions in several measures of credit card use. The fraction of households with at least one credit card fell by 4.3 percentage points, average monthly charges on credit cards fell by $53, and the average debt limit fell by $1,240. Average credit card debt outstanding also fell by $212, though not statistically significant. Surprisingly, opposed to the behavior in the aggregate, automotive loans increased for the sample between 2007 and 2009.

It is difficult to conclude from the evidence in Table 2 if household credit was systematically more difficult to obtain in 2009 compared to 2007 or if households simply decreased their demand for debt. Perhaps the cleanest measure the SCF provides that helps differentiate the two channels is by directly asking respondents (i) if they applied for any credit in the two years previous to the survey and (ii) given they applied for credit, if they were denied. On average, households both applied for less credit and got denied more frequently during the Great Recession. The fraction of households applying for credit fell from 63% to 43%. While we still can not be sure how much of the fall in the rate of credit applications is driven by demand, given that a household might withhold applying for credit if they think they will be denied, we can conclude that those households that wanted credit experienced a fall in their access to it. Conditional on applying for credit, the likelihood of being denied increased from 19% to 25%.

The final two rows of Table 2 show that households’ total income was decreasing, as well as average weekly hours. During a time when income and hours of work are falling, a consumption smoothing motive would suggest that borrowing should increase. However, this is the opposite of what we see in the Great Recession, as consumer debt declined. In the next section, I ask if the

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13 The SCF only asks respondents if they experienced any unemployment spell in the previous 12 months. It is possible that I classify some households as having not entered unemployment but that experienced an unemployment spell between the two survey dates but longer than a year before the last survey date. I drop all households that were employed in 2009, but reported having an unemployment spell in the previous year.

14 I classify a household as having been denied credit if they reported ever being denied a credit application and if they never received that loan upon future applications.
large increase in the number of unemployed households can provide any clarification of the trends we observe in Table 2.

**The Effect of Unemployment on Household Credit, Assets, and Income** This section illustrates the difference in credit, asset, and labor market outcomes for households that entered into unemployment during the initial two years of the Great Recession versus those that did not. To do so, I estimate the following difference-in-difference model:

\[
y_{it} = \beta_0 + \beta_1 EU_i + \beta_2 \mathbb{I}\{t = 2009\} + \beta_3 EU_i \times \mathbb{I}\{t = 2009\} + \beta_4 X_{it} + \epsilon_{it} \tag{1}
\]

The variable \(EU_i\) indicates if the household entered into unemployment between 2007 and 2009. The variable \(\mathbb{I}\{t = 2009\}\) is a dummy for the year 2009. The vector \(X_{it}\) includes observable household characteristics such as age, education, race and sex. The coefficient of interest is \(\beta_3\). It identifies the effect of unemployment on changes in the variable of interest.

The identification relies on the parallel-trends assumption, or that without entering into unemployment, the changes in outcomes for the treatment group would coincide with that for the control group. I argue that the parallel-trends assumption is likely to be valid for two reasons. First, I show in Table 3 the results of a balancing test which suggests the two groups have similar individual characteristics and pre-treatment outcomes. The individuals in the sample who lost their jobs between 2007 and 2009 are no more likely to be male or black, and are weakly more educated than those who maintained employment. Additionally, there is no evidence that either debt outstanding or the incidence of credit use differed between the two groups in 2007. Labor income is also comparable, around $28,000. The notable difference is that those in the treatment group are younger, with an average age of 43 compared to 48 for the control group. This age gap partially explains the differences in total household income, weekly hours worked, and credit card debt limits as younger workers tend to have less non-labor income, work longer hours and have lower credit card limits.\(^\text{15}\)

Secondly, the time period under consideration consisted of a large, exogenous aggregate increase in individuals’ likelihood of being unemployed. The rate of monthly job layoffs increased from 1.3% to 1.7%, or around five-hundred-thousand jobs lost each month for involuntary reasons. This implies that any unobserved characteristics that are correlated with both credit and labor risk are mitigated.

Tables 4 through 7 report the estimated coefficient, \(\hat{\beta}_3\), for different outcome variables. Table 4 shows the results for consumer credit variables. There is a strong negative unemployment effect for

\(^{15}\)One might worry that younger borrowers have higher credit risk as they are the least experienced financially and so would have greater changes in credit constraints during the Great Recession. However, Debbaut et al. (2013) show that young borrowers are among the least likely to experience a serious credit card default.
changes in total consumer debt outstanding. For households that lost their job, credit fell by $2,809. This represents a fall of 60% of debt, on average. This decline is dominated by a fall in credit card debt which decreased by $2,504 more for the treatment group. Table 5 further illustrates the effect of unemployment on credit cards. There is a consistent negative unemployment effect for both the likelihood of having a credit card and the likelihood of using it in a given month. Further, average monthly charges on credit cards fell by $250 more for households that entered into unemployment. These effects are in addition to the evidence that credit use was falling for all households during this time period. There was no significant unemployment effect on the amount of debt outstanding on automotive loans.

Table 6 shows that not only was there a significant negative unemployment effect on the use of consumer credit debt, but that these decreases cannot be explained by a fall in demand. I find that there was a positive unemployment effect on the demand for credit. Households who lost their jobs increased their rate of credit applications by 14% relative to those that maintained employment. However, we see that the rate at which households were being denied for credit increased significantly more for the unemployed. Consumer credit became more difficult to acquire, precisely for the group that should value it the most.

I further test for the possibility of a selection effect driving the rate of credit denials by examining the reason given to the borrower in the event they were denied credit. If we believe the reduction in credit was based on characteristics other than a change in employment status, for instance negative credit history, and if the probability of a household being in the treatment group is correlated with these characteristics, then I could be potentially identifying the selection of households into this group. Columns (3) and (4) of Table 6 show the rate of credit denials for what I term ‘credit-related’ reasons, including having a low credit score and having a history of bankruptcy, and ‘income/employment-related’ reasons, which include lacking a job or insufficient income.\footnote{See Appendix 7.5 for further details.} The rate of denials for credit related reasons showed no differential response for the treatment group. However, there is a positive unemployment effect on the rate of denials for income or employment related reasons. This result, combined with the fact that there are were no differences in any pre-treatment credit denial outcomes, suggests that households who lost their jobs in the Great Recession decreased their credit use as a direct result of facing higher constraints and the primary reason for the increased constraints was (un)employment related.

Finally, Table 7 reports the effect of unemployment on income and assets. First, households who lost their jobs were not able to smooth their income using forms other than labor. Total income for these households fell by $20,000 more than for the control group.\footnote{Total income is defined similarly to the aggregate series in Section 2.1 and includes wages and salaries, income
only partially offset by an increase unemployment benefits of $1,188. Secondly, there is consistent evidence that these households were dis-saving as a result of losing their job. Total liquid assets, measured as all balances in checking, savings, and CD accounts as well as any treasury bills, fell by $5,292 for the unemployed, around half of the 2007 average. Additionally, the unemployment effect explains the entire increase in the fraction of households that were liquidity constrained. I consider a strong measure of liquidity constraints as those households reporting having zero liquid assets at the time of the survey.\textsuperscript{18} Consistent with the evidence in Kaplan \textit{et al.} (2014), there is a considerable amount of hand-to-mouth households in the sample. In 2007, 13\% of the control group had no liquid wealth and the difference for the treatment group was not significant. During the Great Recession the fraction of all households that had no liquid wealth doubled to 26\%, which is entirely explained by those households in the sample that lost their jobs.

\section{A Model of Consumer Credit and Unemployment}

In this section, I present a model of consumer credit and unemployment. In the model, firms and workers meet in a decentralized labor market with search and matching frictions in the style of Mortensen and Pissarides (1994). These firm-worker pairs, then, sell a fraction of their output in a decentralized goods market with search frictions similar to Diamond (1990). I follow Diamond (1990) in assuming that trades in the decentralized goods market occur with pairwise credit. The key friction of the model is that households lack commitment to repay debt. The amount of borrowing depends on the ability of lenders to enforce debt contracts. I assume enforcement constraints are a function of both aggregate credit market conditions, similar to those used in the literature on firm financial constraints (i.e. Jermann and Quadrini (2012) and Monacelli \textit{et al.} (2011)), as well as idiosyncratic income. Similar to Mortensen and Pissarides (1994), a worker that enters into unemployment experiences a fall in their income. This fall causes the enforcement constraint to become tighter which leads to a fall in borrowing.

In the model, firm revenues depend on the extent to which households are credit constrained. A fall in borrowing in the event of a job loss, decreases the demand for the output of a labor match. In equilibrium, this causes a lower number of firms to post vacancies and an increase in unemployment. Household credit constraints generate strategic complementarities, which if strong enough will lead to multiple equilibria as in Kaplan and Menzio (2014) or Bethune \textit{et al.} (2015).

\textsuperscript{18}Holdings of currency is not reported in the SCF. However, the 2010 Survey of Consumer Payment Choice from the Federal Reserve Bank of Boston, suggests that the average amount of cash holdings is $340 and the median is $70. See Foster \textit{et al.} (2013).
3.1 Environment

The model is in discrete time that continues forever. There exists a measure one of households and a large measure of firms. Each period is divided into three stages. In the first stage, households and firms trade indivisible labor services in a labor market (LM). In the second stage, they trade consumption goods with credit in a decentralized market (DM) with search frictions. Finally, in the last stage, wages are paid, debts are settled and trade occurs in a frictionless, competitive market (CM). The consumption good in the CM is treated as the numeraire.

Each household is endowed with one indivisible unit of labor and has expected, lifetime discounted utility of

\[ E \sum_{t=0}^{\infty} \beta^t \left[ \ell(1 - e_t) + v(y_t) + c_t \right], \]

where \( \beta \) is the period discount factor, \( y_t \in \mathbb{R}^+ \) is consumption in the DM, \( c_t \in \mathbb{R} \) is consumption in the CM, \( e_t \in \{0, 1\} \) is time devoted to working and \( \ell \in \mathbb{R} \) can be interpreted as a utility flow from leisure or home production. The utility function in the DM, \( v(y) \), is twice continuously differentiated, strictly increasing, and concave. Further, \( v \) is assumed to satisfy \( v'(0) = \infty \) and \( v(0) = 0 \).HOUSEHOOLDs earn wages, \( w_t \), if employed \( (e_t = 1) \) and income, \( b_t \), if unemployed \( (e_t = 0) \), both in units of the numeraire.

A firm is composed of one job and possesses a technology to transform one unit of labor into \( \bar{z}_t \in \mathbb{R}^+ \) units of intermediate good in the LM. Production occurs at the end of the LM, after matching takes place. Intermediate goods can be costlessly transformed into \( y_t \in [0, \bar{z}_t] \) units of the DM good (determined endogenously) and \( \bar{z}_t - y_t \) units of the CM good. In order to hire in the LM in period \( t \), a firm must post a vacancy at cost \( k > 0 \), in units of the numeraire in period \( t - 1 \).

The LM follows Mortensen and Pissarides (1994) in which households and firms match bilaterally to trade labor services. Let the measure of matches between \( s_t \) searching workers and \( o_t \) job openings be given by \( m(s_t, o_t) \). I assume that the measure of job seekers in period \( t \) is equal to the measure of unemployed households at the end of period \( t - 1 \), \( s_t = u_{t-1} \). The matching function, \( m(s, o) \), has constant returns to scale and is strictly increasing and strictly concave in both of its arguments. Moreover, \( m(0, o) = m(s, 0) = 0 \) and \( m(s, o) < \min\{s, o\} \). Given these assumptions, a worker’s job finding probability is defined as \( m(s_t, o_t)/s_t = m(1, \theta_t) = p(\theta_t) \), where \( \theta_t = s_t/o_t \) is labor market tightness. Similarly, the job filling probability for firms is given by

\[ \text{\textsuperscript{19}} \text{The first assumption is sufficient to guarantee an interior solution to the bargaining problem in the DM. The second assumption is a normalization and helps simplify algebra.} \]

\[ \text{\textsuperscript{20}} \text{For now, I maintain the assumption of no aggregate uncertainty and perfect foresight. In the numerical section, labor productivity will be assumed to follow an AR(1) process } ln(\bar{z}_t) = \rho \ln(\bar{z}_{t-1}) + \epsilon_{z,t}, \text{ where } 0 < \rho < 1 \text{ and } \epsilon_{z,t} \sim N(0, \sigma^2_z). \]
\[ m(s_t, o_t)/o_t = m(1/\theta_t, 1) \equiv f(\theta_t) \]. Matches formed in the LM are exogenously destroyed at rate \( \delta \) at the end of the CM.

The DM has a similar structure to the LM. A measure \( n_t = 1 - u_t \) of retailers (productive firms) and a measure one of households form random bilateral meetings according to the matching technology \( \alpha(n_t) \). Therefore, the probability a household meets a retailer in the DM is \( \alpha(n_t) \) and the probability a retailer meets a household is \( \alpha(n_t)/n_t \). The matching technology is assumed to satisfy \( \alpha'(n) > 0, \alpha''(n) < 0, \alpha(n) \leq \min\{n, 1\}, \alpha(0) = 0, \) and \( \alpha(1) = 1 \). DM matches are destroyed with probability one at the end of the period.

Households trade in the DM through borrowing, but lack commitment to repay their debt. In order to sustain credit relationships, the borrower must face a potential cost of default. I assume that lenders have access to an enforcement technology, which in the event of default allows them to recover up to a fraction \( \nu \) of a household’s current income.\(^{21}\) In the model, \( \nu \) represents aggregate financial conditions that affect all households regardless of employment status.\(^{22}\) Therefore, in the model households are constrained by two dimensions: the ability of lenders to enforce debt contracts, \( \nu \), and the household’s current income. Figure 1 shows the timing of the model.

![Figure 1: Timing](image)

### 3.2 Equilibrium

**Centralized Market (CM)** Consider a household entering the CM in period \( t \). Let \( W_t(d_t, e_t) \) be this household’s value function. Upon entering the CM, a household’s state is comprised of debt

\(^{21}\)It is equivalent to assume that \( \nu \) is the probability that the lender can recover the entire amount of the loan. With probability \( 1 - \nu \), the recovery value is zero. For now I assume that \( \nu \) is constant. In Section 5 I let \( \nu \) be time-varying and follow an AR(1) process and consider aggregate financial shocks as innovations to that process.

\(^{22}\)These could arise from many sources, for instance Herkenhoff and Ohanian (2012) stress the increase in congestion in the foreclosure process during the Great Recession.
obligations $d_t$ owed from trade in the previous DM and employment status $e_t$. Let $U_t(e_t)$ be their value function at the beginning of the LM in period $t$ given by

$$W_t(d_t, e_t) = \max_{c_t} c_t + \ell(1 - e_t) + \beta U_{t+1}(e_t) \tag{3}$$
$$\text{s.t. } c_t + d_t = w_t e_t + b(1 - e_t) + \Delta. \tag{4}$$

Households maximize discounted lifetime utility choosing CM consumption, $c_t$, subject to their budget constraint which states that consumption and debt repayment must equal labor income plus any firm profits, $\Delta$. Substituting the budget constraint (4) into (3), the household’s value function becomes

$$W_t(d_t, e_t) = -d_t + \hat{w}_t(e_t) + \ell(1 - e_t) + \Delta + \beta U_{t+1}(e_t) \tag{5}$$

where $\hat{w}_t(e_t)$ represents labor income, $w_t e_t + b(1 - e_t)$. The use of linearity helps simplify the model in two important dimensions. First, notice from (5) that a household’s lifetime utility is linear in debt, $d_t$. This will help simplify the credit contract in the DM since the surplus from trade will also be linear function of $d_t$. Secondly, linearity implies a household has no desire to smooth the repayment of debt over time and so with-in period debt contracts are weakly optimal in this environment.

The value function of a firm with a filled position at the beginning of the CM with $x_t$ unsold inventories from the previous DM, $d_t$ debt promises, and $w_t$ wage obligations is given by

$$\Pi(x_t, d_t, w_t) = x_t + d_t - w_t + \beta J_{t+1} \tag{6}$$

where $J_{t+1}$ is the value function of the firm at the beginning of the LM in period $t + 1$.

**Decentralized Market (DM) Trade** Next, consider a match between a household and a firm in the DM. The terms of trade are given by the pair $(y_t, d_t)$ which states the amount of DM good the firm transfers to the household, $y_t$, in exchange for $d_t$ units of numeraire to be paid in the subsequent CM.

There are many ways to determine the terms of trade (i.e. proportional or Nash bargaining, Walrasian price setting, etc.). For the benchmark model, I assume that the solution is given by proportional bargaining which guarantees that trade is (pairwise) Pareto efficient and leads to an endogenous firm markup that is convenient in calibrating the degree of firm’s market power.\(^{23}\) The

\(^{23}\)Further, the proportional solution is monotonic in that each individual’s surplus is increasing with the size of the total trade surplus. From Gu et al. (2013), it is known that other, non-monotonic trading mechanisms (i.e. Nash
The proportional bargaining solution is given as the solution to the following problem

\[
\max_{y_t, d_t} u(y_t) + W_t(d_t, e_t) - W_t(0, e_t)
\]

\[\text{s.t. } u(y) + W_t(d_t, e_t) - W_t(0, e_t) = \frac{\mu}{1-\mu} [\Pi_t(\bar{z} - y_t, d_t, w_t) - \Pi_t(\bar{z}, 0, w_t)]
\]

\[d_t \leq \nu_t \hat{w}_t(e_t).
\]

The maximization problem (7)-(8) above is given by Kalai (1977). The solution maximizes the household’s surplus from trade while keeping fixed a proportional split of the total surplus between households and firms. The parameter \(\mu \in (0,1)\) can be interpreted as the household’s bargaining power. Equation (9) is the enforcement constraint. Higher labor income or a better aggregate enforcement technology relaxes the constraint. Notice while \(\hat{w}\) is endogenous, the household and the firm take it as given in the credit contract since wages are determined before the DM.

The bargaining problem (7) - (9) can be simplified by substituting in for \(W_t\) and \(\Pi_t\) from (5) and (6) and combining (7) and (8).

\[
\max_{y_t} \mu [u(y_t) - y_t]
\]

\[\text{s.t. } d_t = (1-\mu)u(y_t) + \mu y_t \leq \nu_t \hat{w}_t(e_t)
\]

The maximand in (10) represents the household’s share, \(\mu\), of the total surplus, \(u(y_t) - y_t\) from DM trade. Equation (11) gives the pricing rule for the transfer from the household to the firm. It says that the wealth the household transfers to the firm is a non-linear function, \((1-\mu)u(y_t) + \mu y_t\), of the firm’s DM output. Let \(y^*\) be the first-best level of output defined as \(u(y^*) = 1\). The solution to (10) - (11) is given by

\[
y_t = y(e_t, w_t) = \begin{cases} y^* & \text{if } (1-\mu)u(y^*) + \mu y^* \leq \nu_t \hat{w}_t(e_t) \\
y_t & \text{s.t. } (1-\mu)u(y_t) + \mu y_t = \nu_t \hat{w}_t(e_t)
\end{cases}
\]

From (11) - (12), we can completely determine the terms of trade from knowledge of the household’s payment capacity \(\hat{w}_t(e_t)\), which depends on their current employment status, \(e_t\), and equilibrium wage, \(w_t\). If the payment capacity is above a certain threshold, \((1-\mu)u(y^*) + \mu y^*\), then the solution to the bargaining problem is to trade the first best level, \(y^*\). Otherwise, households borrow up to or competitive pricing) can lead to endogenous credit cycles in limited commitment economies. With proportional bargaining, it is guaranteed that any endogenous cycles that arise are not due to the trading protocol. See Dutta (2012) for the strategic foundations of the proportional bargaining solution.
their constrained limit and the terms of trade are given by \( \{ y(e_t, w_t), \hat{w}_t(e_t) \} \). In order to simplify notation, I denote the DM consumption of employed and unemployed agents as \( y^1 = y(1, w) \) and \( y^0 = y(0, w) \), respectively.

Let \( V_t(e_t) \) be the household’s lifetime utility upon entering the DM at date \( t \) with employment status \( e_t \). \( V_t \) satisfies

\[
V_t(e_t) = \alpha(n_t)\{ v(y_t) + W_t(d_t, e_t) \} + (1 - \alpha(n_t))W_t(0, e_t)
\]

\[
= \alpha(n_t)\mu[v(y_t) - y_t] + W_t(0, e_t)
\]

where I use (5) and (12) to substitute in for \( W_t(d_t, e_t) \) and \( d_t \) respectively. A household entering the DM gets matched with a firm with probability \( \alpha(n_t) \), upon which they consume a fraction, \( \mu \), of the total surplus from the bilateral relationship. With probability, \( 1 - \alpha(n_t) \), the household does not get matched and enters the CM without any debt. In (16), the household takes the terms of trade (11) - (12) as given. The value function of a firm at the beginning of the DM along the equilibrium path, \( F_t \), is given by

\[
F_t = \frac{\alpha(n_t)}{n_t} \left[ n_t \Pi_t(\hat{z}_t - y_t^1, d_t, w_t) + (1 - n_t)\Pi_t(\hat{z}_t - y_t^0, 0, w_t) \right] + \left[ 1 - \frac{\alpha(n_t)}{n_t} \right] \Pi_t(\hat{z}, 0, w_t)
\]

\[
= \frac{\alpha(n_t)}{n_t}(1 - \mu) \left\{ n_t[v(y_t^1) - y_t^1] + (1 - n_t)[v(y_t^0) - y_t^0] \right\} + \hat{z} - w_t + \beta J_{t+1}
\]

The firm matches with a household in the DM with probability \( \alpha(n_t)/n_t \) and trades \( y_t^e \), \( e = \{0, 1\} \). With probability \( n_t \), they meet an employed household and with probability \( 1 - n_t \) they meet an unemployed household. Further, with probability \( 1 - \alpha(n_t)/n_t \), the firm doesn’t meet a trading partner and carries the full amount of intermediate good, \( \hat{z}_t \), into the CM.

Substituting \( \Pi_t \) from (6) and using the terms of trade (11) - (12), equation (18) gives \( F_t \) as the sum of the firm’s total expected revenue from trade in the CM and DM in terms of the numeraire minus wages, \( w_t \), plus the discounted continuation value of the firm in the following LM, \( J_{t+1} \).

Define the expected revenue as \( z_t \), given by

\[
z_t = \frac{\alpha(n_t)}{n_t}(1 - \mu) \left\{ n_t[v(y_t^1) - y_t^1] + (1 - n_t)[v(y_t^0) - y_t^0] \right\} + \hat{z}
\]

\[24\text{The bargaining contract must also satisfy the household and firm participation constraints given by}
\]

\[
v(y_t) + W_t(d_t, e_t) \geq W_t(0, e_t)
\]

\[
\Pi_t(\hat{z}_t - y_t, d_t, w_t) \geq \Pi_t(\hat{z}_t, 0, w_t)
\]

\[24\text{which never bind given the bargaining solution above.}
\]
Notice that \( z_t \) depends positively on the level of DM trade described by \( y^e \), which is itself a function of wages, \( w_t \).\(^{25}\)

**Labor Market (LM)** Moving to the LM, the value function for a household with access to credit, given employment status, \( e_t \), is given by

\[
U_t(1) = (1 - \delta)V_t(1) + \delta V_t(0) \tag{20}
\]

\[
U_t(0) = (1 - p(\theta_t))V_t(0) + p(\theta_t)V_t(1). \tag{21}
\]

If employed, with probability \( \delta \) the household transitions to unemployment. Likewise, if unemployed, with probability \( p(\theta_t) \), the household finds a job and transitions into employment. Substituting in for \( V_t(e_t) \) in (20)-(21) from (16), yields

\[
U_t(1) = \alpha(n_t)\mu[v(y^1_t) - y^1_t] + (1 - \delta)W_t(0,1) + \delta W_t(0,0) \tag{22}
\]

\[
U_t(0) = \alpha(n_t)\mu[v(y^0_t) - y^0_t] + (1 - p(\theta_t))W_t(0,0) + p(\theta_t)W_t(0,1) \tag{23}
\]

Households have an expected surplus from DM trade equal to the first term in (22)-(23). Otherwise, the progression of a household through the labor market is similar to that in Mortensen and Pissarides (1994). If employed, with probability \( (1 - \delta) \) the household maintains their job or with probability \( \delta \) they get separated. If unemployed, a household finds employment with probability \( p(\theta_t) \) and with probability \( 1 - p(\theta_t) \) they have to continue searching in the following period. The firm’s value function follows similarly. Let \( J_t \) be the expected lifetime value of a firm with a filled vacancy at the beginning of the LM, given by

\[
J_t = (1 - \delta)F_t + \delta V_t \tag{24}
\]

where \( V_t \) is the value function of a vacant firm. From (24), a firm gets exogenously destroyed with probability \( \delta \) and must wait a period before searching for a worker. Otherwise, the firm enters the DM with expected value \( F_t \). Free entry of firms to the matching process guarantees that the value of a vacancy must be zero for all \( t \), \( V_t = 0 \). Substituting in for \( F_t \) from (18), we can write

\[
J_t = z_t - w_t + \beta(1 - \delta)J_{t+1} \tag{25}
\]

Notice, \( z_t \) is a function of DM trades, \( (y^0_t, y^1_t) \), and the current measure of employed workers, \( n_t \). A higher level of DM trade leads to higher expected firm revenue; \( \partial z_t / \partial y^e_t \geq 0 \). On the other

\(^{25}\)Sometimes I will make explicit the dependence of \( z_t \) on wages and refer to \( z(w_t) \).
hand, firm entry has an ambiguous effect on $z_t$. With constant returns to scale matching, the probability an individual firm is matched with a worker is decreasing in the measure of firms, $n_t$. Higher unemployment is good for vacant firms because it becomes more likely to find a match in the LM. However, higher unemployment is bad for firms that stay filled because a higher fraction of unemployed workers implies that firms are more likely to meet a consumer in the DM that is more credit constrained. Hence, the sign of $\partial z_t / \partial n_t$ is ambiguous.

**Wage Determination**  I assume wages are chosen such that the surplus generated in an employment match is proportionally split between the household and firm according to exogenous shares $\lambda$ and $1 - \lambda$, respectively. That is, I assume wages in period $t$ are given by

$$V_t(1) - V_t(0) = \frac{\lambda}{1 - \lambda} J_t \tag{26}$$

The wage outcome in (26) is given as the solution to the proportional bargaining problem in Kalai (1977) where $V_t(1) - V_t(0)$ is the household’s surplus from being employed and $J_t$ is the firm’s surplus from having a filled position. Using (16) and (18), Appendix 7.2 derives the equilibrium wage equation

$$w_t = \lambda[z_t(w_t) + \theta_t k] + (1 - \lambda)(b + \ell - \alpha(n_t)\mu[S^1(w_t) - S^0]) = \Gamma_t(w_t) \tag{27}$$

where $S^0 = v(y_t^0) - y_t^0$ and $S^1 = v(y_t^1) - y_t^1$ represent the joint surplus from a DM match with unemployed and employed households, receptively. Notice in (27), I make explicit the dependence of $y_t^1$ on wages through the loan contract (12). The wage is a weighted average of the firm’s revenue augmented by average recruiting costs per vacancy, $\theta k$, and a household’s flow utility from being unemployed augmented by the net utility cost of potentially losing access to credit. The equilibrium wage is a fixed point of $w_t = \Gamma_t(w_t)$.

A higher wage relaxes credit constraints for employed workers, which has two effects on $\Gamma_t(w_t)$. First, higher credit implies more trade in the DM between firms and employed workers, $\partial y^1 / \partial w > 0$, which increases a firms’ expected revenue, $z_t$. This effect leads to a larger surplus in a labor match which puts upward pressure on wages. Secondly, as credit expands for employed workers, the household’s outside option in labor bargaining is negatively effected. Unemployment not only coincides with a fall in income, but also a shock to credit constraints. Which effect dominates depends on the relative bargaining power of households in labor and goods markets. To see this,
we can substitute in for $z(w_t)$ using (19) and express $\Gamma_t(w_t)$ as

$$
\Gamma_t(w_t) = \lambda \left[ \alpha(n_t) \frac{(1 - \mu) n_t}{n_t} (w_t^0 + (1 - n_t) S^0) + \bar{\alpha}_t + \theta_t k \right] + (1 - \lambda) \left[ (\ell + b) - \alpha(n_t) \mu [S^1(w_t) - S^0] \right]
$$

$$
= w_t^{DMP} + (\lambda - \mu) \alpha(n_t) [S^1(w_t) - S^0] + \lambda \alpha(n_t) (1 - \mu) S^0
$$

(28)

(29)

where $w_t^{DMP} = \lambda (\bar{\alpha} + \theta_t k) + (1 - \lambda)(\ell + b)$, which is identical to the equilibrium wage in the Pissarides (2000) textbook model. It coincides with the equilibrium wage in this model if there were no credit (i.e. $\nu = 0$). The first and last term in (29) are constant with respect to the wage in the match. The second term depends on the surplus in the goods market between a firm and an employed worker. The sign depends on the bargaining power of the household as a worker in the LM relative to their bargaining power as a consumer in the DM. If $\lambda$ is higher, then the first effect discussed above dominates and wages are inflated due to the positive effect on firm revenue. However, in the opposite case when a household’s bargaining power is higher in the goods market, the second effect dominates which creates downward pressure on wages. If this negative effect is large enough, the wage could fall below $w_t^{DMP}$. Under this scenario, the introduction of household credit creates a ‘liquidity discount’ on wages equal to the additional value households place on employment by increasing their access to credit. Figure 2 illustrates the determination of wages under the two cases discussed above.

If $w_t = 0$, then $\Gamma_t(0) = w_t^{DMP} + \left[ \lambda \frac{(1 - n_t)}{n_t} (1 - \mu) + (1 - \lambda) \mu \right] \alpha(n_t) S^0$, denoted as $\zeta_t$ in the figure. In Case (a), $\lambda > \mu$ and the equilibrium wage is higher than in an environment without credit, $w_t^a > w_t^{DMP}$. Case (b) illustrates the opposite when $\lambda < \mu$ and the equilibrium wage is lower than in an environment without credit, $w_t^b < w_t^{DMP}$. Lemma 1 makes this precise.

**Lemma 1** There exists a unique, positive solution to (27). Additionally,

(a) If $\lambda > \mu$, then $w_t \in [\zeta_t, \bar{w}_t]$ where $\bar{w}_t = w_t^{DMP} + \alpha(n_t) (1 - \mu) [u(y^*) - y^*]$. Hence, $w_t \geq w_t^{DMP}$ for any $(u, \theta)$ combination since $\zeta_t \geq w_t^{DMP}$.

(b) If $\lambda < \mu$, then $w_t \in [w_t, \zeta_t]$ where $w_t = w_t^{DMP} - (\mu - \lambda) \alpha(n_t) [u(y^*) - y^*]$. If $\zeta_t - w_t^{DMP} < (\mu - \lambda) \alpha(n_t) S^1(w_t^{DMP})$, then $w_t < w_t^{DMP}$. Otherwise, $w_t \geq w_t^{DMP}$ as in (a).

If household’s have more bargaining power as workers than as consumers, then there is a positive credit externality on wages. In the other case, if households have more bargaining power as workers than as consumers, then there is a positive credit externality on wages. It is also possible under case (b) that the third term in (29) is large enough such that wages are higher than $w_t^{DMP}$.

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Notice it is also possible under case (b) that the third term in (29) is large enough such that wages are higher than $w_t^{DMP}$.

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consumers than as workers, and the net surplus of credit to unemployed households in the labor match isn’t too big, \( \zeta_t - w_t^DMP < (\mu - \lambda)\alpha(n_t)S^1(w_t^DMP) \), then there is a negative credit externality on wages.

**Firm Entry and Unemployment** Plugging in the wage from (27) into (25) we can derive the difference equation for the value of a filled job as

\[
J_t = S^f(n_t, \theta_t) + \beta(1 - \delta)J_{t+1}
\]  

Equation (30) gives a familiar law of motion for the value of a filled job. The function \( S^f \) represents the firms share the total surplus from a labor match equal to

\[
S^f(n_t, \theta_t) = (1 - \lambda)[\bar{z}_t - (b + \ell)] - \lambda \theta_t k
\]

\[
+ (1 - \lambda)\frac{\alpha(n_t)}{n_t}(1 - \mu)[n_tS^1 + (1 - n_t)S^0] + (1 - \lambda)\alpha(n_t)\mu[S^1 - S^0]
\]  

\[\text{Figure 2: Wage Determination}\]
where $S^1$ is a function of $n$ and $\theta$ through its dependence on the wage given by (27).\footnote{\textit{S}^0$ is a function of non-employment income $b$ and other exogenous parameters, such as aggregate financial conditions, $\nu$. See equation (12).} The first two terms in (31) are standard and equal to the firms share of exogenous output minus a worker’s outside option in an environment with no credit, $b + \ell$, adjusted for the costs of vacancy creation. The last two terms are novel. The first is equal to the firm’s share of the additional expected revenue of a labor match from operating in the DM. The second term represents rents the firm collects through wage bargaining, equal to their share of the the household’s cost of loosing access to credit upon unemployment. Lemma 2 characterizes the comparative statics when $\lambda > \mu$.\footnote{This case is informative for the quantitative section because under all of the calibrations considered, $\lambda > \mu$.}

**Lemma 2 (Comparative Statics of $S^f(n, \theta)$)** Let $S^f(n, \theta)$ be given by (31).

(i) $\partial S^f/\partial \theta \leq 0$ for all $n$.

(ii) Suppose $S^0 = 0$ (i.e. $b = 0$). Then $\partial S^f/\partial n \geq 0$ for all $n$.

(iii) Suppose $S^0 \neq 0$. Then the sign of $\partial S^f/\partial n$ is in general ambiguous, but must change sign from positive to negative.

The value of the firm’s labor surplus is weakly decreasing in labor market tightness. In the extreme case, if unemployed households are completely denied credit, $S^0 = 0$, the effect of higher employment on the labor match is always positive. This is because higher employment unambiguously leads to a larger surplus in the DM. If $S^0 > 0$, there are two forces at play when employment increases. First the rate of finding a trading partner in the DM for the firm declines. The sign of $\partial(\alpha(n)/n)/\partial n$ is negative given a constant returns to scale matching function. However in general, and increase in $n$, increases wages which has a positive effect on $S^f$.

The law of motion for unemployment also follows the standard difference equation

$$u_{t+1} = (1 - p(\theta_t))u_t + \delta(1 - u_t). \quad (32)$$

From (32), the measure of unemployed households in period $t + 1$ is equal to the fraction of unemployed households in period $t$ that did not get matched in the previous LM, $(1 - p(\theta_t))u_t$, plus the fraction of employed households that became separated from their job $\delta(1 - u_t)$ between periods. We are now ready to define the equilibrium for the perfect-foresight economy.

**Definition 1** A discrete-time perfect-foresight equilibrium is given by the sequence $\{u_t, J_t\}_{t=0}^\infty$ satisfying (30) and (32) such that $u_0$ is given and $\lim_{t \to \infty} J_t$ is finite.
Given a series for \( J_{t+1} \), we can determine labor market tightness, \( \theta_t \), as the solution to the free entry condition, \( V_t = 0 \) \( \forall t \). This implies that \( k = \beta f(\theta_t) J_{t+1} \) in every period, where \( \partial \theta_t / \partial J_t > 0 \).

Wages are determined by (27) which, given \( b \), pin down the level of trade in DM, \( (y^1, y_0) \).

Letting \( w_t = w(u_t, J_t) \) represent the solution to (27) and the free entry condition \( \theta(J_t) = f^{-1}(k / \beta J_t) \), we can characterize the

\section{Equilibrium Characterization}

\subsection{Steady States in a Simplified Model}

In this section I characterize all steady-state equilibria in the perfect foresight economy under the extreme case that unemployed households receive no credit (i.e. when \( b = 0 \)). For convenience, I also assume that \( \lambda > \mu \) as in case (a) in Lemma 1. A steady-state is a point in \( (u, J) \) space such that the unemployment rate and value of a firm are constant through time. Steady state unemployment is given as a function of \( J \) by (32) setting \( u_t = u_{t+1} \):

\[ u = \frac{\delta}{\delta + p(\theta(J))} \]  

(33)

This is the standard Beveridge curve derived in Mortensen and Pissarides (1994). It equates the flow of households into and out of unemployment. Steady state unemployment is the ratio of total separations to total labor market ‘churn’, given by the total rate of separations and job finding. Let \( \bar{u} \) denote the lower bound for unemployment given as \( u = \delta / (1 + \delta) \). Since for all values of \( J \) above \( \beta k \), \( \theta'(J) > 0 \), in the limit as \( J \to \infty \) we have that \( \theta \to \infty \). Given the assumptions on \( p(\cdot) \), \( \theta \to \infty \) implies \( p \to 1 \). If \( J < \beta k \), the labor market shuts down and \( \bar{u} = 1 \). In Figure 3, (33) is represented as the downward-sloping blue line. Using (30), we can characterize \( J \) as a function of \( u = 1 - n \):

\[ J = \frac{S_f(1 - u, \theta(J))}{1 - \beta(1 - \delta)} \]  

(34)

Given \( u \), the value of a filled job is the solution to the fixed point problem in (34). Lemma 3 this problem is well defined.

\textbf{Lemma 3} There exists a unique, positive solution to (34), \( J^* \). Additionally, for any \( u \in [\underline{u}, \bar{u}] \), \( J^* > J^{DMP} \), where \( J^{DMP} = [(1 - \lambda)(\bar{z} - \ell) - \lambda \theta k] / (1 - \beta(1 - \delta)) \) and \( \partial J^*/\partial u \) takes on the sign of \( \partial S_f / \partial u \).
The right panel of Figure 3 illustrates (33) and (34) for three possible cases. In the first case, the top red dotted line, there exists a unique steady state. In this case, the credit effects of unemployment are not strong enough to generate multiplicity. In the second case, the solid green line, the complementarities between credit and hiring lead to two steady-state equilibria. Across these equilibria unemployment and credit (and vacancies) are negatively correlated. In the third case, the bottom dashed purple line, there also exists two steady-state equilibria, however as unemployment increases credit eventually shuts down and \( J^* = J^{DMP} \). In the figure, this leads to labor market autarky and \( u^* = 1 \). The left panel of Figure 3 illustrates a case when the credit channel is strong enough to generate multiple equilibria, but in the event credit shuts down, the labor market is still active. Under this case, there are three steady-state equilibria.

Consider an exogenous increase in the output of a labor match, \( \bar{z} \). The \( \dot{J} = 0 \) locus shifts up, for instance moving from the solid-green line to the dotted-red line in Figure 3. Unemployment decreases in the ‘good’ steady-state, or the one with the lowest unemployment rate, and the value of a filled job increases. If the increase in \( \bar{z} \) is large enough, the multiplicity of steady-states vanishes. A similar effect occurs with an increase in aggregate financial conditions, \( \nu \). The \( \dot{J} = 0 \) nuclei shifts up for every value of \( u \).

Figure 3: Steady States

4.2 Dynamics in a Simplified Model

In this section, I characterize the perfect-foresight dynamics in the simplified environment considered in Section 4.1. I first focus on properties of equilibria in a neighborhood of the set of stationary
equilibria and then characterize the environment’s global dynamics, or those dynamics beginning anywhere in the \((u, J)\) domain. It is helpful to first transform the equilibrium conditions (30) and (32) from discrete time into continuous time.\(^{29}\) The following defines all perfect-foresight equilibria in continuous time.

**Definition 2** A continuous-time, perfect-foresight equilibrium is given by the path \(\{u_t, J_t\}_t\) such that

(i) For all \(t \geq 0\), \(u_t\) satisfies the law of motion

\[
\dot{u}_t = \delta - [p(\theta(J_t)) + \delta]u_t
\]

(ii) For all \(t \geq 0\), \(J_t\) satisfies the no arbitrage condition

\[
\dot{J}_t = (r + \delta)J_t - S_f(1 - u_t, \theta(J_t)) (36)
\]

(iii) \(\lim_{t \to \infty} J_t\) are finite and \(u_0\) is given.

The functions \(S_f\) is defined in (31) and is independent of time, \(t\). First, consider the dynamics in a neighborhood of the steady state. Approximating the system (35) -(36) by linearizing around a given steady state \((u^*_t, J^*_t)\) yields

\[
\begin{pmatrix}
\dot{u}_t \\
\dot{J}_t
\end{pmatrix} \approx \begin{pmatrix}
u_t - u^*_t \\
J_t - J^*_t
\end{pmatrix},
\]

where the \(M\) represents the Jacobian of the system (35) -(36) given by

\[
M = \begin{pmatrix}
-[p(\theta(J^*)) + \delta] & -p'(\theta(J^*))\theta'(J^*)u^* \\
-S^f_1(1 - u^*, \theta(J^*)) & (r + \delta) - S^f_2(1 - u^*, \theta(J^*))\theta'(J^*)
\end{pmatrix}
\]

The system is characterized by one forward looking ‘jump’ variable, \(J_t\), that is dynamically unstable and one backward looking ‘predetermined’ variable in \(u_t\) that is dynamically stable. The local dynamics can be characterized by solving for the sign of the two eigenvalues of \(M\). The top two elements in (38) are both negative. Part (i) of Lemma 2 guarantees that \(S^f_2 \leq 0\) for all \(u\) and hence the bottom-right element is positive in sign. Additionally, from part (ii) of Lemma 2 and the fact

\(^{29}\)See Appendix 7.4 for a detailed discussion on transforming the model from discrete into continuous time.
that \( b = 0 \) in the environment considered, the sign of \( S_1^f(1 - u^*_i, \theta(J^*_i)) \) is weakly positive and hence the bottom-left element of (38) is weakly negative. Therefore, it is straightforward to show that the eigenvalues must be opposite in sign since \( \det(M) \leq 0 \).

Figure 4 illustrates two examples of dynamic equilibria. The left panel shows the case in which the complementarities between household credit and firm revenue are not strong enough to generate multiple equilibria. In this case there is a unique saddle path that is downward-sloping in the \((u, J)\) domain. This case is illustrative in comparison to the dynamics of the standard Mortensen-Pissarides model in which the saddle path is a horizontal line (i.e. Pissarides (2000), Section 1.7). Suppose unemployment starts above its steady state value. When the saddle path is flat, the value of a filled job (as well as labor-market tightness) remains constant as the economy converges to the steady state. As unemployment decreases, firms post less vacancies since lower unemployment implies a lower job-filling rate. In equilibrium, the decline in vacancy posting exactly corresponds to the decline in unemployment. However if the saddle path is downward-sloping as in the top-left panel of Figure 4, labor market tightness increases along the transition. In this case, vacancies must fall slower than unemployment. The reason is due to the credit effect of unemployment: higher unemployment is good for firms filling vacancies but lowers the value of a filled job.

The right panel provides an example with multiple steady-state equilibria. The dynamics are characterized by two saddle paths that cross through the steady states with the lowest unemployment rate (point A) and the highest unemployment rate (point C). The intermediate steady state is a sink. For any initial values, \((u_0, J_0)\), within the grey-shaded region in Figure 4, the economy makes counter-clockwise cycles converging to point B. For any initial value of \(u_0\), there exist an infinite, bounded measure of equilibria converging to the intermediate steady state and two unique equilibria converging to either the high or low steady state. This example illustrates that if the negative credit effect of unemployment is strong enough, expectations about future unemployment can have real macroeconomic consequences, as in endogenous credit-unemployment cycles.\textsuperscript{30}

\textsuperscript{30}In Section 5 below, all the calibrations considered deliver a unique steady state, as in the left panel of Figure 4.
5 Quantitative Analysis

The primary experiment considered in this section examines if the negative effect of unemployment on consumer credit identified in Section 2.2 has an impact on business cycles in the aggregate. To do so, I consider two sources of exogenous fluctuations. The first are standard: shocks to aggregate labor productivity as in Shimer (2005). The second source are aggregate financial shocks that affect all households. These shocks have been stressed in the literature on credit frictions on the side of firms. For instance Jermann and Quadrini (2012) find that shocks to a firm’s ability to raise funds through debt markets contributes significantly to the dynamics of macroeconomic aggregates. I consider a similar shock, though now on households’ ability to finance consumption, through an exogenous change in lenders’ ability to enforce financial contracts.

5.1 Calibration

The period in the model is set to a quarter. I set agent’s discount factor to $\beta = 0.99$. All empirical targets represent quarterly averages over the time period 1978 Q1 to 2007 Q4. The model is first solved by a projection algorithm in which expectations are computed using Gauss-Hermite quadrature.\footnote{As stressed in Petrosky-Nadeau and Zhang (2013), it is imperative to use global solution algorithm in quantifying the dynamics of the DMP model as log-linearization understates the mean and volatility of unemployment.} I then simulate the model to compute the moments for calibration.
Labor Market  The calibration of the majority of the labor market parameters follows closely to the labor search literature following Shimer (2005), and more recently the literature on financial frictions and unemployment (i.e. Petrosky-Nadeau and Wasmer (2013), Petrosky-Nadeau (2014), and Monacelli et al. (2011)). First, I assume aggregate labor productivity fluctuates over time according to an AR(1) process

\[
\ln(\bar{z}_{t+1}) = \rho \bar{z}_t \ln(\bar{z}_t) + (1 - \rho)\ln(\mu_{z}) + \epsilon_{\bar{z},t} \quad \text{s.t.} \quad \epsilon_{\bar{z},t} \sim N(0, \sigma_{\bar{z}}^2) \tag{39}
\]

I normalize \( \mu_{\bar{z}} = 1.0 \). Using the Bureau of Labor Statistics series on output per worker, I estimate \( \rho_{\bar{z}} = 0.889 \) and \( \sigma_{\bar{z}} = 0.0075 \). For the matching technology, I use a constant returns to scale function as suggested in den Haan et al. (2000), \( m(s,o) = so/(s^{\eta_L} + o^{\eta_L})^{1/\eta_L} \), which has the nice property of binding matching probabilities between zero and one. I set the curvature parameter, \( \eta_L \), to match the average job finding probability. Following Shimer (2012), I first estimate the monthly job finding probability according to \( p_t = 1 - (U_{t+1}/U_t) \), where \( U_L \) represents the number of workers with an unemployment duration above 5 weeks and \( U \) is the total number of unemployed workers.\(^{32}\) I find that \( p = 0.601 \). This implies that \( \eta_L = 1.200 \), consistent with the estimate in den Haan et al. (2000). The household’s bargaining power corresponds to an egalitarian solution, \( \lambda = 0.5 \). Vacancy posting costs, \( k \), are set to match 13% of quarterly wages as suggested by Silva and Toledo (2009). I set the exogenous job destruction rate, \( \delta \), to target an unemployment rate of 6.1%, the average over the sample considered.

The remaining two parameters associated with the labor market are the value of leisure, \( \ell \), and non-employment income, \( b \). In an equilibrium with binding debt constraints, the fall in credit upon job loss exactly coincides with the fall in labor income. To see this, suppose (9) holds with equality for both employed and unemployed households. We can measure the proportional fall in credit as \( 1 - (d^0/d^1) = 1 - (b/w) \). Non-employment income is crucial in disciplining the strength of the complementarities between credit and unemployment in the model. To set \( b \), I use the evidence discussed in Section 2.2 and target a 60.4% decline in a households access to credit in the event of a job loss.I then set the value of leisure such that the total labor surplus is 71% of the average wage as in Hall and Milgrom (2008).

Credit and Goods Market  In general, the frictions in the goods and credit market are designed to capture the inefficiencies of the process of getting produced products into the hands of consumers. Fluctuations in these inefficiencies are what induce fluctuations in unemployment. In the model, matching frictions, \( \alpha \), determine the frequency of a household’s liquidity needs. Alternatively,

\(^{32}\)Data are from the monthly Current Population Survey (CPS).
credit frictions capture the difficulty households face in financing the purchases of goods once the liquidity shock is realized. To calibrate the parameters of the credit and goods market, I use data on household consumer credit use and firms market power in the retail sector.

The matching technology in the DM is also chosen to follow den Haan et al. (2000), \( \alpha(n) = n/(n^{\eta_G} + 1)^{1/\eta_G} \). \(^{33}\) I set the curvature parameter, \( \eta_G \), to target the ratio of consumer credit outstanding to output. During the sample period, this averaged 9.47%. In the model total debt is \( D = \alpha(n_t)[n_t d_t^1 + (1 - n_t) d_t^0] \). Total output, across the DM and CM, is \( Y = n_t z_t - \theta_t u_t k \). Hence I set \( D/Y = 0.097 \), which results in \( \eta_G = 1.27 \). This implies an average matching rate of \( \alpha = 0.56 \), or that liquidity shocks occur every 1.8 quarters.

Utility over DM consumption is given by, \( \nu(y) = y^{1-\gamma}/(1-\gamma) \). The elasticity parameter is set to target the marginal propensity to consume (MPC) out of an increase in debt limits of 14%, as given in Gross and Souleles (2002). In the model the MPC of an agent in employment state \( e \) is given by \( MPC^e = \mu[v'(y^e) - y^e]/[(1 - \mu)v'(y^e) + \mu] \). The aggregate MPC is then given by \( MPC = nMPC^1 + (1 - n)MPC^0 \). I set \( MPC = 0.14 \), which implies \( \gamma = 0.4 \). The household’s bargaining weight, \( \mu \), is set to match a gross retail markup of 39%.\(^{34}\)

To discipline aggregate financial conditions, \( \nu_t \), I follow the approach outlined in Jermann and Quadrini (2012). I first construct a series for \( \nu_t \) using the enforcement constraint (9) under the assumption that it is always binding. That is, I assume

\[
d_t^1 = \nu_t w_t
\]

\[
d_t^0 = \nu_t b
\]

In the aggregate, this implies that total debt, \( D_t \), is equal to

\[
D = \alpha(n_t)[n_t d_t^1 + (1 - n_t) d_t^0] = \nu_t \alpha(n_t)[n_t w_t + (1 - n_t)b] = \nu_t \alpha(n_t)I
\]

Replacing \( d_t^1 \) and \( d_t^0 \), we can express total debt as a fraction of total income as

\[
\alpha(n_t) \nu_t = \frac{\alpha(n_t)[n_t d_t^1 + (1 - n_t) d_t^0]}{[n_t w_t + (1 - n_t)b]}
\]

The numerator is equal to total borrowing in meetings with employed and unemployed agents, multiplied by the measure of agents taking loans, \( \alpha(n)n \) and \( \alpha(n)(1 - n) \), respectively. The de-

\(^{33}\)This specification is also a general form of the matching technology used in the early New Monetarist literature, i.e. Kiyotaki and Wright (1993).

\(^{34}\)Derived from retail gross margins. The average ratio of gross retail margins to sales from 1992-2007 was 0.28. Hence the average markup is calculated by 0.28/(1 – 0.28) = 0.39. See https://www.census.gov/retail.
nominator is total labor income, \( n_t w_t \), plus unemployment benefits, \((1 - n_t)b\). Empirically, for the numerator I use the aggregate time series for consumer credit outstanding, minus education loans, in the household sector from the Flow of Funds Accounts.\(^{35}\) For the denominator, I use the aggregate time series for disposable personal income from the BEA.\(^{36}\) Given an estimate for \( \eta_L \) in the matching function, I am able to fully identify \( \nu_t \) from (43). Notice that disciplining the financial shock in this way, assumes that movements in total consumer credit to disposable income in the data are generated from two sources, \( \alpha(n_t) \) and \( \nu_t \). Feeding the constructed series of \( \nu_t \) back into the model will, by itself, only generate the series of credit to income in the data if doing so also matches the series of \( u_t \), which I don’t calibrate to. After constructing the series for, \( \nu_t \), I estimate the autoregressive process

\[
\ln(\nu_{t+1}) = \rho \ln(\nu_t) + (1 - \rho)\ln(\mu_\nu) + \epsilon_{\nu,t} \quad \text{s.t.} \quad \epsilon_{\nu,t} \sim N(0, \sigma_{\nu}^2)
\] (44)

The estimation yields \( \mu_\nu = 0.928 \), \( \rho_\nu = 0.960 \), and \( \sigma_\nu = 0.0165 \). The top two panels of Figure 5 show the constructed series for \( z_t \) and \( \nu_t \), respectively. The bottom two panels show the innovations, \( \epsilon_{z,t} \) and \( \epsilon_{\nu,t} \). Since 1978, labor productivity has fluctuated around 3-4% of its long-run average. Those fluctuations mostly arise from the persistence parameter in the AR(1) process. However, similar to what Jermann and Quadrini (2012) find with respect to business credit, I find that household financial conditions are largely driven by innovations in the process. Table 8 summarizes the choice of functional forms and Table 9 gives the calibrated parameters.

### 5.2 Quantitative Results: Productivity versus Credit Shocks

In this section, I use the calibrated model to examine if the fall in credit upon job loss is enough to explain the co-co-movement of credit and unemployment in the aggregate. I consider two exogenous sources of fluctuations: aggregate productivity, \( z_t \) and aggregate financial conditions, \( \nu_t \). For each case, I feed the estimated shock process, \( \epsilon_{z,t} \) or \( \epsilon_{\nu,t} \) into the model while keeping constant the other variable at its unconditional mean, \( \mu_z \) or \( \mu_\nu \). The model implied series are then logged and de-trended using a Hodrick-Prescott filter with smoothing parameter of 100.00 for comparison to the data shown in Section 2.1.

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\(^{35}\)Total consumer credit covers most short and intermediate-term credit arrangements. However, data on student loans suggests many of these loans are deferred for several years. Since the objective of the quantitative exercise is to see how borrowing in the current quarter affects sales for same quarter, I exclude this type of debt from the exercise.

\(^{36}\)See NIPA Table 2.1 Personal Income and Its Disposition. This measure of income is broader than that in the model. It includes income from other, non-labor, sources including receipts on assets, dividend or interest payments, and other transfer receipts besides unemployment insurance (i.e. social security). I could alternatively only use data on wages and salaries plus unemployment benefits. Doing so would only increase the estimated volatility in aggregate financial conditions.
Figure 5: Stochastic processes of labor productivity and aggregate financial conditions.

Productivity Shocks  Figure 6 illustrates the effects of productivity shocks on the unemployment rate, vacancies, output, and total consumer credit. The model series are in red and their counterpart in the data is represented in blue. First, productivity shocks have a limited effect on the cyclical movement of unemployment, as has been well documented since Shimer (2005). Only during the 1982 recession does the model predict that unemployment increases close to that in the data. The recessions in 1990, 2001, and 2007 only feature a slight increase in unemployment and a quick reversal back below trend. The predictions for vacancy creation are slightly more in line with the data, though still under represent the magnitude of the deviations. For both series, the model completely misses the expansion between 2004 and 2007. Movements in labor productivity alone imply an increasing unemployment rate and falling vacancy creation between this time period.

A similar story follows with regards to output and consumer credit outstanding. The model implied series and the data coincide in sign for the recessions of the early 1980s, though again misses on the magnitude of the deviations. Each of the recessions post-1990 are not only shallow,
but also short lived. For instance, the data suggest that after the 1990 recession, GDP stagnated relative to trend until 1996. The movement in labor productivity in the model suggests that there should not have been any stagnation. Debt also shows no significant deviations. Again, the model suggests that movements in labor productivity miss the contraction of consumer credit in both the mid-1990s and after the Great Recession. This leads to the conclusion that productivity shocks cannot independently explain either the movement in labor market variables or the movement of consumer credit.

![Figure 6: Productivity Shocks](image)

**Credit Shocks** Figure 7 plots the unemployment rate, vacancies, output, and consumer credit for exogenous fluctuations in aggregate financial conditions, \( \nu_t \). The model implied series are represented in green and their empirical counterpart in blue. Household financial shocks in the model come much closer to explaining movements in the data. The aggregate unemployment rate is in line with the model predictions, particularly for the 1990 recession. Surprisingly, the model
has a difficult time explaining the movements of unemployment during the Great Recession. In the data, unemployment started to increase in the early part of 2007. Movements in household financial conditions lead to an increase only beginning in 2009, after the majority of the change in the data occurred. In magnitude, changes in consumer credit can explain 30 percent of the total change during the Great Recession.

Credit shocks also do a better job explaining the demand for labor. The model predicts the entire fall in vacancies during the early 1980s and 1990s. Regarding the Great Recession, vacancies also fall by the same magnitude though the model predicts that fall starting only in 2009. An additional dimension credit shocks improve on are the persistence of labor market variables, particularly after the 1990s. For instance the slow decline of the unemployment rate in the model, coincides with the data.

The changes in labor market variables as a result of credit shocks also coincide with output and consumer debt. The model predicts a similar fall in output during the 1990s and improves on the length of the contraction. As with the labor market variables, credit shocks miss the beginning of the Great Recession by nearly two years. However, once the recession begins, the model does a good job in predicted the pace of the recovery. Finally, as could be expected, credit shocks do a good job fitting the movements in household debt over the cycle.

6 Conclusion

There is consistent evidence that households face constraints in financing consumption in the face of income shocks. Understanding which groups in the population these constraints affect the most is important in linking the observed movements in debt, employment and consumption during financial crises. The starting point for this paper was illustrating the fact that an income shock, in the form of a job loss, is also a significant credit shock. Credit constraints increase precisely for the group that values credit access the most, the unemployed.

I propose a model that generates an increase in credit constraints for the unemployed that is both analytically tractable, easily quantifiable, and nests into the workhorse model of unemployment, Mortensen and Pissarides (1994). This easily allows comparison to other studies in the literature. I calibrate the model to match the fall in credit for the unemployed and explore to what extent this mechanism generates the observed business cycle co-movement of consumer credit, unemployment, and other macroeconomic variables. I show that productivity shocks do a poor job of generating the patterns in the data. However, I show that shocks to aggregate household financial conditions amplify the drop in credit upon job loss and contribute significantly to the dynamics of both real and financial variables.
Figure 7: Credit Shocks

References


7 Appendix

7.1 Figures and Tables

Figure 8: Consumer credit outstanding to disposable personal income and the civilian unemployment rate. 1978 Q1-2013 Q4. Series are detrended with a Hodrick-Prescott filter with smoothing parameter $\lambda = 100,000$. Sources: Federal Reserve Board Flow of Funds Accounts. Table B.100 and Bureau of Labor Statistics. NBER recessions are shown in grey.
Figure 9: Consumer credit outstanding and macroeconomic time series, 1978 Q1 - 2013 Q3. Each time series is logged then de-trended with a Hodrick-Prescott filter with smoothing parameter $\lambda = 100,000$. The y-axis represents percentage deviations from trend. Consumer credit corresponds to total consumer credit outstanding from the Federal Reserve Board’s Flow of Funds, Table B.100 (FRED Series: TOTALSL). The unemployment rate is the civilian unemployment rate (FRED Series: UNRATE). The employment to population ratio is the fraction of the civilian noninstitutional population employed (FRED Series EMRATIO). Vacancies correspond to the composite Help-Wanted Index calculated by Barnichon (2010). Retail trade sales are given by the OECD Main Economic Indicators (FRED Series: SLRTTO02USA189N). Gross domestic product, personal consumption expenditures, and durable personal consumption expenditures come from the National Income and Product Accounts (FRED Series GDPC1, PCEC, and PCDG, respectively). NBER recessions are shown in grey.
Table 1: Dynamics of Consumer Credit and Macroeconomic Aggregates, 1978-2013

<table>
<thead>
<tr>
<th></th>
<th>(1) Std(Variable)</th>
<th>(2) Corr(Variable,CC)</th>
<th>(3) Std(CC)/Std(Variable)</th>
</tr>
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<tr>
<td>Consumer Credit</td>
<td>0.062</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>0.012</td>
<td>-0.617</td>
<td>5.211</td>
</tr>
<tr>
<td>Employment/Population</td>
<td>0.010</td>
<td>0.646</td>
<td>6.462</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.184</td>
<td>0.511</td>
<td>0.336</td>
</tr>
<tr>
<td>Retail Trade Sales</td>
<td>0.043</td>
<td>0.643</td>
<td>1.423</td>
</tr>
<tr>
<td>Gross Domestic Product</td>
<td>0.025</td>
<td>0.721</td>
<td>2.529</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
<td>0.025</td>
<td>0.543</td>
<td>2.456</td>
</tr>
<tr>
<td>Durable Personal Consumption Expenditures</td>
<td>0.067</td>
<td>0.771</td>
<td>0.918</td>
</tr>
</tbody>
</table>

Note: All series have been logged and detrended using a Hodrick-Prescott Filter with a smoothing parameter \( \lambda = 100,000 \). Consumer credit corresponds to total consumer credit outstanding from the Federal Reserve Board’s Flow of Funds, Table B.100 (FRED Series: TOTALSL). The unemployment rate is the civilian unemployment rate (FRED Series: UNRATE). The employment to population ratio is the fraction of the civilian noninstitutional population employed (FRED Series EMRATIO). Vacancies correspond to the composite Help-Wanted Index calculated by Barnichon (2010). Retail trade sales are given by the OECD Main Economic Indicators (FRED Series: SLRTTO02USA189N). Gross domestic product, personal consumption expenditures, and durable personal consumption expenditures come from the National Income and Product Accounts (FRED Series GDPC1, PCEC, and PCDG, respectively).
### Table 2: Unemployment and Consumer Credit, 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>(1) 2007</th>
<th>(2) 2009</th>
<th>Difference (2)-(1)</th>
<th>(3)</th>
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<tr>
<td>Consumer Debt ($)</td>
<td>4,780</td>
<td>5,441</td>
<td>661*</td>
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<tr>
<td></td>
<td>(150)</td>
<td>(340)</td>
<td></td>
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<td>Credit Card Debt ($)</td>
<td>2,438</td>
<td>2,226</td>
<td>-212</td>
<td></td>
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<tr>
<td></td>
<td>(113)</td>
<td>(113)</td>
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<tr>
<td>Auto Debt ($)</td>
<td>2,513</td>
<td>3,086</td>
<td>573*</td>
<td></td>
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<tr>
<td></td>
<td>(97)</td>
<td>(283)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Card {0,1}</td>
<td>0.67</td>
<td>0.62</td>
<td>-0.43***</td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
<td></td>
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<tr>
<td>CC Monthly Charges ($)</td>
<td>447</td>
<td>393</td>
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<tr>
<td></td>
<td>(16.5)</td>
<td>(14.4)</td>
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<td>CC Debt Limit ($1,000)</td>
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<tr>
<td></td>
<td>(0.52)</td>
<td>(0.48)</td>
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<tr>
<td>Applied</td>
<td>0.63</td>
<td>0.43</td>
<td>-0.20***</td>
<td></td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denied</td>
<td>Applied</td>
<td>0.21</td>
<td>0.27</td>
<td>0.06***</td>
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<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
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<tr>
<td>Income ($1000)</td>
<td>44.72</td>
<td>42.82</td>
<td>-1.90*</td>
<td></td>
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<tr>
<td></td>
<td>(0.61)</td>
<td>(0.61)</td>
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<tr>
<td>Weekly Hours</td>
<td>29.96</td>
<td>25.64</td>
<td>-4.32***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.34)</td>
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<tr>
<td>Observations</td>
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<td>3,820</td>
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</tbody>
</table>

Note: Significance levels: *** p < 0.01, ** p < 0.05, *p < 0.1. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. Dollar values represent real 2007 dollars adjusted using the CPI.
<table>
<thead>
<tr>
<th>Individual Characteristics</th>
<th>(1) $EU = 0$</th>
<th>(2) $EU = 1$</th>
<th>(3) Difference (2)-(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>0.34</td>
<td>0.33</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td>0.20</td>
<td>0.23</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>48.0</td>
<td>43.1</td>
<td>-4.88***</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.77)</td>
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<tr>
<td>High School</td>
<td>0.41</td>
<td>0.39</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Some College</td>
<td>0.22</td>
<td>0.19</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>College Degree</td>
<td>0.37</td>
<td>0.42</td>
<td>0.05*</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome Variables</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Debt ($)</td>
<td>4,711</td>
<td>5,525</td>
<td>814</td>
</tr>
<tr>
<td></td>
<td>(148)</td>
<td>(784)</td>
<td></td>
</tr>
<tr>
<td>Credit Card Debt ($)</td>
<td>2,457</td>
<td>3,304</td>
<td>946</td>
</tr>
<tr>
<td></td>
<td>(102)</td>
<td>(756)</td>
<td></td>
</tr>
<tr>
<td>Auto Debt ($)</td>
<td>2,540</td>
<td>2,221</td>
<td>-319</td>
</tr>
<tr>
<td></td>
<td>(102)</td>
<td>(305)</td>
<td></td>
</tr>
<tr>
<td>Credit Card {0,1}</td>
<td>0.67</td>
<td>0.65</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>CC Monthly Charges {0,1}</td>
<td>0.53</td>
<td>0.58</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>CC Monthly Charges ($)</td>
<td>441</td>
<td>507</td>
<td>66.4</td>
</tr>
<tr>
<td></td>
<td>(16.1)</td>
<td>(89.1)</td>
<td></td>
</tr>
<tr>
<td>CC Debt Limit ($1,000)</td>
<td>16.20</td>
<td>10.75</td>
<td>-5.45***</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(1.08)</td>
<td></td>
</tr>
<tr>
<td>Applied</td>
<td>0.64</td>
<td>0.56</td>
<td>-0.08**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Denied</td>
<td>Applied</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Income ($1000)</td>
<td>45.43</td>
<td>37.10</td>
<td>-8.33***</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(2.36)</td>
<td></td>
</tr>
<tr>
<td>Labor Income ($1000)</td>
<td>28.43</td>
<td>27.85</td>
<td>-0.47</td>
</tr>
<tr>
<td></td>
<td>(0.66)</td>
<td>(2.03)</td>
<td></td>
</tr>
<tr>
<td>Weekly Hours</td>
<td>29.42</td>
<td>35.87</td>
<td>6.45***</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(1.09)</td>
<td></td>
</tr>
</tbody>
</table>

Observations: 3,820 3,820

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. Dollar values represent real 2007 dollars adjusted using the CPI.
Table 4: Impact of Unemployment on Changes in Consumer Debt, 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆Consumer Debt ($)</td>
<td>∆Credit Card Debt ($)</td>
<td>∆Auto Debt ($)</td>
</tr>
<tr>
<td>EU×{t = 2009}</td>
<td>-2,809**</td>
<td>-2,504***</td>
<td>-173</td>
</tr>
<tr>
<td></td>
<td>(941)</td>
<td>(771)</td>
<td>(550)</td>
</tr>
<tr>
<td>Observations</td>
<td>7640</td>
<td>7640</td>
<td>7640</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.01</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table 5: Impact of Unemployment on Credit Card Use, 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆Credit Card {0,1}</td>
<td>∆CC Charges {0,1}</td>
<td>∆CC Charges ($)</td>
</tr>
<tr>
<td>EU×{t = 2009}</td>
<td>-0.06***</td>
<td>-0.15***</td>
<td>-250*</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(89.5)</td>
</tr>
<tr>
<td>Observations</td>
<td>7640</td>
<td>7640</td>
<td>7640</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.20</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.
Table 6: Impact of Unemployment on Credit Applications and Denials, 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆Applied</td>
<td>ΔDenied</td>
<td>Applied</td>
</tr>
<tr>
<td>EU×I{t = 2009}</td>
<td>0.14***</td>
<td>0.05***</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Observations</td>
<td>7640</td>
<td>4112</td>
<td>4112</td>
</tr>
<tr>
<td>R²</td>
<td>0.10</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Demo. Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Note: Significance levels: *** p < 0.01, ** p < 0.05, * p < 0.1. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.

Table 7: Impact of Unemployment on Income and Assets, 2007-2009

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆Income</td>
<td>∆Unemp. Benefits</td>
<td>∆Liquid Assets</td>
<td>∆No Liquid{0,1}</td>
</tr>
<tr>
<td>EU×I{t = 2009}</td>
<td>-20,000***</td>
<td>1,188***</td>
<td>-5,292*</td>
<td>0.15***</td>
</tr>
<tr>
<td></td>
<td>(2,957)</td>
<td>(212)</td>
<td>(3,184)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Observations</td>
<td>7640</td>
<td>7640</td>
<td>7640</td>
<td>7640</td>
</tr>
<tr>
<td>R²</td>
<td>0.20</td>
<td>0.18</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Demographic Controls</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Note: Significance levels: *** p < 0.01, ** p < 0.05, * p < 0.1. Observations are weighted using SCF 2007-2009 probability weights. The sample consists of all primary economic units (PEUs) that were single in both 2007 and 2009 and were employed in 2007. The omitted group consists of white college graduates in 2007 that maintained employment in 2009. Dollar values represent real 2007 dollars adjusted using the CPI.
Table 8: Functional Forms and Stochastic Processes

<table>
<thead>
<tr>
<th>Function</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>labor market matching</td>
<td>$m(s, o) = \frac{so}{(s^{\eta_L} + o^{\eta_L})^{1/\eta_L}}$</td>
</tr>
<tr>
<td>goods market matching</td>
<td>$\alpha(n) = \frac{n}{(n^{\eta_G} + 1)^{1/\eta_G}}$</td>
</tr>
<tr>
<td>DM utility</td>
<td>$u(y) = \frac{y^{1-\gamma}}{1-\gamma}$</td>
</tr>
<tr>
<td>aggregate productivity</td>
<td>$\hat{z}<em>{t+1} = \rho z</em>{t+1} + \epsilon_{z,t}$ where $\epsilon_{z,t} \sim N(0, \sigma_z^2)$</td>
</tr>
<tr>
<td>aggregate financial frictions</td>
<td>$\hat{\nu}<em>{t+1} = \rho \nu</em>{t+1} + \epsilon_{\nu,t}$ where $\epsilon_{\nu,t} \sim N(0, \sigma_{\nu}^2)$</td>
</tr>
</tbody>
</table>
Table 9: Calibration Summary: Parameters and Stochastic Steady State Targets

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Labor Market Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>matching curvature, $\eta_L$</td>
<td>0.710</td>
<td>job filling rate, JOLTS</td>
</tr>
<tr>
<td>separation rate, $\delta$</td>
<td>0.032</td>
<td>unemployment rate, BLS</td>
</tr>
<tr>
<td>vacancy posting costs, $k$</td>
<td>0.150</td>
<td>Silva and Toledo (2009)</td>
</tr>
<tr>
<td>labor bargaining weight, $\lambda$</td>
<td>0.500</td>
<td>normalization</td>
</tr>
<tr>
<td>utility from leisure, $\ell$</td>
<td>0.250</td>
<td>Hall and Milgrom (2008)</td>
</tr>
<tr>
<td>unemployment income, $b$</td>
<td>0.700</td>
<td>average decline in credit upon job loss, estimated in Section 2.2, SCF</td>
</tr>
<tr>
<td><strong>Goods/Credit Market Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>matching curvature, $\eta_G$</td>
<td>1.27</td>
<td>debt to income, FRB Z.1 Flow of Funds</td>
</tr>
<tr>
<td>utility curvature, $\gamma$</td>
<td>0.390</td>
<td>MPC out of credit, Gross and Souleles (2002)</td>
</tr>
<tr>
<td>mean of labor productivity, $\mu_z$</td>
<td>1.000</td>
<td>normalization</td>
</tr>
<tr>
<td>persistence of labor productivity, $\rho_z$</td>
<td>0.884</td>
<td>output per job, BLS</td>
</tr>
<tr>
<td>s.d. of labor productivity shock, $\sigma_z$</td>
<td>0.0075</td>
<td>output per job, BLS</td>
</tr>
<tr>
<td>mean of agg. financial conditions, $\mu_\nu$</td>
<td>0.512</td>
<td>FRB Z.1 Flow of Funds</td>
</tr>
<tr>
<td>persistence of agg. financial conditions, $\rho_\nu$</td>
<td>0.967</td>
<td>FRB Z.1 Flow of Funds</td>
</tr>
<tr>
<td>s.d. of agg. financial conditions, $\sigma_\nu$</td>
<td>0.0083</td>
<td>FRB Z.1 Flow of Funds</td>
</tr>
</tbody>
</table>
7.2 Wage equation

From (16), we can write $V_t(1)$ and $V_t(0)$ combining (5) and (22) as

$$V_t(1) = \alpha(n_t)\mu[v(y_1^t) - y_1^t] + w_t + \beta[(1 - \delta)V_{t+1}(1) + \delta V_{t+1}(0)]$$ (45)

$$V_t(0) = \alpha(n_t)\mu[v(y_0^t) - y_0^t] + (\ell + b) + \beta[p(\theta_t)V_{t+1}(1) + (1 - p(\theta_t))V_{t+1}(0)]$$ (46)

Solving for $V_t(1)$ in (45) and subtracting $V_t(0)$ obtains the surplus of an employed worker

$$V_t(1) - V_t(0) = \alpha(n_t)\mu[v(y_1^t) - y_1^t] - [v(y_0^t) - y_0^t] + w_t - (\ell + b) + \beta(1 - \delta - p(\theta_t))[V_{t+1}(1) - V_{t+1}(0)]$$ (47)

From (26) and the free entry condition $k = \beta f(\theta_t)J_{t+1}$, we can write (47) as

$$V_t(1) - V_t(0) = \alpha(n_t)\mu[v(y_1^t) - y_1^t] - [v(y_0^t) - y_0^t] + w_t - (\ell + b) + (1 - \delta - p(\theta_t))\frac{k}{1 - \lambda f(\theta_t)}$$ (48)

Similarly, from (25), (26), and the free entry condition we can write the value of a filled job as

$$J_t = z_t - w_t + (1 - \delta)\frac{k}{f(\theta_t)}$$ (49)

Combining (48) and (50) using (26) we obtain

$$(1 - \lambda)[\alpha(n_t)\mu[v(y_1^t) - y_1^t] - [v(y_0^t) - y_0^t]] + w_t - (\ell + b)] = \lambda[z_t - w_t + \lambda \theta_t k]$$ (50)

where we have used the result that $p(\theta_t) = \theta_t f(\theta_t)$. Rearranging (50) yields the wage equation (27)

$$w_t = \lambda[z_t(w_t) + \theta_t k] + (1 - \lambda)(b + \ell - \alpha(n_t)\mu[S^1(w_t) - S^0]) = \Gamma_t(w_t)$$ (51)
7.3 Proofs of Lemmas and Propositions

**Proof of Lemma 2:** Part (i): Taking the derivative with respect to $\theta$ in (31) we have

$$
\frac{\partial S^f}{\partial \theta} = -\lambda k + \frac{(1 - \lambda)\alpha(n)\nu[y'(y^1) - 1]}{(1 - \mu)y'(y^1) + \mu} \frac{\partial w}{\partial \theta}
$$

(52)

Taking derivative of (29) with respect to $\theta$ yields

$$
\frac{\partial w}{\partial \theta} = \frac{\lambda k}{1 - (\lambda - \mu)\alpha(n)} \frac{\nu[y'(y^1) - 1]}{(1 - \mu)y'(y^1) + \mu}
$$

(53)

Evaluated at the equilibrium wage, the denominator in (53) is positive, hence $\partial w/\partial \theta > 0$. Let $\frac{\partial S^f}{\partial w} = \nu[y'(y^1) - 1]/[(1 - \mu)y'(y^1) + \mu]$. Combining (52) and (53) we can write $\frac{\partial S^f}{\partial \theta}$ as

$$
\frac{\partial S^f}{\partial \theta} = -\lambda k \frac{1 - (1 - \mu)\alpha(n)\partial S^1/\partial w}{1 - (\lambda - \mu)\alpha(n)\partial S^1/\partial w}
$$

(54)

The sign of (54) depends on the magnitude of $\partial S^1/\partial w$. Its maximum value is $\nu\mu/(1 - \mu)$ when $y^1 = 0$. Since $(1 - \mu)\alpha(n)\nu\mu/(1 - \mu) \leq 1$, then $\partial S^f/\partial \theta \leq 0$ for any $n$.

Part (ii and iii): Taking the derivative with respect to $n$ in (31) yields

$$
\frac{\partial S^f}{\partial n} = (1 - \lambda)\alpha'(n)[S^1 - S^0] + (1 - \lambda)(1 - \mu)\xi S^0 + (1 - \lambda)\alpha(n)\frac{\partial S^1}{\partial w} \frac{\partial w}{\partial n}
$$

(55)

where $\xi = \frac{\alpha'(n)n - \alpha(n)}{n^2} < 0$. If $S^0 = 0$ the result in part (ii) immediately follows. Taking the derivative of (29) with respect to $n$ yields.

$$
\frac{\partial w}{\partial n} = \frac{(\lambda - \mu)\alpha'(n)S^1 + \lambda(1 - \mu)\xi S^0}{1 - (\lambda - \mu)\alpha(n)\partial S^1/\partial w}
$$

(56)

The sign of the denominator is positive at the equilibrium wage. However, the sign of the numerator is ambiguous since $\xi < 0$. Plugging (56) into (55) yields

$$
\frac{\partial S^f}{\partial n} = (1 - \lambda)\alpha'(n)[S^1 - S^0] + (1 - \lambda)(1 - \mu)\xi S^0 + \frac{(1 - \lambda)\alpha(n)(\lambda - \mu)\alpha'(n)S^1\frac{\partial S^1}{\partial w} + (1 - \lambda)\lambda(1 - \mu)\xi \alpha(n)S^0\frac{\partial w}{\partial \theta}}{1 - (\lambda - \mu)\alpha(n)\partial S^1/\partial w}
$$

(57)

**Proof of Lemma 3:** From Lemma 2, we know $\partial S^f/\partial \theta \leq 0$ which implies $\partial S^f/\partial J \leq 0$. Let $\tilde{S}_t$ be the solution to $S^f(1 - u, 0)$. From (31), $\tilde{S}_f > 0$ for any $u$. Since the right-hand side of (34) is
strictly increasing and crosses through the origin, there exists a unique fixed point of the problem. Further since $S^f$ is bounded below by $(1 - \lambda)(\bar{z} - \ell) - \lambda \theta k$, $J^* > J^{DMP}$.

7.4 Continuous Time Derivation

To come. Available upon request.
7.5 Data Appendix

7.5.1 Survey of Consumer Finances, 2007-2009 ‘Denied’ Variable Definitions

Denied because of credit related reasons includes households that were told they haven’t established a credit history, credit rating service reports, credit records/history from another institution, bankruptcy, amounts of debt, size of other payments, or ability to repay loan too high, insufficient credit references, or other credit characteristics of the borrower.

Denied because of asset related reasons includes lack of assets, collateral, property to secure the loan or insufficient collateral or equity.

Denied because of income related reason includes lack of assets, collateral or property to secure the loan, time on current job, the type of job or work (i.e. steady or secure, a good job), lack of job or not working, amount of income or the source of income for retired households, and any other financial characteristics of the household.