A large normative literature on taxation highlights labor supply as the mechanism used by taxpayers to circumvent taxation. However, empirical evidence shows that taxpayers use loopholes and other legal avoidance strategies to reduce their taxable income. This paper seeks to reconcile the normative theory with the empirical evidence. I introduce tax avoidance into a standard optimal taxation model by proposing an informational structure where taxpayers can pay a cost to report something other than the income they produced. This allows them multiple ways to reduce their taxable income, either by changing their labor supply, by using tax avoidance, or by using both. A central theoretical result shows that at the optimum, taxpayers’ reports must equal their income. The government can thus eliminate tax avoidance while increasing tax revenues. I also find that the optimal tax mechanism can be part of three distinct incentive compatibility regions depending on the cost of tax avoidance. I determine which of these regions is consistent with US data by estimating the cost of tax avoidance using IRS Statistics of Income Data.

J.E.L Codes: H21, H24, H26, H31, D82.
Keywords: Optimal Taxation, Personal Income Taxes, Tax Avoidance, Tax Evasion.
1 Introduction

A central idea in the normative literature on taxation is that individuals have private information about their productivity. This allows them to change their labor supply to reduce their taxable income and circumvent taxation. In contrast to the mechanism highlighted by this theoretical literature, evidence from studies on individuals’ responses to taxation shows that taxpayers use loopholes and other tax avoidance strategies to reduce their taxable income. Some taxpayers hire accountants or attorneys to “discover” inconsistencies in the tax code in order to take advantage of deductions or exceptions that were not intended for their use (tax arbitrage). Taxpayers might also reorganize the legal structure of their activities to claim as capital or dividend income what otherwise would be considered labor income (income shifting).

This paper seeks to reconcile the theory of optimal taxation proposed by Mirrlees (1971) with empirical evidence on individuals’ avoidance responses to taxation. Particularly, this paper asks, how does incorporating tax avoidance into a Mirrleesian model of taxation change its prescriptions on optimal marginal taxes? Do these changes have a significant impact on tax revenues? And does the data validate the normative theory proposed in this paper?

To address these questions, I introduce a new information structure into an otherwise standard Mirrleesian model. Along productivity, individuals’ income and consumption are also private information. Taxpayers report their income to the government and pay a cost to report something other than the income they produced. This informational structure gives taxpayers multiple ways to reduce their taxable income, either by changing their labor supply (as conventionally studied), by using costly tax avoidance, or by using both.

I find that when income and consumption are private information, governments face a trade-off between collecting tax revenues and taxpayers’ avoidance activities. A central theoretical finding of this paper, the Non-Falsification Theorem, illustrates this tension. The Non-Falsification Theorem states that, at the optimum, taxpayers’ reports must equal their true income. This is optimal because the government can offset taxpayers’ avoidance costs and incorporate them into tax revenues. Since the cost of tax avoidance is known, the government can reduce taxes levied on misreporting taxpayers until they are marginally better off by paying their lax liability than by paying the cost of tax avoidance. This shifts taxpayers’ expenditures away from tax avoidance and into tax liability. However, to meet the conditions of the Non-Falsification theorem, the government’s objective function is constrained by incentive compatibility conditions that prevent tax avoidance. These incentive constraints set an upper bound on tax revenues because they guarantee that taxpayers’ benefit from

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1 On tax arbitrage, Kleven et al. (2011) found large elasticities of tax avoidance in Denmark for both self-employment and stock income of 0.1 and 1.99 respectively. Their findings suggest that most of taxpayers’ responses to taxation came from tax avoidance rather than tax evasion.

2 In regards to income shifting, Gordon and MacKie-Mason (1990) find that after the implementation of the US Tax Reform Act of 1986, which reduced personal tax rates below corporate tax rates, the rate at which firms elected to be taxed as S-Corporations more than tripled and their reported income increase by 34 times the previous yearly averages.
misreporting are lower than its cost.

In regards to marginal taxes, I find that there are two distinct solutions for optimal marginal taxes depending on taxpayers’ cost of tax avoidance. When taxpayers’ cost of changing their labor supply is lower than their cost of using tax avoidance, the solution to the model is equivalent to the standard Mirrleesian solution. In this case, taxpayers’ labor supply choices are the only mechanism for income report deviation and, for example, optimal marginal taxes on high ability agents are zero. In contrast, when taxpayers’ cost of tax avoidance is lower or equal to their cost of changing labor supply (the tax avoidance solution), optimal marginal taxes on high productivity taxpayers are negative. Negative marginal taxes on high productivity taxpayers allows the government to increase the size of the tax base and offset the limits placed on tax revenues by the Non-Falsification Theorem.

Government revenues collected at the optimum also depends on the relative costs of tax avoidance and changing labor supply. Tax revenues in the tax avoidance solution are lower than those collected at the Mirrleesian solution. When the cost of deviation through avoidance is lower than the cost of changing labor supply, the incentive constraint preventing tax avoidance sets an upper bound on the tax spread between income categories. As the costs of tax avoidance increases, revenues collected in the tax avoidance solution converge to the higher level achieved in the Mirrleesian case.

To validate the model, I quantify the cost of tax avoidance in the US. Because tax avoidance is not directly observable, I use the model to interpret the data. According to the model, the benefit of tax avoidance is an upper bound for the cost of tax avoidance. Using data from the IRS Statistics of Income Data, I find that the benefit of tax avoidance in the US during the decade of 1998-2008 is at most 30% of the income taxpayers “hid” from the government. When the model is calibrated to match US taxpayers’ preference parameters and labor supply responses, this estimate for the upper bound of the cost of tax avoidance is always lower than the comparable costs of deviation through labor supply. This result suggests that the tax avoidance solution is consistent with US taxpayer data.

By studying the connection between tax avoidance and the optimal tax schedule, this paper can inform current policy debates. Many propose to raise tax revenue by closing loopholes and other tax avoidance channels. This paper illustrates the effect of closing loopholes through an increase of the cost of tax avoidance, and provides normative recommendations on marginal taxes and total revenues collected. Furthermore, there are many governments who face low cost of tax avoidance. This paper can provide guidance on the implementation of a progressive income tax. Further extensions of the model could evaluate the relationship between labor income taxes, value added taxes, and proportional taxes on other sources of income. These insights could inform governments facing low costs of tax avoidance about the trade-offs of various types of taxation in conjunction with labor income taxes.

\[ T(x_j) - T(x_i) \]

3 The tax spread between two income categories \( x_j \) and \( x_i \) is \( T(x_j) - T(x_i) \).
Related Literature

Taxpayers’ legal responses to taxation can be defined as real substitution and tax avoidance responses. Real substitution responses are actions that affect an individual’s consumption basket, like labor-leisure choices. Tax avoidance is defined against this standard to be any response that does not affect real variables. This includes the use of accountants and attorneys to reclassify income to tax-preferred categories, reorganizing legal entities to take advantage of tax exceptions and credits, or re-timing income reporting to minimize tax liability.4

A large theoretical literature has studied real substitution responses in the context of optimal taxation, but there are only a couple of examples of normative work on tax avoidance. In the case of real responses, this paper is closely related to the literature dating back to Mirrlees (1971) and to more recent work by Saez (2001).5 As in this paper, the main tenet of these models is that informational asymmetries between governments and taxpayers limit the set of economic outcomes attainable by tax policy. In the static case, this limit is expressed by taxpayers’ ability to use their labor supply to gain an advantageous tax position. This paper contributes to this literature by proposing an alternative information structure to introduce taxpayers’ use of loopholes and tax avoidance strategies. This informational structure is consistent with the mechanism approach in this literature, and can be applied to more general extensions of the static model.6

In the case of normative tax avoidance, this paper builds on the work of Grochulski (2007). The author finds the optimal non-linear tax schedule when taxpayers can use tax avoidance. This paper expands on his work by incorporating a labor-leisure choice to capture real substitution and tax avoidance responses jointly. Additionally, I prove a non-falsification theorem in this informational environment for two productivity types. This paper is also related to the work of Piketty, Saez and Stantcheva (2014). They derive a formula for optimal tax rate for the top of the income distribution when individuals can use multiple channels to respond to taxation, one of which is tax arbitrage.7 In their model, the government can change the effective tax rate of income sheltered from taxation, implicitly controlling taxpayers benefit of tax arbitrage. This gives the government power to eliminate this kind of tax avoidance by equalizing tax rates on income earnings to the effective tax rate on sheltered income. I depart from their set up by including heterogeneity in taxpayers productivity and assuming that governments can’t change taxpayers cost of tax avoidance directly (either by changing the tax rates of other types of income or eliminating tax loopholes completely).8

Additionally, this paper complements the costly state verification models of tax evasion.9

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4 For more details on this definition, see Slemrod and Yitzhaki (2002).
5 These papers review the literature: Mirrlees (1986), Tuomala (1990), Mankiw, Weinzierl and Yagan (2009).
7 A subcategory of tax avoidance, where taxpayers can arbitrage between sources of income that are treated differentially by the tax code.
8 Further discussion of this latter assumption is in the appendix.
in public economics. Starting with Allingham and Sandmo (1972), Yitzhaki (1974), and Sandmo (1981), these models feature a tax collecting institution or government that pays a cost to acquire information about taxpayers’ true income through tax auditing. The government can impose a penalty when it detects a deviation in reported income, and designs its compliance and tax policies by balancing the costs of auditing and taxpayers’ expected costs of facing the penalty. In contrast to tax avoidance models, this paper assumes tax loopholes and tax compliance policy as given. Furthermore, there is no way for the government to acquire information about true income other than what is reported by taxpayers. The optimal tax schedule is then designed to prevent individuals’ tax avoidance and to maximize aggregate welfare. This alternative approach is applicable to situations when tax authorities can’t punish or enforce report deviations because they’re legal and therefore not punishable (tax avoidance), or there exists technological constraints that prevents them from punishing or detecting offenders (undetectable tax evasion).

This paper also incorporates, in a stylized way, lessons from a large literature on taxpayers’ behavioral responses to taxation. Based on their approach, these studies can be divided into two categories. One studies taxpayers’ behavioral elasticities to changes the tax rates, while the second consists of event studies mostly focused around the US Tax Reform Act of 1986 (TRA86). The first group points out that tax avoidance could account for some of the differences between labor supply and taxable income elasticities. A long standing literature finds that labor supply is not responsive to changes in tax rates, leading to estimates of labor supply elasticities close to zero. On the other hand, another set of studies reviewed by Saez, Slemrod and Giertz (2012) find a much larger elasticity of taxable income. Gruber and Saez (2002) find that much of taxable income’s response to tax rates comes from deductions and exceptions applied to broad categories of income. This suggests that differences between the two behavioral responses is partially attributed to tax avoidance. Kleven et al. (2011) finds evidence of large elasticities of tax avoidance to changes in marginal taxes. They find that 14% and 36% of taxpayers, for self-employment and stock income respectively, bunching at the level of income where marginal taxes increase to their top rate. These are equivalent to elasticities of 0.1 and 1.99 of self employment and stock income respectively. Together, this evidence suggests that taxpayers use tax avoidance to minimize their tax liability.

The second category of studies record the avenues for taxpayers use to avoid taxation. For example Maki (1996) and Schoiz (1994) show that after the implementation of TRA86 there was a large shift in the debt instruments used by households, from consumer interest no longer deductive under the new legislation, into tax deductible mortgage or home equity loans. As a product of the same tax reform, the marginal tax rate for personal income was lowered below the corporate marginal tax.

For some examples see Cowell (1990), Cremer and Galvarri (1994), Kaplow (1990), Slemrod (1994). This is the case on the intensive margin for adult males, see Triest (1990), MacCurdy, Green and Paarsch (1990), Heckman (1993). Women are more responsive, specially in the extensive margin, see Mroz (1987). Eissa (1995) shows that 14% and 36% of taxpayers, for self-employment and stock income respectively, bunching at the level of income where marginal taxes increase to their top rate. These are equivalent to elasticities of 0.1 and 1.99 of self employment and stock income respectively. Together, this evidence suggests that taxpayers use tax avoidance to minimize their tax liability.

9 For some examples see Cowell (1990), Cremer and Galvarri (1994), Kaplow (1990), Slemrod (1994).
10 This is the case on the intensive margin for adult males, see Triest (1990), MacCurdy, Green and Paarsch (1990), Heckman (1993). Women are more responsive, specially in the extensive margin, see Mroz (1987).
11 This is evidence from Denmark. All other studies mentioned here focus on the US.
12 These values are net of tax evasion, so can be attributed to tax avoidance.
that this change generated a large shift in the legal structure of firms, from C-Corporations to S-Corporations, to take advantage of lower personal income marginal taxes available to firms established as the latter. This paper models these responses in reduced form by assuming that taxpayers’ avoidance activities are costly, and that regardless of complexity, they eventually reduce taxable income.

The paper is organized as follows. Section 2 sets up taxpayers’ economic and informational environment. It also describes the type of tax mechanism that deliver taxpayers’ maximum aggregate welfare. A social planner’s problem is introduced to find the optimal tax mechanism and closed form solutions for marginal taxes. Section 3 calculates optimal taxes by calibrating the model to US data. Section 4 estimates the costs of tax avoidance for the US to validate the model. I conclude in Section 5, and propose some extensions to the model. Most of the proofs are found in the Appendix.

2 Optimal taxation with incomplete information and tax loopholes

This section starts with an outline of agents’ economic environment and behavior. As with taxpayers, agents can alter their reported income to obtain an advantageous tax position since their income and consumption is not observable. With this informational structure in place, I introduce a social planner’s problem. Per the Revelation Principle, the optimal aggregate welfare outcome of this economy can be found by solving for a direct-revelation mechanism constrained by incentive compatibility conditions. The social planner is tasked with finding such mechanism. The remainder of this section narrows down the requirements for the incentive compatibility conditions through a Non-Falsification Theorem and its corollaries. Using this Theorem, I arrive at a simplified social planners problem which has closed form solutions for marginal taxes. I close the section with a graphical analysis of the solution.

2.1 Economic environment and agents’ behavior

Agents are privately informed about their productivity which can be either high or low, \( \Theta \equiv \{ \theta_L, \theta_H \} \). For \( i \in \{ H, L \} \), agents’ preferences and production functions are represented by:

\[
W(c_i, l_i) = U(c_i) - V(l_i) \quad y_i = \theta_i l_i. \tag{2.1}
\]

\( W(c_i, l_i) \) is the utility derived from consumption and labor effort, and is additively separable. I assume that \( U'(c_i), -U''(c_i), V'(l_i), V''(l_i) \) are positive, exist, and meet the usual regularity conditions: \( \lim_{c_i \to 0} U'(c_i) = \infty, \lim_{c_i \to 0} V'(l_i) = 0 \). Furthermore, an agent’s income, \( y_i \), is produced by multiplying their productivity and labor.

As with productivity, I assume that income, \( y_i \geq 0 \), is private information. However,
agents are required to report their income. They do so by choosing an “income report,” $x_i \geq 0$, which is chosen from a set of believable income reports:

$$X(\theta_i) \equiv \{ x \mid \text{for any } k, x = \arg\max X(\theta_k) \}.$$  \hfill (2.2)

This is the set of income reports that an agent with productivity $\theta_i$ can declare as true income without being discovered. This set contains the utility maximizing income report choices of all types of agents. An agent that reports income outside of this set reveals himself as a deviator, to whom the social planner can impose very large tax penalty.

In this informational environment, agents have the option of reporting a level of income that gives them an advantageous tax position, pay the costs associated with making this report and keep the remainder as tax-free consumption. The agent’s budget constraint reflects these possibilities:

$$c_{i}^{\text{reported}} = x_i - T(x_i)$$
$$c_i = y_i - g(y_i, x_i) - T(x_i).$$  \hfill (2.3)

Since income is not observable, consumption is not observable either. Furthermore, taxes for agents of productivity $i$ are based on reported income, $T(x_i)$. The tax function is set outside of the agent’s control, so they take it as given when maximizing their utility.

The cost of tax avoidance function, $g(y_i, x_i)$, is the cost of reporting $x_i$ while privately producing $y_i$. This cost represents tax avoidance expenditures on accountants or tax attorneys, time spent in learning the tax code and filling more complicated tax forms, or any economic cost that would result in a lower tax liability without altering labor choices. I assume that the cost of tax avoidance is a deadweight loss since these expenses represent a reallocation of resources to activities that wouldn’t take place had the opportunities for tax avoidance never existed.

For the purposes of this paper the cost of tax avoidance function is the same for both type of agents, it takes non-negative values $g: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, is zero when agents’ income report matches their income, $g(z, z) = 0$ for any $z \in \mathbb{R}$, and is monotonically increasing for any income report that is not equal to income produced:

$$g_x(y, x) < 0, g_y(y, x) > 0 \text{ if } x < y$$
$$g_x(y, x) > 0, g_y(y, x) < 0 \text{ if } x > y.$$

In summary, agents act optimally given their preferences, production technology, and their

13 Here I follow the costly state falsification approach of Lacker and Weinberg (1989) in optimal contract theory. Like in this paper, Lacker and Weinberg consider the interaction between a risk neutral agent and a risk averse agent that can “falsify” the information the other agent receives about its random endowment. This paper applies this informational structure to a setting where the endowment is endogenous and there is more than one type of agent with a concave objective function.

14 See the appendix for a discussion on the difference between messages, commonly used in the literature, and income reports. This discussion also covers the relationship between income reports and the Revelation Principle.
opportunities for tax avoidance by choosing the amount of income they are going to produce and report:

\[
\max_{y_i, x_i} U(y_i - g(y_i, x_i) - \bar{T}(x_i)) - V\left(\frac{y_i}{\theta_i}\right) \\
\text{s.t.} \\
c_i = y_i - g(y_i, x_i) - T(x_i)
\]

(2.3)

2.2 Social Planner’s problem with costly tax avoidance

A benevolent social planner is tasked with finding the combination of income, income reports, and taxes that deliver the greatest expected aggregate welfare. The planner knows the agents’ preferences and the distribution of productivity across agents. Additionally, the planner knows the functional form of the cost of tax avoidance function, \(g(\cdot)\), but can’t observe individual realizations (preventing the social planner from backing out information about agents’ types from the cost they pay to report their income).

The planner maximizes aggregate welfare through the mechanism \(\Gamma^* = \{y_i^*, x_i^*, T^*(x^*)\}_{i=H,L}\) consisting of recommendations to agents for income earned and reported, and a lump-sum tax based on agents’ income report. For ease of exposition, the notation for the lump-sum tax imposed on agent’s with productivity \(i\) is \(T_i = T(x_i)\):

\[
\max_{y_i, x_i, T_i} \pi_H [U(c_H) - V(l_H)] + \pi_L [U(c_L) - V(l_L)] \\
\text{s.t.} \\
x_i \geq T_i \\
\pi_H c_H + \pi_L c_L = \pi_H c_H^{\text{reported}} + \pi_L c_L^{\text{reported}}
\]

(2.4)

(2.5)

To calculate aggregate welfare each agent’s utility is weighed by \(\pi_i = \mathbb{P}(\theta_i)\), where \(\sum_i \mathbb{P}(\theta_i) = 1\).\(^{15}\) And finally, taxes are collected based only on income reports and may not exceed them.\(^{16}\)

The Revelation Principle narrows the search for the optimal mechanism to direct-revelation incentive compatible mechanisms. In this type of mechanism, all agents share their private information and find it optimal to follow the recommendations of the social planner. This rules out mechanisms that lead to a pooling Bayesian equilibrium. In this equilibrium high and low ability agents report the same level of income. However, if both types of agents report the same level of income, taxes are the same for all types of agents. Noting that the feasibility

\(^{15}\) Since the social planner knows the functional form for the cost of tax avoidance function. Given a distribution of productivity across the population, a Pareto weight \(\pi_i = \mathbb{P}(x_i(\theta_i))\) can be imputed for agent’s of type \(i\) based on their income report \(x_i\). Hence, to avoid burdensome notation, it will be written as \(\mathbb{P}(\theta_i)\)

\(^{16}\) Another way to write (2.5) is \(\pi_H T_H + \pi_L T_L \geq 0\)
condition for this problem maintains that taxes are purely redistributive, \( \pi_H T_H + \pi_L T_L = 0 \), either all agents request a tax transfer (leaving no one to provide tax revenue) violating the feasibility condition, or all agents pay taxes (leaving no agents to receive a transfer) violating optimality due to wasted resources. Either of these alternatives violates the optimality required of the social planner’s recommendations. Hence, I restrict the mechanisms to direct-revelation incentive compatible mechanisms that result in a separating equilibrium of income reports.

**Incentive compatibility and non-falsification**

The set of separating Bayesian equilibria can be characterized by incentive compatibility constraints that limit the optimum to mechanisms where the income reports of high and low ability agents are different. There are three such conditions for each type of agent \( i \neq j \in \{H, L\} \). Note that each equation takes into account the agent’s possible income report strategies outlined in equation (2.2). The first set of equations, the tax avoidance constraints (TAC’s), reduces the set of mechanism to those where agents don’t profit by reporting an income level different from their realized income:

\[
U(y_i - g(y_i, x_i) - T_i) - V\left(\frac{y_i}{\theta_i}\right) \geq U(y_i - g(y_i, x_j) - T_j) - V\left(\frac{y_i}{\theta_i}\right) \quad \text{or}
\]

\[
g(y_i, x_j) - g(y_i, x_i) \geq T_i - T_j. \tag{2.6}
\]

From the perspective of a high productivity agent, this constraint prevents the social planner from choosing a mechanism where this agent could report the same income as a low productivity agent, receive a tax transfer from other high productivity agents, pay a cost of tax avoidance, and keep the difference as private consumption.

The second set of equations, the labor supply constraints (LSC’s), is the same found in Mirrleesian models of optimal taxation,

\[
U(y_i - g(y_i, x_i) - T_i) - V\left(\frac{y_i}{\theta_i}\right) \geq U(y_j - g(y_j, x_j) - T_j) - V\left(\frac{y_j}{\theta_i}\right). \tag{2.7}
\]

It restrains the social planner from choosing a mechanism where agents alter their labor supply in response to taxation. The common analogy used to explain this constraint is that it prevents high productivity agents from “shirking.”

The third set of equations contains the double deviation constraints (DDC’s). These expressions rule out mechanisms where an agent could use both methods (tax avoidance and changes in labor effort) to appear to be an agent of the other productivity-type:

\[
U(y_i - g(y_i, x_i) - T_i) - V\left(\frac{y_i}{\theta_i}\right) \geq U(y_j - g(y_j, x_i) - T_i) - V\left(\frac{y_j}{\theta_i}\right). \tag{2.8}
\]
This equation prevents mechanisms where, for example, a low productivity agent works like a high productivity agent while reporting the income level corresponding to low productivity agents. This is a profitable deviation if the low productivity agent’s costs of tax avoidance and exerting more labor effort are more than compensated by the additional consumption from their true income and tax transfers.

These conditions outline the set of separating equilibria. However, having a set of three incentive conditions per agent type can be burdensome. The following theorem can further narrow down the search for the optimum.

**Theorem 2.1 (Non-Falsification Theorem).** The solution to the social planner’s problem is a non-falsification mechanism.

The intuition behind this result rests on the assumption that the cost of tax avoidance is deadweight loss. Tax avoidance has negative effects on the social planner’s budget constraint because it reduces tax revenues. This reduction is not directly offset by an increase in agents’ consumption—by engaging in tax avoidance agents are are paying an unproductive cost that reduces their welfare and the net benefit of tax avoidance. These two forces contribute to welfare losses that could be averted if there was no tax avoidance. Hence, the social planner should reduce these welfare losses by choosing another mechanism that gives agents incentives to report all of their income. Starting from a mechanism that induces tax avoidance, this can be accomplished by modifying taxes to make agents indifferent between tax avoidance and complete income reports. This new mechanism will collect more tax revenue and increase welfare since it captures the deadweight loss costs of tax avoidance.

The Non-Falsification Theorem simplifies the problem by reducing the number of decision variables in the social planner’s problem. The set of mechanism the social planner has to search for the optimum is now restricted to mechanisms where the recommendations for income report and income are equal. Accordingly, the social planner’s constraints should be re-written and simplified.

**Corollary 2.2.** The tax avoidance constraint, (2.6), simplifies to

\[ g(y_i, y_j) \geq T(y_i) - T(y_j) \]

**Corollary 2.3.** The double deviation constraint is redundant if the other incentive compatibility constraints are satisfied

\[ g(y_i, y_j) \geq T_i - T_j \]

(TAC’s)

and

\[ U(y_i - T_i) - V \left( \frac{y_i}{\theta_i} \right) \geq U(y_j - T_j) - V \left( \frac{y_j}{\theta_i} \right). \]

(LSC’s)

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17 The proof of this theorem and the following corollaries are in the appendix.

18 Taxes are now based on truthful income reports. I will still use the simplified notation \( T(y_i) = T_i \), but note the change.

10
**Corollary 2.4** (Downward Binding tax avoidance Constraint). *The tax avoidance constraint on low productivity agents never binds,\[ g(y_l, y_h) \geq 0 \geq T_L - T_H.\]*

Rewriting the incentive compatibility constraints brings some insights into the maximization problem. Corollary 2.2 shows that the cost of tax avoidance sets an upper bound on tax revenues by imposing limits on the tax spread, or the difference between the tax liability of high and low productivity agents. As will be explained later, this limitation is a major driving force in the calculation of optimal taxes. Corollary 2.3 points out that a double deviation strategy is more costly than a single deviation strategy. Because agents pay costs associated with each deviation, it’s clear that making two deviations, rather than one, will make the agent worse off. Finally, Corollary 2.4 follows directly from the functional form of the objective function. A concave, utilitarian objective function guarantees that \( T_H \geq 0 \geq T_L \), implying that the tax avoidance constraint is only binding for the high productivity type agents. This fact, together with the “single crossing property,” reduces the set of binding incentive compatibility constraints to those imposed on high productivity agent’s recommendations and taxes.

**Simplified Social Planner’s Problem**

After reducing the number of incentive compatibility constraints, the social planner’s problem can be restated as searching for the mechanism \( \Gamma^* = \{y_H^*, y_L^*, T_H^*, T_L^*\} \) that solves:

\[
\max_{y_i, T_i} \pi \left[ U(y_H - T_H) - V\left(\frac{y_H}{\theta_H}\right)\right] + (1 - \pi) \left[ U(y_L - T_L) - V\left(\frac{y_L}{\theta_L}\right)\right]
\]

\[\text{s.t.}\]

\[
\begin{align*}
[\lambda] & \quad \pi T_H + (1 - \pi) T_L = 0 \\
[\gamma] & \quad g(y_H, y_L) \geq T_H - T_L \quad \text{(TAC)} \\
[\mu] & \quad U(y_H - T_H) - V\left(\frac{y_H}{\theta_H}\right) \geq U(y_L - T_L) - V\left(\frac{y_L}{\theta_L}\right) \quad \text{(LSC)}
\end{align*}
\]

where \( \lambda, \gamma, \mu \) are the multipliers on the feasibility, tax avoidance, and labor supply constraints respectively.

\[\text{19} \quad \text{The single crossing property of the indifference curves of high and low productivity agents guarantees that only the LSC on high productivity agents bind. The intuition behind this result rests on understanding the labor-leisure tradeoffs of agents with different productivity. A high productivity agent can produce income at a lower cost than a low productivity agent. Hence, high productivity agents may reduce their labor effort to qualify for low ability agents’ consumption-income bundle and still remain indifferent between their bundle and that of low productivity agents. The same strategy is not advantageous to low ability agents since producing the income level of high productivity agents is prohibitive. Raising their labor supply to match high productivity agents’ income levels will lead to a reduction in utility that can’t be offset by an increase in consumption.}\]
Optimum

The solution to this problem is characterized by the Kuhn-Tucker Theorem. The complementary slackness conditions on the incentive constraints allows for a total of four alternatives. However, one of these can be ruled out by noting that the first best allocation can’t be achieved in this economic environment.

**Proposition 2.5.** The social planner’s allocation is always constrained by at least one incentive compatibility constraint.

This result can be illustrated by changing the way the incentive compatibility constraints are written. Instead of inequality constraints, I can add two slack variables, \( \chi^{\text{avoidance}} \) and \( \chi^{\text{labor}} \) to the system of incentive compatibility constraints, and rewrite the former as equalities. These new variables are defined as the difference between high productivity agents’ utility level if they deviate from their income allocation and taxes. For the sake of clarity, assume that the utility functions are quasi-linear, \( W(c_i, l_i) = c_i - V(l_i) \):

\[
g(y_H, y_L) - T_H + T_L = \chi^{\text{avoidance}} \geq 0
\]

\[
\Delta \text{Consumption} - \frac{T_H + T_L - (V(y_H/\theta_H) - V(y_L/\theta_H))}{\Delta \text{Taxes}} = \chi^{\text{labor}} \geq 0
\]

The first variable, \( \chi^{\text{avoidance}} \), is high productivity agents’ net utility cost of tax avoidance. This variable is the difference between their costs of tax avoidance with the tax benefit of reporting the income level of low productivity agents. The second variable, \( \chi^{\text{labor}} \), is high productivity agents’ cost of changing their labor supply to match that of low productivity type agents. The first term in this equation is the change in consumption of a high productivity agent that changes his labor supply to mimic low productivity agents. The next two terms are the tax benefit (not paying taxes and receiving transfers) and extra leisure gained by working less.

For rational agents, the optimal deviation strategy is the least costly. Therefore, the binding constraint for the social planner for any level of \( g(\cdot) \) is

\[
\min \{ \chi^{\text{avoidance}}, \chi^{\text{labor}} \}.
\]

This implies that the social planner will be constrained by at least one incentive compatibility constraint. If not, the marginal benefit of tightening the least costly constraint is positive—when the Lagrange multipliers \( \gamma \) and \( \lambda \) are positive—and the social planner could increase welfare by changing the income allocation and taxes until high productivity agents are indifferent between deviating through the cheapest option.

\[\text{The proof is in the appendix.}\]
This logic narrows the solution to three complementary slackness alternatives, either:

**Region 1:** \( \gamma > 0, \mu = 0 \) - only the tax avoidance constraint binds

**Region 2:** \( \gamma > 0, \mu > 0 \) - the tax avoidance and labor supply constraints bind.

**Region 3:** \( \gamma = 0, \mu > 0 \) - only the labor supply constraint binds

These three conditions outline three incentive compatibility regions, each is a different solution to the social planner’s problem. As illustrated by condition 2.9 which of these alternatives is the solution depends on the relative levels of the costs of income deviation. The following analysis will vary the cost of tax avoidance to obtain a different outcome to condition 2.9 generating different solutions to the social planners problem.

### 2.3 Marginal Taxes and Redistribution

In each of the incentive compatibility regions, the optimal mechanism proposes a “bundle” for each type of agent that consists of a triple \( \{y_i^*, x_i^*, T_i^*\} \). In the consumption-income \((c, y)\) space, this mechanism can be implemented using a tax schedule that passes through a consumption and income point \((c_i, y_i)\) in each type of agents’ indifference curves, as illustrated in Figure 1.

![Figure 1: A hypothetical tax schedule](image-url)
Such tax function would guarantee that each type of agent chooses the bundle the social planner recommends for their productivity type. If this tax schedule was differentiable, the first order condition of the agents would be:

\[
\frac{V'(y_i)}{U'(c_i)} = (1 - \tau_i)
\]

The left-hand side is the agents’ marginal rate of substitution; the right-hand side is the after tax marginal return to working an extra hour. This expression defines marginal tax rates \(\tau_i\) for type \(i\). While it is clear that in general the tax schedule will not be differentiable, I will refer to \(\tau_i\)’s as marginal tax rates.

For \(g_{yH}, g_{yL} \in (0, 1)\) marginal taxes are:

\[
\begin{align*}
\text{Region 1:} \\
\tau_H &= -(1 - \pi)g'_{yH} \left(\frac{V_H'}{V_H} - 1\right) < 0 \\
\tau_L &= \pi g'_{yL} \left(1 - \frac{V_L'}{V_L}\right) > 0
\end{align*}
\]

\[
\begin{align*}
\text{Region 2:} \\
\tau_H &= -(1 - \pi)g'_{yH} \left(\frac{(1-\pi-\mu)\pi U''_H}{(1-\pi-\mu)(1-\pi)} - 1\right) < 0 \\
\tau_L &= 1 - \left(\frac{1-\pi-\mu \pi U''_L}{1-\pi-\mu \pi U''_H}\right) \left[1 + \pi g'_{yL} \left(1 - \frac{(\pi+\mu)(1-\pi)U''_L}{(1-\pi-\mu)\pi U''_H}\right)\right] > 0
\end{align*}
\]

\[
\begin{align*}
\text{Region 3:} \\
\tau_H &= 0 \\
\tau_L &= 1 - \left(\frac{1-\pi-\mu \pi U''_L}{1-\pi-\mu \pi U''_H}\right) > 0.
\end{align*}
\]

The connection between these regions lies on the cost of tax avoidance. In Region 1 the cost of avoidance is lower than the cost of changing labor supply, \(\chi_{\text{avoidance}} < \chi_{\text{labor}}\). As the cost of avoidance reaches the same level as the cost of changing labor supply, the expressions for marginal taxes in Region 1 converge to their counterparts in Region 2. In Region 2, high productivity agents are indifferent between using avoidance or labor supply as their avenue for report deviation. As the costs of avoidance increase, \(\chi_{\text{avoidance}} > \chi_{\text{labor}}\), the expressions for marginal taxes in Region 2 converge to those in Region 3. In this region, marginal taxes for high and low productivity match the expressions for marginal taxes in the Mirrleesian

\[\text{The tax schedule is no uniquely identified, but as in Figure 1 in some cases marginal taxes can be defined as the left-hand side derivative of } T(x) \text{ at each point } (c_i, y_i).\]

\[\text{If the marginal cost of deviation is zero, the social planner couldn’t prevent tax avoidance. On the other hand, values above one mean tax avoidance is too costly. In this latter case the model is equivalent to a two agent Mirrlees model.}\]
model with two productivity types.

The relationship between the marginal tax rates under the different regions can be best understood through a graphical analysis. For this analysis, assume that \( g(y - x) = \eta|y - x| \) for any \( y > x \in \mathbb{R} \), and \( \eta \in (0, 1) \). This functional form assumes that the cost of tax avoidance is proportional to the absolute distance between income produced and reported.

**Region 3**

Figure 2: Region 3 - Binding labor supply constraint.

![Figure 2: Region 3 - Binding labor supply constraint.](image)

Figure 2 illustrates the solution for Region 3. This graph is characterized by the sloping indifference curves of high and low productivity agents. Because agents’ marginal rate of substitution is proportional to their productivity, \( V'(y_i/\theta_i)/U'(c_i) = \theta_i \), the high productivity agent’s curve is flatter than the low productivity agent’s curve for any level of income. In this region \( \chi_{\text{avoidance}} > \chi_{\text{labor}} \), and the LSC is binding. Graphically this is represented by the intersection of the indifference curves, or Point A. At this intersection high productivity agents are indifferent between their consumption-income bundle and that of the low...
productivity agents, limiting the social planner’s income redistribution. Graphically, the vertical distance from Point A and the 45 degree line is the lump-sum transfer given to low productivity agents. Consequently, the tax assessed to high productivity agents is the same vertical distance since in absence of a revenue requirement income is redistributed from high to low ability agents. This tax is the vertical distance between the 45 degree line and Point B, where the slope of the indifference curve of high productivity agents is one. Point B maximizes tax revenue coming from high productivity agents and maintains incentive compatibility. If high productivity agents’ taxes were any larger (ceteris paribus) their consumption-income bundle would be below the optimal allocation, giving them the incentives to deviate and produce the income of low productivity agents. Similarly, any income allocation to the right or left of Point B on the indifference curve would yield less tax revenue, making it suboptimal. As mentioned before, marginal taxes are defined as one minus the slope of the indifference curves at Points A and B. For high productivity agents, this implies zero marginal taxes since their indifference curve is parallel to the 45 degree line at Point B. For low productivity agents the slope is below one, resulting in positive marginal taxes.

Figure 3: Region 3 - Non-binding tax avoidance constraint.

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23 This presumes that there are equal proportions of low and high productivity agents. The relative size of these distances will generally depend on the distribution of types.
In Figure 3, I superimpose on Figure 2 a line representing the region of incentive compatible allocations delimited by tax avoidance constraint. I call this line the TAC line. Any allocation that between the TAC line and the 45 degree line meets the TAC’s incentive compatibility condition, while any point on it represents an allocation where the TAC holds with equality. To illustrate why the TAC is not binding in this region, take an alternative allocation Points C and D. These points mark an allocation that would allow more income redistribution than Points A and B—as evidenced by their greater vertical distance from the 45 degree line to the level of consumption of both agents at the same level of income. However, choosing this alternative allocation would result in a violation of the incentive compatibility conditions of the LSC: given their cost of changing labor supply and of tax avoidance, high productivity agents would be better off by changing their labor supply to produce and consume the income-consumption bundle of low productivity type agents. Therefore, this alternative allocation does not meet all of the requirements of the optimal mechanism. Contrast this to points A and B, which meet the conditions of both TAC and LSC and is binding for the LSC. This allocation meets all incentive compatibility conditions and maximizes tax revenue.

**Region 2**

Figure 4: Region 2 - Binding tax avoidance and labor supply constraints.
For Region 2, the TAC line illustrated in Figure 4 is steeper due to a lower value for \( \eta \), and runs through the intersection of the agents’ indifference curves, Point E. This intersection marks the equivalence of either constraint in how they limit the redistribution efforts of the social planner. The vertical distance from Point E to the 45 degree line marks the transfers to low productivity agents, just like the vertical distance from Point F marks the amount of taxes high productivity agents need to pay. Note that the slope of the indifference curve at Point F is above one, implying negative marginal taxes for high productivity agents. They need this incentive to increase the amount of income they produce. Since all tax revenues are collected from high productivity agents, increasing the amount of income they produce relaxes the limits to redistribution set by the TAC. This is illustrated by the growing difference between the TAC line and the 45 degree line as income increases. This difference captures the size of the tax base: as high productivity agents’ income increases, there is a larger pool of income to be taxed, which in turn can be redistributed to low productivity agents. It’s worth noting that the tax spread, \( T_H - T_L \), in this region is reduced from their levels in Region 3. This directly translates into lower tax revenue levels compared to the Mirrleesian solution. Across the regions where tax avoidance is binding, as \( \eta \) decreases, the limit on tax revenues collected becomes tighter leading to lower levels of tax collections.

**Region 1**

In the last region, Figure 5, the TAC line is below the intersection of the indifference curves making it the binding constraint on redistribution. As in the previous cases, the intersections of this line, Points G and I, with the indifference curves of both high and low productivity agent marks the income-consumption bundle for both types. Any allocation between the TAC line and the 45 degree line is incentive compatible, but tax revenue is maximized when the TAC line intersects the indifference curves. Just like in Region 2, the slope of the indifference curves at the intersection points defines marginal taxes. High productivity agents face negative marginal taxes, since Point I is almost always above the point in the indifference curve where its slope is equal to one. Low productivity agents almost always face positive marginal taxes. I say almost always because throughout this region as the cost of tax avoidance tends towards zero the slope of this line increases towards one, making income transfers more difficult. At the extreme, when \( \eta = 0 \), the slope of the TAC line is one (overlapping the 45 degree line) resulting in zero tax revenue and marginal taxes for both types of agents. Anywhere else in this region, negative marginal taxes on high productivity agents encourage truth-telling while increase the size of the tax base. High productivity agents gain by reporting any additional unit of income, while on the other hand, hiding that unit income would cost them resources. By increasing these agents’ labor supply through negative marginal taxes, the social planner increases the pool of income to redistribute across income types by relaxing the left hand side of the tax avoidance constraint.

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3 Numerical Solution

The complete solution for this model requires the numerical computation of lump-sum taxes and multipliers. For this purpose, I assume the same functional form for the cost of tax avoidance as in the graphical analysis in the previous section, \( \eta|y - x| \), and the utility from consumption and labor takes the form:

\[
U(c_i) - V(l_i) = \frac{c_i^{1-\gamma} - 1}{1 - \gamma} - \frac{\alpha}{\sigma} y^{\sigma}.
\]

The parameter \( \gamma \) determines the degree of curvature of the utility from consumption, \( \alpha \) is the relative weight of leisure to consumption utility, and \( \sigma \) determines the agents’ compensated labor supply elasticity, \( \epsilon_c = \frac{1}{1 - \sigma} \). The values for these parameters are \( \gamma = 1.5, \alpha = 2.55, \sigma = 3 \) respectively. The value of sigma implies an elasticity of labor supply somewhere in between the upper limit of micro-estimates, and conventional macro estimates, \( \epsilon_c = 0.5 \). The value of alpha is chosen to match the average percentage of hours that are spent on work from the NLSY - approximately 8 hours. The probability of being a high productivity agent is \( \mathbb{P}(\theta_H) = \pi = 0.5 \). Finally, workers productivity, will be represented by hourly wages:
\( \theta_L = 40 \) and \( \theta_h = 100 \). These values of \( \theta \) are the 90\textsuperscript{th} and 99\textsuperscript{th} percentiles of the US wage distribution respectively.\(^{25}\)

**Optimal mechanism as a function of the cost of tax avoidance**

Just as in the previous section, the optimal mechanism can be expressed as a function of \( \eta \). Figure 6 shows marginal taxes for different values of this parameter. When \( \eta = 0 \), agents can falsify their income at no cost, hence marginal taxes for high and low productivity agents are zero, corresponding to the autarkical solution. For high productivity agents, as \( \eta \) increases through the range \((0, 1)\), marginal taxes become negative and then return to zero. This path is shaped by two forces working in opposite directions: the marginal cost of tax avoidance \( g_{yH} = \eta \), and the utility inequality among the agents \((U'_L/U'_H - 1)\). As \( \eta \) rises, the utility inequality among agents is reduced. Initially, \((U'_L/U'_H - 1)\) increases

\[^{25}\text{These values from Mankiw and Weinzierl (2010)}\]
slowly, leading to a rapid decrease of marginal tax rates. However, as the social planner’s limits on redistribution are loosened by an increasing cost of tax avoidance, utility inequality is reduced, pushing for an increase in marginal taxes. On the other hand, low productivity agents face monotonically increasing marginal taxes as the values for $\eta$ increase, reaching their height at Region 3. Their income process works in a similar fashion, except that both forces previously mentioned work in the same direction, leading to a monotonic increase in taxes.

Note that after $\eta$ reaches the point where $\chi^{labor} \geq \chi^{taxavoidance}$ (Region 3) there is no change in taxes and $\eta$ becomes irrelevant. In Region 3 high productivity agent’s costs of changing their labor supply to reduce their tax liability is lower than the cost of tax avoidance. Furthermore, in the same figure, graphed along with marginal taxes is a line representing $\eta$. Its purpose is to illustrate that high productivity agents’ marginal tax may never exceed the marginal cost of tax avoidance. The logic is simple, if the social planner imposes marginal taxes above the marginal cost of tax avoidance, agents would have incentives to hide their income. As stated by the Non-falsification Theorem, tax avoidance is not optimal, and the social planner could improve the welfare of at least one type of agents by encouraging truthful income reports.

![Figure 7: Indirect utility for different values of $\eta$.](image-url)
Figure 7 shows the path of aggregate welfare and high and low productivity agents’ indirect utility. As $\eta$ increases the agents’ indirect utility gets closer to one another—as expected when using a utilitarian objective function. Aggregate welfare slowly increases to reach its peak in Region 3. This welfare equalization process hinges on the social planner’s capacity to redistribute income, which is in itself a function of the cost of tax avoidance. Throughout Region 1, welfare increases as the agents’ indifference curves shift away from the 45 degree line in the consumption-income space.

Figure 8: Income-consumption bundle for different values of $\eta$.

This process is a combination of how the income allocation and taxes change as $\eta$ increases. As the TAC line in Figure 5 becomes flatter, the tax avoidance constraint is relaxed, and the social planner can impose larger lump-sum taxes on high productivity agents. To increase the size of these lump-sum taxes the social planner also encourages more labor effort from them. The result of this incentive can be seen in Figure 8. Because high productivity agents are encouraged to increase their labor effort for low values of $\eta$, income increases monotonically throughout Region 1, reaches its peak while passing through Region 2, and then retreats to the level in Region 3—precisely as negative marginal taxes approach zero. This figure also shows the opposite effect on low productivity agents, who receive transfers and work less as $\eta$ increases.
Figure 9: Aggregate expected income for different values of $\eta$.

Figure 10: Consumption and income inequality for different values of $\eta$. 
This process has a direct impact on aggregate output, which is illustrated in Figure 9. Aggregate output increases monotonically throughout Region 1, and stays above its level in autarky ($\eta = 0$) through most of Region 2. As $\eta$ increases towards Region 3 output falls below autarky levels. Another consequence of the welfare equalization process is the path of income and consumption inequality. As $\eta$ moves in the $(0, 1)$ range, income inequality between high and low productivity agents increases, see Figure 10. Similarly, consumption inequality is almost always decreasing. Although it’s difficult to see in the graph, consumption inequality increases for very low values of $\eta$ and then starts to descend until its lowest level in Region 3. For these values, high productivity agents have to be compensated to report all of their income, which might result in greater consumption inequality as compared to autarky (or $\eta = 0$).

4 The cost of tax avoidance

The previous section shows that incorporating tax avoidance into a Mirrleesian model has substantial effects on the optimal tax schedule. In particular, depending on the cost of tax avoidance, the optimal mechanism belongs to one of three distinct incentive compatibility regions. When this cost is low enough, taxpayers’ use tax avoidance to circumvent taxation and the optimal tax schedule is designed to prevent their income report deviations. On the other hand, when the cost of income falsification is high enough, the tax schedule is designed to prevent agents’ labor supply deviations. Since taxpayers’ costs of deviating through their labor supply is fixed by the parametrization of the model, determining where the solution lies within the incentive compatibility regions depends on $\eta$, the proportional cost tax avoidance. This section uses data on taxpayers tax returns to quantify $\eta$. This estimate can be used to find which incentive compatibility region is consistent with US data.

4.1 Empirical strategy

Tax avoidance costs are difficult to observe since their are obscure by nature. Measuring them directly would require a sample of taxpayers with detailed records of their tax avoidance strategies and their costs. Instead, I use the agents’ optimality conditions to estimate an upper bound for $\eta$.

For any optimal tax avoidance outcome, agents’ choices of income and income reports, $y^*$ and $x^*$, imply that

$$y^* - g(y^*, x^*) - T(x^*) \geq y^* - g(y^*, x) - T(x) \quad \forall x \neq x^*.$$

The parameters $\alpha$ and $\sigma$ determine the curvature of the dis-utility of labor. The cost of changing labor supply directly depends on these parameters.
Since \( y^* \neq x^* \), this expression can be written for any optimal deviation from a truthful report:

\[
y^* - g(y^*, x^*) - T(x^*) \geq y^* - g(y^*, y^*) - T(y^*)
\]

or

\[
T(y^*) - T(x^*) \geq g(y^*, x^*).
\]

This equation sets the benefit of tax avoidance, \( B(y^i, x^i) \), as the upper bound of the cost of tax avoidance:

\[
B(y^i, x^i) = T(y^i) - T(x^i) \geq g(y^i, x^i).
\]

From this inequality, the upper bound for the proportional cost of tax avoidance for a given level of reported income, \( \bar{\eta} \), is simply the total benefit of tax avoidance divided by hidden income

\[
\bar{\eta} \leq \frac{B(y^i, x^i)}{y^i - x^i}.
\]

This upper bound can be estimated using two components: an estimate of true income \( \hat{y}^i \) and a tax function \( T(z) \).

An estimate of \( \bar{\eta} \) can determine which of the incentive compatibility regions is empirically relevant. The numerical solution to the model from Section 3 provided the thresholds for each region: \( \eta_1 = (0, 0.58) \), \( \eta_2 = [0.58, 0.66] \), and \( \eta_3 = [0.66, 1) \) for Region 1, Region 2, and Region 3 respectively. If the upper bound for the cost of income falsification for income in the 99th percentile is within Region 1 or 2, \( \bar{\eta}_{99} \in (0, 0.66) \), then the tax avoidance solution proposed by this paper is is consistent with US data. On other hand, if \( \bar{\eta}_{99} > 0.66 \), this statistic can’t rule out Region 3 as the solution to the social planners problem.

### 4.2 Estimating true income \( y^* \)

I follow the “expenditure approach” to estimate tax avoidance rates. Once obtained, these rates can adjust income reports \( x^* \), to estimate true income, \( y^* \).

The expenditure approach compares the income and expenditures of two subsets of taxpayers to determine relative levels of income misreporting. It uses individuals that report their income accurately as counterfactuals for misreporting taxpayers. Assuming that both groups have the same consumption preferences on average, hence similar expenditures at any given level of income, average differences in reported income across groups can be attributed to income misreporting.

Estimates of income misreporting rates using the expenditure approach are usually interpreted as measuring tax evasion. However, these estimates capture both tax evasion and tax avoidance. The assumption justifying their interpretation as tax evasion holds as long as the relationship between true income and income reports over different sources of income.

27 Using a composition of two estimates will, on average, overestimate \( \bar{\eta} \). This is not a problem for the purpose of estimating \( \bar{\eta} \) as it will be used as an upper bound.
does not change. However, taxpayers that earn income other than wages and salaries have greater income reporting flexibility than wage earners. Taxpayers with self-employment, rent income, farm income, etc are not subject to stringent information reporting or income withholding. Additionally, there are many deductions, exceptions and offsetting expenditures these taxpayers use to reduce their taxable income. This translates into different tax treatment between taxpayers earning wages and salaries and those with other sources of income. Therefore, the relationship between true income is not the same across income sources in part because of tax avoidance.

Because the estimates for true income using the expenditure approach include tax avoidance and evasion responses, I interpret them an upper-bound estimate of tax avoidance. From now on I will refer them as tax avoidance, but note this distinction. Furthermore, since the model can also accommodate tax evasion, the usage of misreporting rates as a proxy for tax avoidance is consistent with the numerical characterization of the model.

To estimate tax avoidance rates for taxpayers in the US, I use an application of the expenditure approach by Feldman and Slemrod (2007). As in their paper, I use tax return data to divide taxpayers into two subgroups, those earning only wage and salary income and those earning income from at least one other source (non-farm proprietors income, rents and royalties, parterships/S-Corp income, or capital gains). I compare the income reports of these two groups to the charitable donations itemized on their tax forms as a measure of their expenditures. Assuming that charitable preferences are similar across income categories, differences in reported income by taxpayers who report similar charitable donations across groups measures relative levels of tax avoidance.

The identification of estimates derived this way rests on two key assumptions. First, wage and salary earners report their income truthfully. Second, the propensity to make and report charitable donations at any given level of income is not associated with the source of income. The first assumption is true in the case of tax evasion. This is confirmed by evidence that taxpayers that earn wages and salaries have high compliance rates. In the case of tax avoidance, this assumption implies that the estimated rates of tax avoidance are calculated relative to the level of tax avoidance of wage earners. In regards to the second assumption, one could argue that self-employed individuals might use charitable donations as way to

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28 Because the expenditure approach also captures legal activities that reduce taxable income, its estimates could also include tax planning. Tax planning are the expected outcome of the incentives provided by tax laws. This would overstate the magnitude of income misreporting rates.

29 The appendix contains a discussion on this idea.

30 Since I am using these estimates to quantify an upper bound for \( \eta \), overestimating tax avoidance only makes a difference if my estimates of \( \eta \) are in Region 3. The ranges for \( \eta \) within this regions used in Section 3 correspond to \( \eta_1 = [0, 0.58) \), \( \eta_2 = [0.58, 0.66) \), and \( \eta_3 = [0.66, 1) \) for Region 1, Region 2, and Region 3 respectively. Changing the spread between income reports and true income by 10% on either side of the estimates obtained in Section 4 increase (decrease) \( \eta \) by less than 1% on average, and 5% at most. Since the estimated \( \eta \) is roughly 25%, changing the spread between true income and income reports does not affect the results. Changes of 20%, 50% to either side of the estimates didn’t change the qualitative results using the parametrization from Section 3 either.

31 The IRS periodically reports compliance rates for different sources of income, for an example see Internal Revenue Service (1996).
promote their business or signal its profitability. This behavior could explain differences in charitable expenditures across sources of income. If this was true, the connection between charitable donations and income source should be evident on data collected for purposes other than revenue collection. This is not the case. Feldman and Slemrod (2007) find no evidence that self-employed individuals’ charitable donations are different from those earning wages and salaries when analyzing data from the Independent Sector’s Giving and Volunteering National Survey. Hence, while it is possible that charitable contributions might differ across income sources, these differences are not large enough to be significant.

Data and econometric model

The information on taxpayers’ income tax returns comes from the IRS Statistics of Income Data. The data I use spans the decade between 1998 and 2008, and consists of yearly stratified samples of the population of taxpayers. Each of the 2.2 million records records in the sample contains anonymous, yet detailed information on almost every field of a taxpayer’s tax return. Most importantly for the purposes of this paper, each entry provides information about taxpayers’ reported income by source: wages and salaries, self-employment income, capital gains income, etc. In addition, for taxpayers that itemize their deductions, the data contains their reported charitable conditions.

Since the relationship between true income and charitable expenditures can’t be quantified for taxpayers who don’t itemize, I restrict the sample to those that do. I keep taxpayers that are married filing jointly or single to avoid additional biases due to the blurring technique used by the IRS to protect the identify of the individuals in the sample. Furthermore, the data has been harmonized by field across all years. The original version of the data distributed by the IRS comes in yearly batches, and since tax returns forms and field definitions change over time, a data set containing multiple years has to be matched to reflect these changes across years. Lastly, for any pooled estimates using more than one year, all records have been adjusted for inflation using 2000 as the base year. The final sub sample contains a total of 886,456 observations.

The data will be used to estimate tax avoidance rates based on the relationship between taxpayers’ income and charitable contributions described by the equation:

\[ \ln(G) = a_0 + a_1 \ln(I). \] (4.2)

---

32 Not all taxpayers who itemize give to charity. Out of itemizing taxpayers, 94% give some form of charitable contributions. The average of these contributions is $77,360.

33 Taxpayers who itemize in their tax forms are 62% of the sample. They are more likely to be married and claim a larger number of non-personal exceptions on average that those who claim the standard deduction (82% are married compared to 56%, and claim an average 0.93 non-personal exceptions to 0.54). They also earn more income and pay more taxes. For example, in the year 2000, the average income and tax liability for itemizing taxpayers is $2,304,613 $638,826 (median: $695,500 $189,479) respectively. Compare this to the average income and tax liability for taxpayers claiming the standard deduction $117,974 $86,414 (median: $24,020 $2,464).

34 For a discussion on this blurring technique, see Slemrod (1985).
The dependent variable $ln(G)$ is the log of charitable contributions $G$. The variable $I$ represents total income. This linear functional form assumes a constant income elasticity of charitable donations: for any increase in income, charitable giving changes proportionally to $a_1$. This coefficient captures the relationship between income and charitable giving absent of any tax avoidance. To incorporate tax avoidance, I expand the sources (types) of income taxpayers receive. Following the broad definition of tax avoidance in this paper, I divide total income, $I$, into two categories: ordinary income $O$ from wages and salaries, and income subject to tax avoidance, $F$, which sums income from all sources of income other than ordinary income.

$$ln(G) = a_0 + a_1 ln(O + kF) \quad (4.3)$$

Income $F$ will be multiplied by the adjustment factor $k$. This adjustment factor can reconcile differences in income reported among income sources to match a constant income elasticity of charitable giving, $a_1$. A positive value for $k$ suggests that income $F$ has to be larger on average than what is reported by taxpayers to be consistent with their average charitable preferences. Consequently, the fraction $1 - 1/k$ is the average tax avoidance rate. This implies that the relationship between income reports and true income is linear:

$$y^* = kx^*.$$  

The relationship in equation 4.3 will be estimated using non-linear least squares using equation 4.4 below. It departs from equation 4.3 because there are additional adjustments that should be made to account for idiosyncrasies in tax return data.

$$ln(G_{\text{cash}} + 100) = a_0 + a_1 ln(O + kF + \sum b_i S_i) + c_1 ln(Price) + c_2 NPEX + c_3 MAR + \epsilon. \quad (4.4)$$

First, charitable contributions are divided into cash and non-cash gifts. Because these two categories are treated differently in the tax code and the value of non-cash gifts is difficult to assess, I only include charitable contributions made in cash. Additionally, I add a scalar (100) to cash charitable contributions in order to include those taxpayers who itemize but don’t make charitable contributions.

Total income is Adjusted Gross Income (AGI) as defined by the IRS plus an adjustment

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35 About 12% of taxpayers in the sample earn only ordinary income. This statistic doesn’t take into account observation weighting.

36 $1/k$ is the percentage of true income that is reported by taxpayers with $F$ income.

37 Tax law requires that total charitable contributions for a particular year can’t exceed 50% of AGI. Whatever is left over can be itemized in the following year. The amount of taxpayers that hit that limit is 0.38% of the total, so I did not make an adjustment.

38 About 92% of taxpayers itemize cash donations at an average value of $39,452. 98% of taxpayers give both cash and non-cash donations, 1.8% only give non-cash donations, and 0.2% only donate cash. Over all the itemizing observations, the proportion of the value of cash contributions to total contributions is 82%.

39 Estimates are robust to different scalar factors ranging from 20 to 120.
for capital gains income. This adjustment is necessary because there is a cap on capital losses for any given year. The remainder of a loss greater than the cap can be carried forward to another year, up to the cap, until the carryovers are exhausted. Hence, to obtain a precise measurement of reported gains and losses for a given year, I define capital gains and losses (Schedule D income) as the sum of short and long term gains and unrestricted losses.

Additionally, I add a dummy variable, \( S_i \), for the source of reported income \( i \in \{ C, E, F \} \), for business income (Schedule C), farm income (schedule F), and supplemental income and loss (Schedule E) respectively. These dummy variables correct for level differences in reported income for those sources. It’s coefficient, \( b_i \), estimates the level of hidden income of taxpayers that file a schedule \( i \) in their tax forms but report zero income for that source. A positive estimate of this coefficient indicates that taxpayers that file a schedule \( i \) earn \( b_i \) more income than those who don’t.

Furthermore, because charitable contributions are tax deductible, the relationship between income and charitable giving should be controlled for changes in the relative price of charitable giving. Therefore, I add the variable \( \text{Price} \), defined as the marginal after tax costs of donating to charity, or \( 1 - T'(x) \), where \( T'(x) \) is the effective marginal tax on reported income.\(^{40}\) I also include the variables NPEX and MAR as demographic controls. NPEX represents total non-personal exceptions and MAR is indicator variable for married couples filing jointly.

Results

Table 1 contains results from pooling all available years under different specifications of equation 4.4. By using all available years, estimates capture the overall pattern of taxpayers’ tax avoidance in the U.S. during the decade of 1998-2008. For the first two columns, I only use observations with a positive value of F income. This means, however, that there is a loss of information calculating the income elasticity of charitable donations since there are 131,450 observations with a negative value for F income. To include this information into the calculation of \( a_3 \), I divide F income into positive and negative components and estimate coefficients \( k_p \) and \( k_n \) for positive and negative values respectively. Furthermore, in the last specification I add a quadratic term for negative and positive F income, with coefficients \( q_p \) and \( q_n \). These coefficients capture the differences in tax avoidance across the income distribution, and serve as a robustness check for the model’s specification.

The results for \( k \) in column 3 (equation 4.4) suggests that on average, income reported by those who have access to tax avoidance is 69% of their true income, or equivalently they hide

\(^{40}\) In order to calculate effective marginal taxes for each itemizing taxpayer, I adjust tax liability to be consistent across income sources. Self-employment income and farm income are subject to paying a full contribution of Social Security and Medicare taxes. In contrast employers match the 50% contribution to Social Security and Medicare of their employees. I make the necessary correction on tax liability for taxpayers with self-employment and farm income to make sure that there is no bias on their estimates. Additionally, I do not use the state marginal income tax rate to calculate effective tax rates, since the data does not contain state identifiers for those with income above $200,000. Effective tax rate calculations do take into account Alternative Minimum Tax.
Table 1: Estimated Regression Coefficients, Equation 4.4

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ln($G_{\text{cash}} + 100$)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.647***</td>
<td>0.686***</td>
<td>0.682***</td>
<td>0.683***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$k^1$</td>
<td>1.602***</td>
<td>1.602***</td>
<td>1.446***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_p^1$</td>
<td></td>
<td></td>
<td>1.588***</td>
<td>1.590***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$k_n^1$</td>
<td></td>
<td></td>
<td>-1.887***</td>
<td>-1.892***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.062)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>$q_p^2$</td>
<td></td>
<td></td>
<td></td>
<td>-0.003***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>$q_n^2$</td>
<td></td>
<td></td>
<td></td>
<td>-0.020***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.001)</td>
</tr>
<tr>
<td>$b_1$ (Schedule C)</td>
<td>7,351.558***</td>
<td>6,460.606***</td>
<td>6,596.206***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(550.356)</td>
<td>(563.600)</td>
<td>(566.444)</td>
<td></td>
</tr>
<tr>
<td>$b_2$ (Schedule E)</td>
<td>34,676.793***</td>
<td>30,982.682***</td>
<td>31,187.202***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(992.643)</td>
<td>(1,021.676)</td>
<td>(1,022.440)</td>
<td></td>
</tr>
<tr>
<td>$b_3$ (Schedule F)</td>
<td>79,778.823***</td>
<td>77,935.870***</td>
<td>78,037.248***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4,003.341)</td>
<td>(4,208.583)</td>
<td>(4,204.255)</td>
<td></td>
</tr>
<tr>
<td>$c_1$ (Log(PRICE))</td>
<td>-1.239***</td>
<td>-1.359***</td>
<td>-1.354***</td>
<td>-1.358***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$c_2$ (NPEX)</td>
<td>-0.015***</td>
<td>-0.013***</td>
<td>-0.012***</td>
<td>-0.012***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$c_3$ (MAR)</td>
<td>0.500***</td>
<td>0.492***</td>
<td>0.492***</td>
<td>0.491***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Observations</td>
<td>646,946</td>
<td>646,946</td>
<td>886,456</td>
<td>886,456</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.324</td>
<td>0.326</td>
<td>0.328</td>
<td>0.328</td>
</tr>
</tbody>
</table>

Estimated using unweighted non-linear least squares. Robust standard errors in parentheses.

*** p<i>0.01, ** p<i>0.05, * p<i>0.1

1 Significantly different from one

2 Corresponding coefficients are multiplied by one million.
about 30% of their income. Furthermore, the values for the coefficients \( b_i \) suggests that given similar charitable inclinations, those that report business, supplemental, or farm income earn $7,627, $34,676, $79,778 respectively more true income than taxpayers who don’t file these forms.

The negative coefficient on the log of the price of charity, \( c_1 \), suggests that an increase in the price of charity, or a decrease in the effective marginal tax rate, due to charitable donations, leads to a decrease in charitable donations. Additionally, the coefficient \( c_2 \) shows that charitable contributions decrease as the number of household dependents grow. The coefficient \( c_3 \) indicates that married taxpayers tend to donate more than similar single households. And finally, the estimate for \( a_1 \), or the True income elasticity of charitable giving is 0.68. All of these estimates are consistent and within the range of similar estimates found in the literature of tax incentives on charitable donations.\(^{41}\)

The estimated values of \( q_p \) and \( q_n \) suggests that as income rises (decreases) the percentage of income reported decreases, or that tax avoidance rises as proportion of true income. However, the values for these coefficients are very small. In order to scale the coefficients for the tables, the quadratic income terms were divided by one million. This implies that the relationship between income reports \( x^* \) and income \( y^* \) is approximately linear.\(^{42}\) Incorporating the coefficient estimates from Table 1, the relationship between observed income reports and true income estimated from equation 4.2 is:

\[
\hat{y}^* = 1.446 x^*.
\]

### 4.3 Estimating the tax function \( T(z) \)

I use the method by Gouveia and Strauss (1994) to estimate effective average tax rates. They proposed an axiomatically justified equation that describes the relationship between effective income tax rates and taxpayers’ reported income. Their method is particularly appropriate for estimating the cost of tax avoidance since their equation for average tax rates incorporates incentive effects influence taxpayers’ choice of observable income level. In fact, the equation can be interpreted as a “classical equal sacrifice function when there are substantial costs, other than the tax payment proper, that are born by the taxpayer,” Gouveia and Strauss (1994). Furthermore, their results have been successfully compared to non-parametric estimates and other structural estimates.\(^{43}\)

To be consistent with the counterfactual method used in estimating \( \hat{y}^* \), the tax function used to calculate \( B(y^*_i, x^*_i) \) should be the effective tax function levied on taxpayers who earn ordinary income. After restricting the data sample to taxpayers who only earn ordinary income, I estimate the equation:

\(^{41}\) Please refer to Clotfelter (1985) and Steinberg (1990).

\(^{42}\) I also tried other different polynomial specifications with similar results.

\(^{43}\) A more recent study by Guner, Kaygusuz and Ventura (2014) note that the Gouveia-Strauss provides the best fit to the data, but suggest that it understates marginal taxes at the top of the income distribution as compared to other alternatives.
\[ T(z) = b - b(sz^\rho + 1)^{-\frac{1}{\rho}} + \epsilon. \] (4.5)

This expression relates average effective tax rates to income reports, \( z \). The parameter \( b \) measures the maximum asymptotic effective tax rate, \( 1 + \rho \) is the elasticity of the marginal utility of consumption, and \( s \) is the sacrifice, or utility cost from reporting income \( z \). I use the same definitions for total income and tax liability as in the previous section, namely that income reported is equal to capital gains adjusted AGI and average tax rates is total tax liability after credits divided by reported income.

The parameter estimates from the regression routine are \( b = 0.31 \), \( \rho = 0.76 \), and \( s = 0.0000482 \).\(^{44}\) This suggests that the maximum average effective tax rates under the current schedule for the period of 1998 to 2008 was approximately 31%.

### 4.4 The proportional cost of tax avoidance

![Figure 11: Upper bound for the proportional cost of tax avoidance: \( \eta_i \).](image)

Note that only taxpayers who earn ordinary wages are used for this estimation, hence income reports are equal to true income.\(^{44}\)

These coefficients are significant at a 95% level. I use robust standard errors and unweighted non-linear least squares estimation.\(^{45}\)

The estimate for the elasticity of the marginal utility of consumption \( 1 + \rho \) is consistent with the parametrization of the model in Section 3.\(^{46}\)
Having obtained an estimate for the effective tax function and taxpayers’ optimal choices of true income and income report, I can calculate the upper bound for the proportional cost of tax avoidance using equation 4.1. For every choice of income $y^*$, Figure 11 shows $\bar{\eta}_i$.

Regardless of the level of income chosen, the proportional cost of tax avoidance is no higher than 30% of hidden income. Maintaining the productivity dispersion between agents in the numerical solution to the model to match that of the tax avoidance gap $1 - 1/k$, this level for $\bar{\eta}$ is always within Region 1 (compare to $\eta$ in Figure 6 for $y_H \approx \$200,000$). This implies that the solution to the simplified social planners problems lies somewhere within this Region, and that this Region is consistent with US data.

5 Conclusion

Evidence shows that taxpayers respond to the labor incentives created by the tax system, both by the tax schedule and the opportunities for tax avoidance found in the tax code. Yet the normative tax literature has mostly focused on the effect of labor supply choices in the design of the optimal tax schedule. This article addresses this gap by developing a costly tax avoidance framework for the normative analysis of taxation.

The theoretical results derived from the model illustrate how incorporating tax avoidance changes the design of optimal taxes. The solution to the model can be belong to one of three incentive compatible regions. Which of these region is empirically relevant depends on the relative costs of income deviation, either through labor supply or tax avoidance. The empirical section of this paper quantifies this relationship by estimating the cost of tax avoidance in the US. I find that the cost of tax avoidance is at most 30% of taxpayers hidden income. This estimate lies within the tax avoidance solution, and shows that the cost of tax avoidance is lower than the taxpayers’ costs of changing their labor supply implied by the model.

This highlights the importance of tax avoidance in the normative analysis of taxation. In particular, the design of optimal marginal tax rates should take into consideration the laws and income reporting structure that generate opportunities for tax avoidance. This would allow normative prescriptions to incorporate taxpayers’ responses to taxation which determine the success of the implementation of tax policy.
References


6 Appendix

Income reports, messages, and the Revelation Principle

The income report structure is consistent with the Revelation Principle and is without loss of generality. First, it’s consistent with the Revelation Principle because in this environment any arbitrary mechanism can be replicated by a direct revelation mechanism where agents truthfully report their productivity types via messages. Truth-telling, however, does not rule out income falsification through tax avoidance. An agent can send a message stating to be of a particular productivity type while reporting an income level that is not equal to her produced income. As long as the message sent and income reported are in accord with the income reports of all other agents of her type (those who sent the same message), the social planner can’t verify whether the income reported is the entirety of her produced income. Second, the structure of income reports, as previously described, is without loss of generality because the message space is extraneous. To prove this, suppose that instead of imposing taxes solely on reported income, $T(x_i)$, taxes are based on both the reported income and a message sent to the social planner $T(x_i, m_i)$, $T : X \times M \rightarrow \mathbb{R}$. In this mechanism, $m \in M$ is an element of the message space $M$. Assume that for two different income realizations $y$ and $y'$ the income reported is the same, or $x(y) = x(y') = x$, but the message sent is different, $m(y) = m$, $m(y') = m' \neq m$. For the pair $(x, m)$ to be optimal when $y$ realized it must be that
\[
y - g(y, x) - T(x, m) \geq y - g(y, x) - T(x, m') \\
T(x, m) \leq T(x, m').
\]
Similarly, the optimality of $(x, m')$ when $y'$ is realized requires that
\[
y' - g(y', x) - T(x, m') \geq y' - g(y', x) - T(x, m) \\
T(x, m') \leq T(x, m).
\]
So that two income realizations with the same reported income must lead to the same allocation, making messages extraneous.

Additionally, I want to make an important distinction between sending a message and reporting income. By reporting income, agents are taking an action to make information publicly available and readily verifiable. In contrast, in the context of mechanism design, a message can be any form of communication between agents and the social planner. This communication may include information about income levels, but is not verifiable and does not necessarily reveal private information. Also, an income report doesn’t necessarily have to equal total produced income, it may not; reporting income determines how much income is observed by the social planner, and implies the possibility that a fraction of agents’ income may remain unobserved.

This argument follows Lacker and Weinberg (1989) who developed costly state falsification in the context of optimal contract theory.
Discussion on the assumption of exogenous cost of tax avoidance

One may argue that the social planner should be able to “fix” the tax avoidance problem first by eliminating loopholes, and then search for the optimal tax schedule. This argument suggests that the social planner should have power to manipulate all the structure of the tax system, including the tax code. If one subscribes to this argument, then taking portions of the structure of the tax system (like tax loopholes and opportunities for tax avoidance) as exogenous is inconsistent with a description of how the world should be.

This argument can be debated using the same justification of the assumption that agent’s productivity is private information. A.B. Atkinson, noted that information varies in its relevance to optimal income tax design. If information about a person’s productivity is either very difficult to collect or is not admissible for tax design purposes, then it’s justified as exogenous when designing the optimal tax schedule. Information about taxpayers’ income meets similar criteria. Like determining the productivity of a taxpayer, gathering accurate information about taxpayers’ income can be difficult. The obstacles can be technological or legislative. A government needs to establish effective institutions and methods of collecting income information, like enacting proper compliance statutes and auditing organizations. Assuming that these institutions are already in place and that they function perfectly goes against the realities of most countries.

There are also political barriers to the free flow of income information between the government and the taxpayer. The tax code is generally designed by legislative bodies under the influence of many political, social, and economic forces. It’s only appropriate to take these features as exogenous if these forces are not explicitly modeled. The normative focus of this paper is not to describe the optimal tax system, rather it’s to find optimal tax schedule, and outline some of the consequences of implementing it within the limitations of an already existing tax system.

7 Proofs

Proof of the Non-falsification Theorem [2.1]. The proof consists of showing that for any falsification mechanism mechanism (where \( x_i \neq y_i \) for at least one type of agent) there is a non-falsification mechanism (where \( x_i = y_i, \forall i \)) that delivers no less aggregate utility and increases government revenue. This proves that the solution to the social planner is characterized by non-falsification mechanisms.

Starting with high productivity agents, suppose the falsification mechanism

\[
\Gamma_f = (x_H, x_L, y_H, y_L, T_H, T_L),
\]

where \( x_H < y_L \), is a solution to the social planner’s problem, so it satisfies the incentive compatibility constraints (2.6), (2.7), and (2.8).

\[\text{Tuomala (1990), pg. 58.}\]
Define a non-falsification mechanism

\[ \Gamma_{nf} = \left( \hat{x}_H, x_L, y_H, y_L, \hat{T}_H, T_L \right), \]

where \( \hat{x}_H = y_H \), and \( \hat{T}_H = g(y_H, x_H) + T_H \). Having high productivity agents’ income reports equal their income implies

\[ g(y_H, \hat{x}_H) = g(y_H, y_H) = 0. \]

This alternative mechanism still maintains the inequality required by the tax avoidance constraint (equation (2.6)) for high productivity agents, and the labor supply constraint (equation (2.7)) for high and low productivity agents:

\[
\begin{align*}
U(y_H - g(y_H, x_H) - T_H) &\geq U(y_H - g(y_H, x_L) - T_L) \quad (2.6-H) \\
U(y_H - g(y_H, x_H) - T_H) - V \left( \frac{y_H}{\theta_H} \right) &\geq U(y_L - g(y_L, x_L) - T_L) - V \left( \frac{y_L}{\theta_L} \right) \quad (2.7-H) \\
U(y_L - g(y_L, x_L) - T_L) - V \left( \frac{y_L}{\theta_L} \right) &\geq U(y_H - g(y_H, x_H) - T_H) - V \left( \frac{y_H}{\theta_H} \right), \quad (2.7-H)
\end{align*}
\]

since it provides the same level of consumption for high ability agents,

\[
\begin{align*}
\hat{c}_H &= y_H - g(y_H, \hat{x}_H) - \hat{T}_H \\
&= y_H - \hat{T}_H \\
&= y_H - g(y_H, x_H) - T_H \\
&= c_H.
\end{align*}
\]

\( \Gamma_{nf} \) also satisfies equation (2.8) for high ability agents and equation (2.6) for low ability agents:

\[
\begin{align*}
U(y_L - g(y_L, x_L) - T_L) &\geq U(y_L - g(y_L, x_H) - T_H) \quad (2.6-L) \\
U(y_H - g(y_H, x_H) - T_H) - V \left( \frac{y_H}{\theta_H} \right) &\geq U(y_L - g(y_L, x_H) - T_H) - V \left( \frac{y_H}{\theta_H} \right), \quad (2.8-L)
\end{align*}
\]

since consumption of agents who work \( y_L \) but report \( x_H \),

\[ c_{L,H} = y_L - g(y_L, x_H) - T_H, \]

is lower in \( \Gamma_{nf} \) than in \( \Gamma_f \):

39
\[ \hat{c}_{L,H} = y_L - g(y_L, \hat{x}_H) - \hat{T}_H \]
\[ \hat{c}_{L,H} = y_L - g(y_L, y_H) - g(y_H, x_H) - T_H \]
\[ \hat{c}_{L,H} = c_{L,H} + g(y_L, x_H) - g(y_L, y_H) - g(y_H, x_H) - T_H \]
\[ < y_L - g(y_L, x_H) - T_H \]
\[ = c_{L,H} \]

since

\[ g(y_L, y_H) > g(y_L, x_H) \]
\[ \text{and} \]
\[ g(y_H, x_H) > 0 \]

because \( g(\cdot) \) is monotonically increasing in \( x \) and non-negative for any \( x \neq y \). Additionally, \( \Gamma_nf \) meets the double deviation constraint (equation (2.8)) for low ability agents trivially, since it doesn’t involve high productivity agents’ income reports or taxes. Hence, it satisfies all incentive compatibility constraints. Finally, since

\[ T_H < g(y_H, x_H) + T_H = \hat{T}_H \]

because \( g(y_H, x_H) > 0 \), \( \Gamma_nf \) satisfies the feasibility constraint and collects more tax revenue. This contradicts the supposition that \( \Gamma_f \) is optimal. The proof for low ability agents follows by symmetry.

\[ \square \]

**Proof of Corollary 2.2** To simplify the TAC, eliminate the utility of labor \( V(\cdot) \) from both sides of the inequality, and set \( g(y_i, x_i) = g(y_i, y_i) = 0 \) per theorem 2.1

\[ U(y_i - T(y_i)) \geq U(y_i - g(y_i, y_i) - T(y_i)) \]
\[ y_i - T(y_i) \geq y_i - g(y_i, y_i) - T(y_i) \]
\[ -T(y_i) \geq y_i - g(y_i, y_i) - T(y_i) \]
\[ g(y_i, y_i) \geq T(y_i) - T(y_i) \]

\[ \square \]

**Proof of Corollary 2.3** The relationship illustrated in equation (2.8) is contained within the expressions in (2.7) and (2.6).
\[ U(y_i - g(y_i, x_i) - T_i) - V\left(\frac{y_i}{\theta_i}\right) \geq U(y_j - g(y_j, x_j) - T_j) - V\left(\frac{y_j}{\theta_i}\right) \quad (2.7) \]

\[ \geq U(y_j - g(y_j, x_i) - T_i) - V\left(\frac{y_j}{\theta_i}\right) \]

since
\[-g(y_j, x_j) - T_j \geq -g(y_j, x_i) - T_i \quad (2.6)\]

**Proof of Proposition 2.4.** The concavity of the objective function guarantees that \( T_H \geq 0 \geq T_L \). Together with the non-negativity restriction on the cost of avoidance, this condition implies that the report constraint never binds for the low ability agent.

\[ T_l - T_h \leq 0 \leq g(y_t, y_h) \]

**Proof of Theorem 2.5.** Rewrite the incentive compatibility constraints as slack variables representing the net cost of income falsification:

\[ U(y_h - T_h) - V\left(\frac{y_h}{\theta_h}\right) - U(y_h - g(y_h, y_l) - T_l) + V\left(\frac{y_h}{\theta_h}\right) = \chi_{IF} \geq 0 \]

\[ U(y_h - T_h) - V\left(\frac{y_h}{\theta_h}\right) - U(y_l - g(y_l, y_l) - T_l) + V\left(\frac{y_l}{\theta_h}\right) = \chi_{LS} \geq 0 \]

Now, suppose that the mechanism \( \Gamma^* = (y_H^*, y_L^*, T_H^*, T_L^*) \) solves the social planner’s problem when the incentive compatibility constraints defined as above are non-binding or \( \chi_{LS}^* > 0 \), and \( \chi_{IF}^* > 0 \). If this is true, the social planner can increase aggregate welfare by choosing an alternative recommendation \( \hat{\Gamma}^* \) that tightens the incentive compatibility constraints until

\[ \min \{ \hat{\chi}_{IF}^*, \hat{\chi}_{LS}^* \} = 0. \]

This is a contradiction of the optimality of \( \Gamma^* \).