A Tale of Two Countries: 
Sovereign Default, Trade, and Terms of Trade

Grace W. Gu*

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Abstract

This paper explores the interactions of sovereign defaults with income and trade through a terms-of-trade channel, in a DSGE model of two risk-averse open economies with production. It makes contributions in two main aspects. First, the paper models the impact of sovereign defaults on trade, especially bilateral trade flows that are largely absent in theoretical sovereign default literature. Moreover, unlike existing models, this paper endogenously generates nonzero trade balances and trade balance reversal upon a sovereign default. Second, this model differs from others by endogenizing terms of trade and real exchange rate that interact with default risk. It captures a defaulting country’s terms-of-trade deterioration that significantly contributes to its income and trade losses, as in the data. In a quantitative analysis, the model produces countercyclical trade balances, procyclical bilateral trade, countercyclical bond spreads with a data-consistent average, and other empirical features of Mexican business cycles and its sovereign default episodes. The model results also imply that greater trade openness supports a higher debt-to-GDP ratio, and lowers terms of trade volatility, default frequency, and default-triggered welfare losses.

Keywords: sovereign default, terms of trade, real exchange rate, trade, DSGE.
JEL code: F34 - F41 - F44

*Gu: Department of Economics, UC Santa Cruz, Engineering 2 Building, Room 463, Santa Cruz, CA 95060, grace.gu@ucsc.edu. I thank seminar participants at Cornell, UC Davis, UC Riverside, and UC Santa Cruz, for their helpful comments.
1 Introduction

Sovereign default events are associated with three empirical regularities: (a) deep recessions, (b) international goods trade declines with reversed trade balances, and (c) deteriorating terms of trade and real exchange rates. Recent evidence shows that, across countries, default episodes have on average been accompanied by a GDP drop of 5 percent below trend, a bilateral trade value decline of 8 percent, a net export increase of 10 percentage points of GDP, and real depreciation of 30-50 percent.\footnote{See Rose (2005), Cuadra and Sapriza (2006), Reinhart and Rogoff (2011), and Mendoza and Yue (2012).}

The existing quantitative sovereign debt literature, such as Aguiar and Gopinath (2006), Arellano (2008), and Mendoza and Yue (2012), based on Eaton and Gersovitz (1981), has made significant contributions in endogenizing default risk and income. They also account for countercyclical net exports, output losses, and other key empirical patterns of developing countries’ business cycles and default episodes. However, those small open economy models do not address sovereign defaults’ endogenous interactions with 1) bilateral trade flows, and 2) terms of trade and real exchange rates. Using a two-country DSGE model with defaultable bonds, this paper studies how sovereign default risk interacts with income and international goods trade endogenously through a terms-of-trade channel, which is largely left out of current literature.

First, why is it important to theoretically model trade during sovereign defaults? There are at least two reasons. On one hand, as emphasized by the empirical literature, it is not only that trade surpluses improve during default episodes, but also that bilateral trade declines (Rose, 2005). The latter stylized fact has not yet been captured by existing models. This paper, using a two-country model with separate import and export flows, is able to do that. It also helps us understand how a country’s consumer preferences regarding home goods and imports affect its propensity to default (Rose and Spiegel, 2004; Rose, 2005).\footnote{Past empirical research suggests that less outward-oriented sovereigns are more willing to default. Therefore, if a sovereign government internalizes its citizens’ desire for imported goods, we can begin to consider how a country’s reduced desire for foreign goods can spur defaults, or how we can motivate the country to service its debt on time.}

On the other hand, even though previous sovereign default models address the countercyclical trade balances, they generate zero trade balances upon default, which is inconsistent with the data.\footnote{In those models, bond trading is the only capital flow that finances trade balances, once default-triggered financial autarky happens, the trade balances become zero. Mendoza and Yue (2012) is an exception in that their paper produces nonzero trade balances, but through exogenous capital flows upon default interactions, the trade balances remain nonzero even after default.} This paper’s trade balances, however, result from capital flows of...
both bond trading and global vertical integration (e.g., vertical FDI, offshoring, and other global sourcing). This feature allows the trade balances to be nonzero (and turn positive) endogenously during default episodes, even though countries are no longer trading bonds.

Second, endogenizing terms of trade and real exchange rate is crucial to sovereign default literature. The logic lies twofold. The foremost reason is that the well-known deterioration of terms of trade and real exchange rate upon default has rarely been captured by existing sovereign default models. However, the deterioration contributes significantly to a defaulting country’s income and trade losses. For instance, Figure 1 shows that upon default (year 0), on average GDP volume growth rate declined a lot less than GDP value, if at all. The majority of the defaulting countries’ losses of output value growth came from real depreciation. Similar patterns exist for export value and volume. On the contrary, about half of the decline in import value can be attributed to declines in volume, and the other half to the real depreciation. It is clear that the deterioration of terms of trade and real exchange rate plays an important role in affecting a defaulting country’s GDP, import, and export values, thus should be captured by a sovereign default model.

Figure 1: Growth around Sovereign Default Episodes (in percent)

![Graphs showing growth around sovereign default episodes](image)

Note: The statistics are based on 45 sovereign default episodes of 27 developing countries over 1977-2009. Due to data limitation, the sample period and/or the number of default episodes vary slightly for some variables. Raw data sources are detailed in the Appendix.

Therefore, endogenizing terms of trade and real exchange rate in my model results in an endogenous default penalty on income and trade. That is to say, this model does not default.
have to rely on an exogenous output loss as in many previous sovereign default models. In this respect, this paper is similar to Mendoza and Yue (2012), where it endogenizes the output loss by a production efficiency loss due to a default-triggered decline of trade credit for imported inputs. However, it is unclear how consistent their paper’s terms of trade and real exchange rate movements are with the data. In this paper, terms of trade are instead explicitly modeled as the ratio of two countries’ final goods prices and influenced by their trade flows according to consumer preferences. Meanwhile, this paper’s real exchange rate is calculated as the ratio of domestic and foreign countries’ aggregate price indices (weighted sum of prices of domestic and imported final goods).

The other reason for endogenizing terms of trade and real exchange rate is that it also allows their endogenous two-way interaction with default risk prior to sovereign default occurrences. I elaborate the mechanism when describing the model below.

This paper contributes to existing literature by filling the aforementioned gaps, including (1) modeling trade flows during default episodes, (2) generating endogenous nonzero trade balances and trade balance reversal upon default, (3) endogenizing terms of trade and real exchange rate that interact with default risk, and (4) capturing a defaulting country’s deterioration of terms of trade and real exchange rate that significantly contributes to its income and trade losses.

The model features endogenous default risk, endogenous income and trade, as well as endogenous terms of trade and real exchange rate. In the model, as the borrower country’s debt accumulates and default risk increases, I show that its terms of trade deteriorate along with its consumption adjusting towards home goods and away from imports according to home bias preference. The deterioration of terms of trade prevents the borrower country from real appreciation to improve its budget and from easing its debt burden that is denominated in the creditor country’s final goods. Hence, the default risk further rises. This is how the default risk interacts with terms of trade and real exchange rate prior to a sovereign default.

In the model, once the borrower country defaults due to a large enough adverse productivity shock, the event affects foreign firms’ activities with the defaulting country (e.g., FDI, offshoring, and other global sourcing) more than it affects the defaulting country’s domestic firms, as evidenced by many empirical studies (Brennan and Cao, 1997; Tille and van Wincoop, 2008; Milesi-Ferretti and Tille, 2010; and Broner, Didier, Erce, and Schmukler, 2013). In particular, this model specifies that a sovereign default triggers

4In Mendoza and Yue (2012), its price index for imported inputs declines during default episodes, since parts of the imports are no longer used. This implies better terms of trade upon default.
5These papers have provided empirical evidence for asymmetric crisis impacts on domestic and foreign
an efficiency loss of foreign-firm-related intermediate goods production in the defaulting country, which results in a decline of intermediate goods exports from the defaulting country to the creditor country. This mechanism is consistent with the data, where the impact of defaults on intermediate goods export growth distinguishes itself from that on final goods export growth through a more severe decline. The detailed data are provided in the next section of this paper.

This decline in intermediate goods exports upon default triggers an income loss additional to that from the initial adverse productivity shock in the defaulting country. As the overall income loss overpowers what is gained from the defaulted debt repayment, I show that the defaulting country’s imports further decline and trade balances improve. Meanwhile, its terms of trade and real exchange rate deteriorate, which takes a third toll on the defaulting country’s income. Hence, it builds into the model an endogenous terms-of-trade mechanism by which a sovereign default amplifies the effects of adverse productivity shocks on income and trade.

In a quantitative exercise, I apply the model to study the Mexican debt crises in the 1980s and the country’s business cycles for the period of 1981Q1-2012Q4. This paper generates three important stylized empirical features of emerging markets’ business cycles and their sovereign default episodes. First, it delivers countercyclical trade balances and procyclical bilateral trade flows over business cycles. Second, the model supports high bond spreads that are also countercyclical. Third, the model accounts for terms of trade deterioration, real depreciation, trade balance reversal, and bilateral trade flow and GDP declines during and right after a sovereign default.

Then, to further study the role of terms of trade, this paper examines the model results where trade and GDP losses are partially due to terms of trade deterioration and partially due to volume changes, as in the data. I also evaluate the welfare of creditor country that is largely left out by existing sovereign default models. I find that the creditor country’s welfare experiences long-lasting losses from an income decline triggered by a default event, even though the losses are relieved by favorable terms of trade. Moreover, the model can predict the time series of Mexican output and bond spreads in the sample period, as well agents due to crisis-elevated information asymmetry and risk aversion. Moreover, Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent. Aizenman and Marion (2004) also documents that greater supply uncertainty reduces the expected income from vertical FDI.

It is worth emphasizing that, like in previous sovereign default models, this paper’s default also arises in equilibrium as an optimal decision of a benevolent government who takes into account all the positive and adverse consequences.

I choose Mexico for this two-country model because Mexico has a relatively large open economy among the countries that recently defaulted, as well as relatively large vertically integrated sectors involved with foreign production, including its maquiladora sector (Zlate, 2012).
as its crises around 1995. Lastly, this paper conducts sensitivity analyses on productivity shock process, default penalty, consumer patience and preferences, and investigates their implications to default incentives and countries’ post-default welfare.

In explaining the defaulting country’s trade balance reversal and deteriorating real exchange rate and terms of trade, this model is related to papers in the international business cycle literature, such as Backus, Kehoe, and Kydland (1992, 1994), Mendoza (1995), Stockman and Tesar (1995), Heathcote and Perri (2002), Kehoe and Perri (2002), Kose (2002), Broda (2004), Iacoviello and Minetti (2006), Bodenstein (2008), and Raffo (2008). These papers have addressed many important international business cycle features. Some have assumed a complete market for financial assets, while others have incorporated enforcement or borrowing constraints with the result that actual defaults are ruled out at equilibrium. Using a non-state-contingent bond, my model endogenously generates default in equilibrium. Hence, this paper is closely associated with previous small-open-economy sovereign default models. As explained earlier, however, those models do not focus on default-triggered changes to trade and terms of trade.

Another related strand of literature also focuses on the connection between international trade and sovereign defaults, but empirically. For instance, Rose (2005) documents that a default can reduce real bilateral trade value (in USD) by 8 percent for an extended period after the event. However, it remains unclear why trade declines. The four hypotheses of trade sanctions, trade credit collapse, asset seizures, and reputation are commonly mentioned, but their empirical evidence remains ambiguous (Martinez and Sandleris, 2011; Tomz and Wright, 2013). This paper uses a theoretical model to examine the interaction between trade and sovereign defaults through a terms-of-trade channel.

A few recent sovereign default papers (Cuadra and Sapriza, 2006; and Bleaney, 2008; Popov and Wiczer, 2014) have examined the roles of exogenous terms-of-trade shocks and exogenous terms-of-trade default penalty in small open economy models. The inclusion of endogenous terms of trade and real exchange rate distinguishes this paper from them. Na, Schomitt-Grohe, Uribe, and Yue (2014) also includes endogenous exchange rate but in nominal terms, so that they can focus on optimal exchange rate policy. Like this paper, their model achieves concurrent default and depreciation. However, their nominal depreciation is driven by wage rigidity and government’s intention to reduce unemployment, whereas this paper’s real depreciation is associated with changes to capital and trade flows. Most recently, another working paper (Asonuma, 2014) has also endogenized real exchange rate in a two-country sovereign default model, but through a different mecha-
anism in endowment economies. This paper uses production economies to incorporate richer business cycle fluctuations.

The remainder of this paper is organized as follows. Section 2 describes the model environment, equilibrium, and properties. Section 3 provides the model calibration and quantitative results. Section 4 offers concluding remarks.

2 Model

2.1 Environment

In this section, I study sovereign default, international goods trade, terms of trade, and the real exchange rate in a dynamic model of two risk-averse open economies. In the model, these two countries \((i = 1, 2)\) trade one-period discount bond, produce two unique final goods \((j = 1, 2)\), respectively, and consume both through trade. The two final goods are imperfect substitutes with constant elasticity, and \(c_{ij}\) stands for country \(i\)’s consumption of final goods \(j\).

In final goods market, \(p_j\) stands for final goods \(j\)’s price, and I normalize country 1’s final goods price \(p_1 = 1\). Therefore, \(p_2\) is terms of trade for country 2. When \(p_2\) declines, it means the deterioration of terms of trade for country 2. I assume that the nominal exchange rate between the two countries is 1, and thus the real exchange rate is the ratio of country 2’s over country 1’s aggregate price index (weighted sum of domestic goods price and import price). When it decreases, it means country 2 experiences real depreciation.

Without loss of generality, I set country 1 to be the creditor who never defaults and has constant productivity \(e_1\), whereas country 2 is set to be the borrower who has an option to default on its sovereign bonds and faces stochastic productivity \(e_2\) that follows Markov chain. One way to interpret the creditor country having constant productivity is that it always can smooth its production through other financial channels that are not in this model, regardless of the situation in the bond market with the borrower country. Moreover, since the creditor country never defaults, it is not of this paper’s interest to complicate the model results by including its productivity shocks.

Creditor country 1’s firms have a fixed amount of capital, \(\bar{k}_1\). The firms can pair the capital either with a fixed amount of domestic labor \(\bar{n}_1\) to produce final goods 1, or with

\[\text{8Asonuma (2014) uses traded and non-traded goods in achieving real depreciation in his model.}\]

\[\text{9Having said that, it is still of future research interest to study spillover effects when a creditor country’s productivity shocks may trigger a borrower country’s sovereign default.}\]
imported intermediate goods produced by borrower country 2’s labor to produce the same final goods 1. I use $k_1$ to denote capital used with domestic labor, $k_1^*$ to denote that used with imported intermediate inputs, and $k_1 + k_1^* = \bar{k}_1$. Borrower country 2 has a fixed amount of labor, $\bar{n}_2$. It is divided into $n_2^*$ who produce intermediate inputs for creditor country 1, and $n_2 = \bar{n}_2 - n_2^*$ who produce final goods 2 with a fixed amount of domestic capital $\bar{k}_2$.

Three reasons stand out for this asymmetric model setup, where creditor country 1 allocates capital and borrower country 2 allocates labor and produces intermediate goods for exporting. First, many countries that defaulted in the past are developing or emerging economies, where labor is abundant and used for producing intermediate goods for exporting, through foreign firms’ activities such as vertical FDI, offshoring, and other global sourcing. Second, even though creditor country 1 does not produces intermediate goods, its domestic labor input is an imperfect substitute to intermediate goods from borrower country 2 and can be considered as creditor country 1’s own implicit intermediate inputs. Third, the impact of a sovereign default on borrower country 2’s intermediate goods exports serves as one of the default penalties in the model, as detailed below. Having both countries exporting intermediate goods complicates the model and makes it difficult to single out the impact of borrower country 2’s intermediate goods exports upon default as a penalty\(^{10}\). The current setup makes the model tractable, yet still has a sense of reality.

It is also worth noting that borrower country 2 produces intermediate goods only for exporting, not for domestic use. The key purpose to single out those intermediate goods for exporting is to distinguish the foreign-firm related activities from the domestic activities in the borrower country. It is because the two types of activities are impacted differently by sovereign defaults in both data and the model, as shown below. Meanwhile, we can consider those intermediate goods for domestic use embedded in the value of borrower country 2’s final goods 2.

In the bond market, a non-state-contingent one-period bond denominated in the creditor country’s final goods 1 is traded between the two countries. The bond is denoted as $b_i$ for country $i$’s asset holding. The bond contracts reflect the borrower country’s default probabilities that are endogenous to its debt holding and its fundamental. Hence, the equilibrium interest rate is linked to the borrower country’s default risk. Default can be triggered by negative productivity shocks and can happen along the equilibrium. The risk-averse creditors in country 1 are willing to offer debt contracts that in some states

\(^{10}\)However, it may be interesting to include more channels of global integration in the future research.
may result in default by charging a higher interest rate. Hence, the equilibrium interest rate is associated with the default risk, as well as the creditor’s consumption changes and risk aversion.

Here is how default risk is linked with trade and terms of trade prior to a default. As the borrower country accumulates debt, its default risk and equilibrium bond interest rate rise. Higher cost of debt reduces the borrower country’s available funds to smooth consumption, and thus due to consumption home bias, it will consume fewer imports. Meanwhile, there is deterioration pressure on its terms of trade and real exchange rate, preventing the borrower country from real appreciation to improve its budget constraint and from easing its debt burden. This, in turn, increases the borrower country’s default risk.

Once borrower country 2 does default due to an adverse productivity shock, the event affects foreign firms’ activities with the defaulting country (e.g., FDI, offshoring, and other global sourcing) more than it affects the defaulting country 2’s domestic firms.\textsuperscript{11} In particular, the model specifies that the default triggers an efficiency loss of foreign-firm-related intermediate goods production in the defaulting country, which results in a decline of intermediate goods exports from defaulting country 2 to creditor country 1. Here, I use the term “foreign-firm-related” because the intermediate goods production is only for exporting in the model.\textsuperscript{12}

This story is consistent with the data. Table 1 shows the average annual growth rates of exports of 14 countries for the sample period of 1989-2013, as well as for their 16 default episodes during that period (see Table 6 in the Appendix for detailed data availability). On average, final and intermediate goods exports grow at similar rates during the sample period, but intermediate goods exports experience a much larger decline in growth than final goods exports do during default episodes.

Back to the model, this decline in intermediate goods exports upon default harms creditor country 1’s production of those final goods 1 that involve such imported inputs from defaulting country 2. Therefore, the creditor country’s firms reallocate capital away from combining with the imported intermediate goods, but more towards its domestic

\textsuperscript{11}Brennan and Cao (1997), Tille and van Wincoop (2008), Milesi-Ferretti and Tille (2010), and Broner, Didier, Erce, and Schmukler (2013) have provided evidence for asymmetric crisis impacts on domestic and foreign agents due to crisis-elevated information asymmetry and risk aversion. Moreover, Aizenman and Marion (2004) also documents that greater supply uncertainty reduces the expected income from vertical FDI. Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent.

\textsuperscript{12}This is not the only story that this model setup can support. Other variations of global vertical integration being affected by sovereign defaults that can be studied using this model setup are discussed in the last section of this paper.
Table 1: Average Growth Rate of Exports During a Default (in percent)

<table>
<thead>
<tr>
<th>Variable</th>
<th>1989-2013</th>
<th>During Default</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intermediate Goods Exports</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>7.51</td>
<td>−2.22</td>
</tr>
<tr>
<td>Volume</td>
<td>4.92</td>
<td>0.57</td>
</tr>
<tr>
<td>Share of GDP</td>
<td>3.60</td>
<td>−1.87</td>
</tr>
<tr>
<td><strong>Final Goods Exports</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>7.65</td>
<td>−1.39</td>
</tr>
<tr>
<td>Volume</td>
<td>5.96</td>
<td>5.39</td>
</tr>
<tr>
<td>Share of GDP</td>
<td>3.74</td>
<td>−1.02</td>
</tr>
</tbody>
</table>

Note: I collected annual growth data of final goods and intermediate goods export value (in USD), volume (as value in local currency), and as a ratio to GDP for 16 sovereign default episodes (14 countries) over the period of 1989-2013. *Here, due to data limitation, the volume is of total exports.

labor to produce final goods 1. Since there is no unemployment in this model, some workers in defaulting country 2 will then shift from the intermediate goods sector to domestic production of final goods 2. This labor shifting enables the country to produce and export more of its own final goods 2 given the initial adverse productivity shocks, than in a case without such labor shifting.

Importantly, the per-worker wage paid in the defaulting country 2 also declines due to the initial adverse productivity shock and the oversupply of labor to the domestic production sector. This reduced labor income serves as part of the defaulting country’s default costs. In addition, when the borrower country 2 defaults on its sovereign bonds, both countries face financial autarky and only with a certain probability can they resume bond trading again.\(^\text{13}\) There is no other direct penalty, such as exogenous output loss or trade sanctions.\(^\text{14}\)

The overall income loss after default overpowers what is gained from the defaulted debt repayment. As the defaulting country’s budget constraint tightens, its consumption portfolio is once again adjusted away from imports, and its final goods exports rise. Consequently, its trade balances become positive, while terms of trade and the real exchange rate deteriorate sharply, which in turn induces more income losses in the defaulting country. The following sections describe the model specifications.

\(^\text{13}\)The creditor country has no productivity shock, hence it is not lacking the bond market with the defaulting country that affects its production.

\(^\text{14}\)Trade sanctions after default lack of empirical support in the literature (Martinez and Sandleris, 2011; Tomz and Wright, 2013).
2.2 Country 1: Creditor

In the model, there are two types of agents in creditor country 1: representative firms, and households.

2.2.1 Firms

Creditor country 1’s firms choose capital allocation $k_1, k_1^*$, and how many foreign workers to hire in the intermediate goods sector in borrower country 2, $n_2^*$. Here, I combine the decisions of imported intermediate inputs and final goods 1 production to reflect the practice of vertical integration. This arrangement is similar to those used in global sourcing literature where firms have a choice to internalize its input supply (Antras and Helpman, 2004). Firms’ goal is to maximize their income after deducting the cost of intermediate goods paid to the borrower country 2 (i.e., wage payment $p_2 w_2^* n_2^*$, where the real wage is denominated in final goods 2), taking $w_2^*$ and $p_2$ as given:

$$\Pi_1 = \max_{n_2^*, k_1^*} \left\{ e_1 (\varepsilon_2 n_2^*)^{a_1} k_1^{1-a_1} - p_2 w_2^* n_2^* + e_1 \bar{n}_1 \bar{k}_1^{1-a_1} \right\}$$

where $k_1 + k_1^* = \bar{k}_1$. The first two terms are the income the firms gain from using intermediate inputs $\varepsilon_2 n_2^*$ (produced in borrower country 2 using only labor $n_2^*$) to produce final goods 1, after deducting intermediate goods costs (or wage cost). The last term is the total income the firms gain from using domestic labor $\bar{n}_1$ to produce final goods 1. Notice that the production of final goods 1 using intermediate goods from borrower country 2 uses the same technology as it uses to produce the same final goods 1 with domestic labor.

The intermediate goods production is a linear function of borrower country 2’s workers in this sector, $n_2^*$, and is associated with its productivity $e_2$. Importantly, $\varepsilon$ symbolizes the efficiency of intermediate goods production for exporting by borrower country 2. During non-default time, $\varepsilon = 1$, meaning there is no efficiency loss compared with the borrower country’s domestic goods sector. During default periods, $\varepsilon = \min(\frac{\bar{e}_2}{e_2}, 1)$, where $0 < \varepsilon < 1$ and $\bar{e}_2$ is borrower country 2’s average productivity. It ensures that during default episodes intermediate goods production has lower than average productivity. It also generates inefficiency losses increasing with the defaulting country 2’s productivity state, such that all else equal the country has a larger incentive to default at a lower productivity state. Importantly, here $\varepsilon$ reflects asymmetric crisis impacts on defaulting country 2’s domestic and foreign-related activities, as evidenced by many empirical studies (Brennan and Cao, 1997; Aizenman and Marion, 2004; Tille and van Wincoop, 2008; Milesi-Ferretti and Tille, 2010; and Broner, Didier, Erce, and Schmukler, 2013).
The default-triggered efficiency loss in country 2’s intermediate goods production decreases the number of workers that firms in creditor country 1 hire, \( n^*_2 \). Meanwhile, the firms shift capital to the domestic labor sector, by reducing \( k^*_1 \) and increasing \( k_1 \). These decrease the marginal product of capital in creditor country 1 and the overall production of final goods 1.

### 2.2.2 Households

Households in creditor country 1 work at and own the firms. They use the proceeds for consumption to maximize a standard time-separable utility function \( E[\sum_{t=0}^{\infty} \beta^t U(c_{11t}, c_{12t})] \), where \( 0 < \beta_1 < 1 \) is the discount factor and \( U(\cdot) \) is a one-period utility function which is continuous, strictly increasing and concave, and which satisfies the Inada conditions. More specifically, based on Krugman (1980), I use an additive separable utility function

\[
U(c_{11t}, c_{12t}) = \rho_1 c_{11t}^{\theta_1} + (1 - \rho_1) c_{12t}^{\theta_1},
\]

where \( 0 < \rho_1, \theta_1 < 1 \). The degree of home bias increases with \( \rho_1 \), and the elasticity of substitution is constant at \( \frac{1}{1-\theta_1} \). This utility function assumes independence between domestic final goods and imported final goods in marginal utility, and brings tractability and computability to this model.

Households also choose how many of the one-period non-state-contingent bonds issued by borrower country 2 to purchase, given the bond pricing function \( q(b^1_1, s) \). Here, \( b^1_1 \) is the creditor country 1 tomorrow’s bond asset holding, and \( s \) is the aggregate state of the two economies. Hence, their expected lifetime utility also depends on borrower country 2’s default decisions. When the borrower country does not default in the current period, the creditor country’s optimization problem can be written recursively as:

\[
V_{1c}(s, b_1) = \max_{b^1_1, c_{11}, c_{12}} \left\{ U(c_{11}, c_{12}) + \beta_1 \left[ \int_{s' \in D(b^1_2)} V_{1c}(s', b^1_1) dF(s'|s) + \int_{s' \in D(b^1_2)} V_{1d}(s') dF(s'|s) \right] \right\}
\]

(2)

where \( D \) is the default set for the borrower country that I will explain in the next section. The problem is subject to:

\[
\Pi_1 + b_1 = c_{11} + p_2 c_{12} + q b^1_1.
\]

(3)

where \( q(s', b^1_1) = \beta_1 \frac{\int_{s' \in D(s', b^1_2)} \partial V_{1d}/\partial b^1_1 dF(s'|s)}{\lambda_1} \), and \( \lambda_1 \) is the multiplier of the budget constraint.

When the borrower country defaults, both countries have to undergo financial autarky
for a certain period of time\textsuperscript{[15]} The creditor country’s constrained maximization problem becomes:

\[ V_{1d}(s) = \max_{c_{11}, c_{12}} \{ U(c_{11}, c_{12}) + \beta_1 E[reV_{1x}(s', 0) + (1 - re)V_{1d}(s')] \} \]  \hspace{1cm} (4)

where \( V_{1x} = [V_{1d}(s, b_1) \text{ or } V_{1c}(s)] \) [borrower country 2 defaults or not], and \( re \) is the probability of both countries’ return to the bond market. The problem is subject to

\[ \Pi_1 = c_{11} + p_2 c_{12}. \]  \hspace{1cm} (5)

The main costs to the creditor when the borrower defaults are the missed debt repayment, the production inefficiency with imported intermediate goods, and the capital reallocation. These tighten the creditor country’s budget constraint. However, the creditor also gains from more favorable terms of trade and real appreciation to allow more imports.

Given the above setup, I calculate creditor country 1’s GDP as the gross production of final goods 1 minus the cost of imported intermediate goods, i.e., \( \Pi_1 \). Notice that its GDP value and volume are the same in the model, because its goods price \( p_1 = 1 \).

### 2.3 Country 2: Borrower or Defaulter

There are three types of agents in country 2: representative firms, households, and a government.

#### 2.3.1 Firms

Country 2’s firms passively hire all the remaining workers that are not hired by the intermediate goods sector, \( n_2 \). They maximize their profits according to the following first order condition:

\[ e_2 \alpha_2 n_2^{\alpha_2 - 1} k_2^{1-\alpha_2} = w_2 \]  \hspace{1cm} (6)

where \( w_2 \) is domestic sector wage. When the borrower country defaults due to an adverse productivity shock, intermediate goods export sector suffers from an efficiency loss, and creditor country 1 hires fewer workers in defaulting country 2. Hence, labor shifts from

\textsuperscript{[15]}Some may argue that it is not realistic to also exclude the creditor from the international financial market. But since the creditor country has no productivity shock, its loss from the bond market exclusion is minimal.
the intermediate goods sector to the domestic production sector, which has two important
effects. First, the oversupply of labor to the latter sector reduces \( w_2 \). Since in equilibrium
the two sectors has equal wages, \( w_2 = w_2^* \), the overall wage income in the defaulting coun-
try decreases additional to the income loss from the initial adverse productivity shock.
Second, the influx of labor to the domestic production sector supports final goods 2’s
production and exports, which contributes to the defaulting country’s trade balance re-
versal. Yet, it is worth noting that this model does not have unemployment. It would be
interesting to include more labor market dynamics in the future.

2.3.2 Households
Households in borrower country 2 derive income from two sources: their wage from pro-
ducing intermediate goods for creditor country 1, and all the revenue of domestic firms
that are owned by the households. They also choose a consumption bundle to maximize a
standard time-separable utility function \( E[\sum_{t=0}^{\infty} \beta_2^t U(c_{21t}, c_{22t})] \), where \( 0 < \beta_2 < 1 \) is the
discount factor. Similar to that of creditor country 1’s consumers, the one-period utility
function is specified as \( U(c_{21t}, c_{22t}) = (1 - \rho_2)c_{21t}^{\theta_2} + \rho_2 c_{22t}^{\theta_2} \), where \( 0 < \rho_2, \theta_2 < 1 \). Again,
the degree of home bias increases with \( \rho_2 \), and the elasticity of substitution is constant at
\( \frac{1}{1-\theta_2} \).

As in Mendoza and Yue (2012), households do not borrow directly from abroad, but
the government borrows, pays transfers to the households, and makes default decisions
internalizing its citizens’ utility. Therefore, taking as given the wage from the intermediate
goods sector, \( p_2 w_2^* n_2^* \), the revenue from domestic firms, \( p_2 e_2 n_2^2 k_2^{1-\alpha_2} \), and government
transfers, \( T \), the households solve the following optimization problem:

\[
\max_{c_{21t},c_{22t}} E[\sum_{t=0}^{\infty} \beta_2^t U(c_{21t}, c_{22t})] \tag{7}
\]

subject to

\[
p_2 e_2 n_2^2 k_2^{1-\alpha_2} + p_2 w_2^* n_2^* + T_t = c_{21t} + p_2 e_2 c_{22t} \tag{8}
\]

where \( n_{2t} + n_{2t}^* = \bar{n}_2 \).

2.3.3 Government
Country 2’s sovereign government issues one-period non-state-contingent discount bonds,
so the asset market is incomplete. It cannot commit to repaying its debt. It compares
the value of repaying debt \( V_{2c} \) and that of default \( V_{2d} \), and chooses the option that gives
a bigger value, that is:

\[ V_{2c}(s, b_2) = \max \{ V_{2c}(s, b_2), V_{2d}(s) \} \]  

(9)

The nondefault value is given by the choice of \((b'_2, c_{21}, c_{22})\) that maximizes the following problem, where the benevolent government internalizes its citizens’ preferences for domestic final goods 2 and imported final goods 1:

\[ V_{2c}(s, b_2) = \max_{b'_2, c_{21}, c_{22}} \{ U(c_{21}, c_{22}) + \beta_2 EV_{2x}(s', b'_2) \} \]  

(10)

subject to

\[ p_2 c_{2} n_2^\alpha \bar{k}_2^{1-\alpha} + p_2 w_2^* n_2^* + b_2 = c_{21} + p_2 c_{22} + q b'_2. \]  

(11)

where \( n_2 + n_2^* = \bar{n}_2 \) and \( q(s', b'_2) = \beta_2 \int_{s' \in D(s', b'_2)} \frac{\partial V'_2(s', b'_2)}{\partial b'_2} dF(s'|s) \), and \( \lambda_2 \) is the multiplier of borrower country 2’s budget constraint.

The definitions of the default set \( D \) and the probability of default are standard from Eaton-Gersovitz type models (also see Arellano, 2008). Default set \( D \) at each current debt level \( b_2 \) is a collection of exogenous states when borrower country 2’s government strategically chooses to default to maximize its value:

\[ D(s, b_2) = \{ s \in S : V_{2c}(s, b_2) < V_{2d}(s) \} \]  

(12)

Because no one is certain about the aggregate state tomorrow, the default probability \( \pi_2 \) is the sum of all the probabilities of tomorrow’s states where the borrower country will choose to default, given the current debt level. This default probability exists even if the borrower country itself does not consider the default risk when issuing bonds:

\[ \pi_2(s, b'_2) = \int_{s' \in D(s', b'_2)} f(s, s') ds' \]  

(13)

In the event of a default due to an adverse productivity shock to the borrower country, both countries’ bond assets are set to zero. The defaulting country gains by not having to repay the debt. However, it also suffers from several costs as follows. First, both countries are temporarily excluded from the bond market, and the defaulting country can not smooth its consumption by borrowing any more. Second, the production of intermediate goods in the borrower country suffers from an efficiency loss as explained earlier. It causes workers to overflow into the domestic goods sector, lowering the wage
in the defaulting country.

Overall, due to the initial adverse productivity shock and the additional wage reduction from intermediate goods sector inefficiency loss and labor shifting, the defaulting country’s income declines even though it does not repay the debt. Hence, its consumers have to adjust their consumption portfolio according to home bias preference to import fewer final goods, while the labor shift supports more exports of final goods. The defaulting country’s trade balances improve. Meanwhile, its terms of trade and real exchange rate deteriorate, further lowering the income, which is the third cost to the defaulting country in the model. Taking into account all these consequences, the borrower country’s default value is as follows:

$$V_{2d}(s) = \max_{c_{21}, c_{22}} \{ U(c_{21}, c_{22}) + \beta_2 E[reV_{2x}(s', 0) + (1 - re)V_{2d}(s')] \}$$

(14)

subject to

$$p_2e_{2n_2}^{\alpha_2} k_2^{1-\alpha_2} + p_2 w_2^* n_2^* = c_{21} + p_2 c_{22}$$

(15)

where $n_2 + n_2^* = \bar{n}_2$. Therefore, borrower country 2’s sovereign default decision is an optimal one after considering all the positive and negative consequences.

Given the above setup, I calculate the borrower country 2’s GDP value as the gross production of final goods plus the intermediate goods exports, $p_2e_{2n_2}^{\alpha_2} k_2^{1-\alpha_2} + p_2 w_2^* n_2^*$, and its GDP volume as $e_2 n_2^{\alpha_2} k_2^{1-\alpha_2} + w_2^* n_2^*$.

### 2.4 Equilibrium

Finally, in equilibrium all goods and bond markets clear for both countries in default and nondefault regimes. Also, in the borrower country, the intermediate goods sector per-worker wage equals to the wage paid in its domestic production sector, so that there is no labor flowing between the two sectors. Therefore, when the borrower country defaults, apart from the initial adverse productivity shock, the additional efficiency loss in intermediate goods sector and the labor shifting to the domestic goods sector reduce the defaulting country’s wage in both sectors. The equilibrium conditions are formulated and defined as follows:

$$e_1(s) n_1^{\alpha_1} (k_1 - k_1^*)^{1-\alpha_1} + e_1(e_2 n_2^*)^{\alpha_1} k_1^{\alpha_1} = c_{11} + c_{21}$$

(16)

$$e_2(s) (\bar{n}_2 - n_2^*)^{\alpha_2} k_2^{1-\alpha_2} = c_{12} + c_{22}$$

(17)
\[ b_1'(s, b_1) + b_2'(s, b_2) = 0 \quad \text{in nondefault regime,} \quad (18) \]
\[ \text{or} \quad b_1'(s, b_1 = 0) = 0 \quad \& \quad b_2'(s, b_2 = 0) = 0 \quad \text{in default regime} \quad (19) \]

\[ w^*_2 = w_2. \quad (20) \]

**Definition 1.** A recursive competitive equilibrium is defined as a set of functions for (a) creditor country 1’s capital allocation and hiring decisions; (b) both countries’ households’ consumption policy \( c \) and saving policy \( b' \); (c) welfare value \( V \) at default and repayment; and (d) the law of motion for the aggregate state \( s \), such that: (i) the borrowing and lending policies satisfy the problem’s first order conditions; (ii) the two countries’ value functions satisfy Bellman Equations; (iii) \( p_2 \) and \( q \) clear the goods and bond markets; (iv) \( w^*_2 \) and \( w_2 \) stabilize labor flows between the two sectors in borrower country 2; and (v) the law of motion is consistent with the stochastic processes of \( e_2 \).

Now I illustrate how bond prices are determined in this model. Figure 2 plots bond price \( q \) against creditor country 1’s asset level tomorrow \( b'_1 \) (i.e., borrower country 2’s borrowing tomorrow) for a given productivity state \( s \) and current asset level of the creditor country’s \( b_1 \) (i.e., the borrower country’s current borrowing). Let’s backpedal to a simpler case where there is no default risk. In such a case, the bond price is determined by the following equation:

\[ q(b, b', s) = \beta_1 \frac{\partial U}{\partial c_{11}} \lambda_1 = \beta_2 \frac{\partial U}{\partial c_{21}} \lambda_2 \quad (21) \]

where \( \lambda_1 = \frac{\partial U}{\partial c_{11}} \) and \( \lambda_2 = \frac{\partial U}{\partial c_{21}} \). The first equation is creditor country 1’s bond demand function, and the second equation is borrower country 2’s bond supply function. As shown in the left panel of Figure 2 in the case of no default risk, the bond demand curve and bond supply curve (dash lines) are close to linear and intersect at point \( E_1 \). Point \( E_1 \) pins down the market equilibrium bond price and tomorrow’s bond quantity.

If the current bond holding \( b_1 \) is at a higher level, as in the right panel of Figure 2, then creditor country 1’s bond demand curve will shift up to the thicker dash line because of a lower current marginal utility of domestic consumption (i.e., \( \lambda_1 \)), according to Equation 21. Meanwhile, borrower country 2’s bond supply curve will shift down because of a higher current marginal utility of imported consumption (i.e., \( \lambda_2 \)). This results in a new intersect point \( E'_1 \), which provides a larger equilibrium bond quantity and a slightly lower price, depending on the two countries’ risk aversion.

\[ ^{16}\text{Even in this risk-free bond case, the bond supply and demand curves are not exactly linear, because} \]
\[ \text{of the agents’ risk aversion.} \]
Figure 2: Bond Price (given aggregate productivity state $s$)

Note: The x-axis in the above plots is $b'_1$. As $b'_1$ is positive (right hand side) and becomes larger, borrower country 2 accumulates more and more debt.

Now let’s consider default risk. In this case, bond supply and demand curves take into account the borrower and the creditor’s perspectives on default probability, respectively, as in the following equations:

$$q(b, b', s) = \beta_1 \int_{s' \notin D(s', b'^0)} \frac{\partial U}{\partial c_{11}} dF(s'|s) = \beta_2 \int_{s' \notin D(s', b'^0)} \frac{\partial U}{\partial c_{21}} dF(s'|s)$$  \hspace{1cm} (22)

The first equation represents the bond demand curve of creditor country 1, and the second equation is the bond supply curve of borrower country 2. As default risk increases, both the demand and supply curves imply lower bond prices, since both countries take into account a larger default set $D$. In particular, for borrower country 2, the intuition is that taking default risk as given the government knows it has to lower bond price in order to issue more bonds. But, as proven by Arellano (2008), there is a lower bound for bond price $q$, up to which a borrower country is willing to take on debt. That is to say, for any bond price $q$ below the threshold, a borrower country is able to find a higher bond price $q$ and a lower liability $b'_1$ to finance the same amount of consumption (of final goods 1).

As shown by the solid lines in the left panel of Figure 2, given current $b_1$, both curves bend downward as tomorrow’s $b'_1$ (i.e., country 2 tomorrow’s borrowing) becomes larger, reflecting a higher default risk; and borrower country 2’s bond supply curve terminates at the lower bound for $q$ after bending downward. This results in a different equilibrium.

\hspace{1cm} 17\text{At which levels of } b'_1 \text{ the supply and demand curves will bend down, however, may differ depending on which country is more observant and/or more sensitive about the default risk. In Figure 2 I draw}
point from the no-default-risk case, at \( E_2 \): both the equilibrium bond quantity and price are lower than those at \( E_1 \).

Again, if the current bond holding is at a higher level, as by the solid lines in the right panel of Figure 2 then creditor country 1’s bond demand curve will shift upward and borrower country 2’s bond supply curve will shift downward. The new intersect point \( E_2’ \), again, provides a larger equilibrium bond quantity and a lower price. However, due to the default risk, the increase in the quantity is much smaller and the decrease in bond price is much larger than the no-default-risk case.

Computationally, the inclusion of default risks in both bond supply and demand functions poses a difficult challenge in solving this problem numerically. This is because if both curves bend downward at similar levels of \( b_1’ \), they may create multiple equilibria. Therefore, in practice I solve the model with creditor country 1’s expectation about borrower country 2’s default probability, but assume that borrower country 2 itself is not concerned about its own default risk. Hence, creditor country 1’s bond demand curve is the solid bending curve, while borrower country 2’s bond supply curve is the dash line in Figure 2.

**Theorem 1** Given a productivity shock to \( e_2 \) and a pair of borrower country 2’s bond assets \( b_0^2 < b_1^2 \leq 0 \), if default is optional for \( b_1^2 \), then default is also optimal for \( b_0^2 \) and the probability of default at equilibrium satisfies \( \pi_2(s, b_0^2) \geq \pi_2(s, b_1^2) \).

**Proof 1** Given a productivity shock to \( e_2 \), the value of default \( V_{2d}(s) \) is independent of \( b_2 \). The value function of repaying debt \( V_{2c}(s, b_2) \) is increasing in \( b_2 \). Therefore, if \( V_{2c}(s, b_1^2) \leq V_{2d}(s) \), then it must be the case that \( V_{2c}(s, b_0^2) \leq V_{2d}(s) \) since \( V_{2c}(s, b_0^2) \leq V_{2c}(s, b_1^2) \). Hence, for any \( s \in D(s, b_2^0) \), we must also have \( s \in D(s, b_1^2) \), that is, \( D(s, b_1^2) \subseteq D(s, b_0^2) \). Therefore, by definition, we have \( \pi_2(s, b_0^2) \geq \pi_2(s, b_1^2) \).

3 Quantitative Results

3.1 Baseline Calibration

In this section, I study the quantitative implications of the model by conducting numerical simulations at the quarterly frequency and using a baseline calibration based on the data largely from Mexico and Canada. Table 2 shows the calibrated parameter values.
The probability of reentering the international financial market after a default is 0.083, which implies that the country stays in exclusion for an average of three years after default. This is the estimate obtained by Dias and Richmond (2007) for the median duration of exclusion periods. It is also consistent with the finding by Gelos, Sahay, and Sandleris (2011) and is applied by Mendoza and Yue (2012).

The parameters $\rho_1$ and $\rho_2$ in the model control the degree of home bias. According to the World Bank (WDI), the average shares of domestic products in final consumption for Canada and Mexico for the period of 1981 – 2012 are 0.6 and 0.7, respectively. Hence, I use Mexico’s 0.7 in calibrating $\rho_2$ such that at steady state the borrower country 2’s domestic goods share in final consumption is 0.7. As for Canada, not all 40 percent of its final consumption is necessarily imported from Mexico, in fact it is a lot smaller than that.
Therefore, I calibrate $\rho_1$ such that at steady state the creditor country 1’s domestic goods share in final consumption is 0.89. Other values of $\rho_1$ and $\rho_2$ are explored in sensitivity analyses.

The next pair of parameters $\theta_1, \theta_2$ have to do with the elasticities of substitution between domestic goods consumption and imported goods consumption for developed and developing countries. The literature provides a large range of estimates for the elasticity of substitution. Backus, Kehoe, and Kydland (1994) document that the U.S. elasticity is between 1 and 2 and values in this range are generally used in empirical trade models. Their benchmark model adopts a value of 1.5. Later papers have used similar values, e.g., Chari, Kehoe, and McGrattan (2002), Bergin (2006), and Ruhl (2008). A recent paper by Feenstra, Luck, Obstfeld, and Russ (2014) estimates both micro and macro elasticities of substitution together and uses an additional moment condition to overcome small-sample bias in the macro estimate. They find point estimates for the macro elasticity exceeding unity in almost all sectors.

However, few paper has studied the Armington elasticity for developing countries. Ostry and Reinhart (1992) find the elasticity of substitution between traded and nontraded goods in the range of 1.22 to 1.27, and significant regional differences with less developed countries displaying higher values. Yet, it remains unclear about the cross-country comparison of Armington elasticities. This paper does not take a stand on the value of the elasticity. As a starting point, I adopt 1.33 as the elasticity of substitution for the creditor country to match that for developed countries on average, and a higher value of 2.5 for the borrower country to indicate that less developed countries may have a higher elasticity of substitution between home and foreign goods as well. In sensitivity analyses, I explore other values for the elasticities.

The labor share in final production is set at 0.63 for Canada and 0.45 for Mexico, which are the average labor income shares using annual data for the period of 1981-2009 (1981-2008 for Canada) from OECD Statistics. The input share of the imported intermediate goods to produce final goods 1 is the same as the domestic labor input share to produce the same final goods 1. This is because no matter using intermediate goods or domestic labor the production of final goods 1 uses the same technology, and intermediate goods is a function of labor as well.

I calibrate the sizes of the two countries assembling some regularities of Canada and Mexico. The capital and labor endowments of borrower country 2 are normalized to 1. Therefore, creditor country 1’s labor size is $\bar{n}_1 = 2.5$ to match the average CA-to-Mexico employment ratio for 1981Q1-2012Q4. Creditor country 1’s capital endowment

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$k_1$ is chosen such that at steady state its capital used with intermediate goods, $k_1^*$, is 65 percent of borrower country 2’s GDP. This is approximated by the average of FDI-to-GDP ratio for Mexico during 1981Q1-2012Q4, assuming the majority of the FDI to Mexico is vertical. However, it is important to note that this approximated target is by no means a complete calibration for the actual amount of foreign capital used with Mexican intermediate goods to produce foreign final goods.

The only productivity shock in the model is to the borrower country 2’s productivity $e_2$, whose steady state is normalized to 1. It follows an AR(1) process:

$$\log e_{2,t} = \rho \log e_{2,t-1} + \eta_t$$

with $\eta$ being iid and following $N(0, \sigma^2)$. I estimate the process using the model’s production functions, and HP-filtered Mexican data of GDP, (average) capital stock, and employment at both domestic sector and FDI sector for 1981Q1-2012Q4. Using Tauchen and Hussey (1991), I further construct a Markov approximation to this process with 5 states of productivity realization for $e_2$. For creditor country 1, its constant productivity $e_1$ is calibrated to be 0.7106, so that at steady state the CA-to-Mexico GDP ratio is 2, equal to the average CA-to-Mexico GDP ratio for 1981 – 2011 according to annual data from IMF.

The targets for setting $\beta_1$, $\beta_2$, and $\epsilon$ are, respectively, the risk free interest rate from US or Canadian treasury bills, quarterly frequency of Mexican defaults, and the loss in Mexican intermediate goods exports upon default. Both U.S. and Canadian treasury bills bear real interest rates that are below 1 percent on average, hence, we have $\beta_1 = 0.99$. Mexico’s quarterly default frequency is about 1%, since it had eight default episodes between 1828 and 2012 according to Reinhart (2010). To study the dynamics around Mexican sovereign defaults during 1980s, I also include more such episodes from Paris Club data for the sample period of 1981Q1-2012Q4. They are 1982Q3, 1986Q1, and 1989Q1. Upon these three most recent onsets of sovereign default, Mexico’s intermediate goods export value, on average, was about 8.6 percent below trend. Given these two targets, the simulated procedure yields $\beta_2 = 0.9697$ and $\epsilon = 0.833$.

I solve the model with a discretized state space of 5 realizations for borrower country 2’s productivity and 107 points for asset holdings. The model is considered as solved when the convergence distance diminishes to $1.0000e-06$. In the following sections, I first examine the properties of the calibrated model, then study the simulated results both along business cycles and around default events.
3.2 Policy Functions

The properties of bond quantity and its price in this model are in line with existing sovereign default papers. Figure 3 left plot graphs the next period assets for the borrower country against its current assets, at a high productivity state and a low productivity state in the current period. As the borrower country accumulates debt (to the left of the bottom axis), its marginal borrowing capacity diminishes. Moreover, when the country is at a low state, its bond function starts to flatten out at a lower current debt amount than if it were at a high state. That is to say, all else equal, a higher productivity state supports a higher debt level.

Figure 3 right plot graphs the bond price functions. It shows that the bond price decreases with debt level (i.e., the interest rate rises). Across productivity states, the bond price is significantly higher for a high state, which implies countercyclical interest rates.

3.3 CyclicalMovements in the Baseline Model

This section starts the assessment of the quantitative performance of the model by comparing moments from the data with moments from the model’s dynamics. To compute the latter, I feed borrower country 2’s productivity process into the model and conduct 1000 simulations, each with 600 periods. Then I truncate the first 100 observations and use the rest to compute the statistics of the model results.

Table 3 compares the moments produced by the baseline model with those from Mexico data and Mendoza and Yue (MY, 2012). Notice that Mendoza and Yue (2012) calibrate
Table 3: Statistical Moments of Borrower Country 2’s Business Cycles

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline</th>
<th>M&amp;Y (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average debt/GDP ratio (in percent)</td>
<td>74.94</td>
<td>20.53</td>
<td>22.88</td>
</tr>
<tr>
<td>Average bond spreads (in percent)</td>
<td>4.35</td>
<td>4.66(5.52)</td>
<td>0.74</td>
</tr>
<tr>
<td>Bond spreads std. dev. (in percent)</td>
<td>4.71</td>
<td>0.79(2.32)</td>
<td>1.23</td>
</tr>
<tr>
<td>Real exchange rate std. dev. (in percent)</td>
<td>17.30</td>
<td>1.29</td>
<td>n.a.</td>
</tr>
<tr>
<td>Terms of trade std. dev. (in percent)</td>
<td>6.21</td>
<td>5.45</td>
<td>n.a.</td>
</tr>
<tr>
<td>Domestic product consumption std. dev./GDP std. dev.</td>
<td>1.23</td>
<td>0.70</td>
<td>n.a.</td>
</tr>
<tr>
<td>Total consumption std. dev./GDP std. dev.</td>
<td>1.12</td>
<td>1.07</td>
<td>1.05</td>
</tr>
<tr>
<td>Trade balance/GDP std. (in percent)</td>
<td>2.08</td>
<td>1.14</td>
<td>n.a.</td>
</tr>
<tr>
<td>Final goods export std. dev./GDP std. dev.</td>
<td>0.82</td>
<td>0.05</td>
<td>n.a.</td>
</tr>
<tr>
<td>Intrm. goods export std. dev./GDP std. dev.</td>
<td>1.13</td>
<td>0.42</td>
<td>n.a.</td>
</tr>
<tr>
<td>Total import std. dev./GDP std. dev.</td>
<td>1.02</td>
<td>0.44</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

**Correlation with GDP**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline</th>
<th>M&amp;Y (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond spreads</td>
<td>−0.39</td>
<td>−0.67</td>
<td>−0.17</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.53</td>
<td>0.39</td>
<td>n.a.</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.25</td>
<td>0.38</td>
<td>n.a.</td>
</tr>
<tr>
<td>Trade balance/GDP</td>
<td>−0.65</td>
<td>−0.15</td>
<td>−0.54</td>
</tr>
<tr>
<td>Total exports</td>
<td>0.21</td>
<td>0.82</td>
<td>n.a.</td>
</tr>
<tr>
<td>Intermediate goods exports</td>
<td>0.18</td>
<td>0.83</td>
<td>n.a.</td>
</tr>
<tr>
<td>Total import</td>
<td>0.75</td>
<td>0.90</td>
<td>n.a.</td>
</tr>
<tr>
<td>Wage</td>
<td>0.65</td>
<td>0.92</td>
<td>n.a.</td>
</tr>
<tr>
<td>GDP volume</td>
<td>0.65</td>
<td>0.79</td>
<td>n.a.</td>
</tr>
<tr>
<td>Default occurrence</td>
<td>−0.14</td>
<td>−0.24</td>
<td>−0.09</td>
</tr>
<tr>
<td>Default duration</td>
<td>−0.39</td>
<td>−0.62</td>
<td>n.a.</td>
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</tbody>
</table>

**Correlation with bond spreads**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline</th>
<th>M&amp;Y (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real exchange rate</td>
<td>−0.76</td>
<td>−0.85</td>
<td>n.a.</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>−0.13</td>
<td>−0.85</td>
<td>n.a.</td>
</tr>
<tr>
<td>Trade balance/GDP</td>
<td>0.30</td>
<td>0.49</td>
<td>0.15</td>
</tr>
<tr>
<td>Total exports</td>
<td>−0.02</td>
<td>−0.50</td>
<td>n.a.</td>
</tr>
<tr>
<td>Intermediate goods exports</td>
<td>−0.08</td>
<td>−0.63</td>
<td>n.a.</td>
</tr>
<tr>
<td>Total import</td>
<td>−0.28</td>
<td>−0.82</td>
<td>n.a.</td>
</tr>
<tr>
<td>Wage</td>
<td>−0.35</td>
<td>−0.50</td>
<td>n.a.</td>
</tr>
<tr>
<td>GDP volume</td>
<td>−0.19</td>
<td>−0.16</td>
<td>n.a.</td>
</tr>
<tr>
<td>Default occurrence</td>
<td>0.18</td>
<td>0.29</td>
<td>n.a.</td>
</tr>
<tr>
<td>Default duration</td>
<td>0.56</td>
<td>0.94</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Note: Except bond spreads, and default occurrence and duration, all other data in the table are HP-filtered. Bond spreads are calculated over U.S. government bond real interest rates that are negative sometimes. All data are in real terms and at quarterly frequency.

their model partially to Argentine data and partially to Mexican data. All the data used in this model are quarterly from 1981Q1 to 2012Q4. The data sources are provided in the Appendix. First, this model produces a debt-to-GDP ratio of about 20 percent on average while matching the 1 percent default frequency observed in the data. Because the time discount factor of the borrower country 2 is lower than that of the creditor country 1 in calibration, the difference in consumption patience can support a certain level of debt. The lower debt-to-GDP ratio result than the data is common to the literature of strategic
sovereign default. This result is also close to that in Mendoza and Yue (2012). Their paper adopts a lower default frequency and a lower time discount factor (\(\beta = 0.88\)) that make it more likely to produce a higher debt-to-GDP ratio than mine.

Moreover, unlike Mendoza and Yue (2012) and many other sovereign default models, this paper incorporates creditors’ risk aversion, which limits this model’s ability to generate data-matching debt-to-GDP ratios. As default risks mount, creditors’ risk aversion can potentially cause bond interest rate to increase faster than in a model with risk neutral creditors. This feeds back to further increase the default risk, and suppresses the borrower country’s debt level. However, risk aversion does help my model support a higher average bond spread, which is more consistent with the data than existing literature. I will elaborate on this point below. Additionally, the debt level in the model also increases with the sovereign default’s costs to the borrower country. Here, the costs include exclusion from the bond market and endogenous income losses as explained in the model section.

Model statistics for bond spreads are tricky in that during default periods the modeled interest rate goes to infinity. I report in Table 3 the modeled bond spread statistics along business cycles without the default episodes (not in parentheses). The mean of bond spreads is close to the data. Here, the bond price reflects not only the expected return due to the probability of default, but also compensation to the risk-averse creditors for bearing sizable consumption risk\

As explained in the next section, the creditor country suffers from a long-lasting welfare decline once the borrower country defaults. Therefore, unlike previous papers using small open economy with risk-neutral investors, this model breaks the close link between the probability of default and bond pricing by including the creditor country’s welfare loss and risk aversion. It enables the result to be consistent with the data, where the average bond spread is several times higher than historical default frequency.

Meanwhile, the modeled volatility of bond spreads, 0.79, is much lower than data. The main reason is that I exclude the infinitely large interest rate during default episodes from the model result. However, if in the model result I cap the infinite interest rate at Mexican average real interest rate around default episodes from the data (10 percent) and include them to calculate the same statistics (in parentheses), the modeled volatility of bond spreads becomes, 2.32, closer to the data.

The volatility of terms of trade in the model is consistent with the data, while that of real exchange rates is much smaller. Terms of trade fluctuations are greatly influenced

\(^{18}\)The impact of the creditor country’s welfare loss on bond spreads follows a similar rationale of “rare disaster” as in Barro (2006) and Gabaix (2008).
by home bias preference (see Sensitivity Analysis section). In the model, when a large share of the borrower country’s utility depends on its consumption of foreign goods, it is optimal for the country to not reduce imports too much when income lowers and to have a stable terms of trade. However, real exchange rate is influenced by many other factors along business cycles, such as monetary and nominal exchange rate policies, which are not taken into account by this model.

Domestic goods consumption is much smoother in the model than in the data because of borrowing, home bias preference, and labor movement from the intermediate goods sector to the domestic sector in default crises. These three factors support domestic goods production and their consumption in spite of adverse productivity shocks. Total consumption is less smoothed than domestic goods consumption in the model on account of the variations in imports and terms of trade along business cycles. It is also slightly more volatile than output, as in the data.

In terms of the volatility of trade balances, imports, and exports, the model results are lower than those of the data. Other sovereign default models have generated similar results for trade balances, for example, Aguiar and Gopinath (2007) produce trade balance standard deviation to be 0.95, Arellano (2008) 1.5, and Yue (2010) 2.81. But the difference here is that the trade balances in this paper are not just residuals of output after consumption. Instead, they are the combined result of capital flows and consumers’ preference regarding home and foreign final goods. Hence, this model differs from previous default models in separating exports and imports. In both the data and the model, final goods exports are less volatile than imports and intermediate goods exports.

Next, Table 3 shows that this model does a good job of delivering the correlation between GDP value and bond spreads, as well as their correlations with other variables. The model yields a negative correlation between bond spreads and GDP, consistent with the data, because the bond bears higher default risk in bad states. As in Mendoza and Yue (2012), this model produces countercyclical default risk in a setting where both income and default risk are endogenous and affect each other, unlike in the models of sovereign default alone or of business cycles alone.

However, this model distinguishes itself from Mendoza and Yue (2012) in that the endogenous income and default risk interact also through the movement of terms of trade. Meanwhile, in my model, both terms of trade and the real exchange rate have positive relations with GDP and strong negative relations with bond spreads, which is consistent with the data. As the borrower country accumulates debt, the bond interest rate rises, resulting in a tighter budget constraint with more and more funds flowing out of the
borrower country into the creditor country. Therefore, due to the home bias preference, the borrower country’s consumers adjust their consumption portfolio to reduce imports, and terms of trade are under pressure to fall. This prevents real appreciation from easing the borrower country’s budget constraint and from helping it to pay back debt that is denominated in the creditor country’s final goods. Therefore, default risk is further elevated.

Once the borrower country defaults due to an adverse productivity shock, it is penalized by exclusion from the bond market and another endogenous income loss that is additional to that from the initial adverse productivity shock. This second income loss comes from a per-worker wage decline paid in the defaulting country due to the oversupply of labor to the domestic production sector as analyzed in the model section. The overall income loss overpowers what is gained from the defaulted debt repayment. As the borrower country’s budget constraint further tightens, its consumption portfolio is once again adjusted away from imports and its final goods exports rise. Consequently, the trade balances turn positive, while terms of trade and the real exchange rate deteriorate sharply. This deterioration takes a third toll on the borrower country’s income.

This mechanism, in which increased borrowing and default risk raise the real interest rate, and decrease income and imports, also explains why this model generates a negative relation between trade balances and GDP, while producing a positive relation between trade balances and bond spreads. It is consistent with the stylized business cycle features of Mexico and other developing countries.

More importantly, this model also delivers procyclical bilateral trade flows and a negative relation between bilateral trade flows and bond spreads consistently with the data, which have not been captured by previous sovereign default models. This is also a result of having the procyclical terms of trade. During downturns, the values of both imports and exports decline partly due to the deterioration of terms of trade.

Furthermore, the model predicts the correlation between the borrower country’s exported intermediate goods and its GDP or bond spreads, qualitatively in line with the correlations observed in Mexican data. More broadly, the business cycle correlations between output and intermediate goods export value vary across countries but are usually positive. For instance, using annual growth data (1988-2013), I compute the correlation for 16 countries, for which I have intermediate goods exports data. On average, their correlation between output growth and intermediate goods exports growth is 41 percent.

The correlation for Argentina is 34.8%, Brazil 66.4%, Croatia 13.1%, Ecuador 5.2%, Greece 17.1%, Iceland 41.8%, Indonesia 24.4%, Moldova 26.2%, Peru 52.6%, Russia 66.1%, South Africa 56.4%, Thailand 55.5%, Turkey 7.2%, Ukraine 81.8%, Uruguay 58.5%, and Venezuela 43.3%.
As discussed earlier, the wage in borrower country 2 declines with productivity and even more during sovereign default episodes. This is confirmed by the model results where the wage strongly positively correlates with GDP and negatively with bond spreads, as in the data. These results are also consistent with the finding of Li (2011) for the wage cyclicality of emerging markets in general.

Additionally, this model disentangles the default-related loss of GDP volume in GDP value. As I show in the next section, during default periods, about one half of GDP value loss is due to GDP volume declines, while the other half is attributed to real depreciation. In Table 3, even though GDP value is positively correlated to GDP volume, it is not a perfect correlation—only 65 percent in the data and 79 percent in the model. The real exchange rate and terms of trade do play a role in GDP value variations in both the model and the data. Also, consistently with the data, the model generates declining production activities when bond spreads increase along business cycles.

Lastly, I report in Table 3 the correlations between default and output, and between default and bond spreads. In particular, the onset of a default event is positively correlated with output and negatively correlated with bond spreads in both the data and the model. As for the duration of a default episode, it is more closely related to bond spreads than to output in the model, which is also the case in the data.

3.4 Dynamics around Default Events

Next I study the model’s macroeconomic, trade, and welfare dynamics of borrower country 2 around default events by comparing the simulated results with the time series data for Mexico. This paper identifies three sovereign default occurrences by Mexico since 1981, in 1982Q3, 1986Q1, and 1989Q1. Those dates are inferred from Paris Club data, which shows that Mexico was treated on 1983 June 22nd, 1986 September 17th, and 1989 May 30th by foreign creditors for its sovereign debts. Each episode window covers 6 quarters before and after a default onset. Date 0 is the quarter of default occurrences. I plot the mean of these three default episodes for each of the variables from the data, as well as the mean from the model simulation surrounded by a one-standard-deviation band.

20 Slight date adjustments according to GDP fluctuations have been made with regard to the Paris Club dates to reflect delayed treatments after defaults. The results are not sensitive to the default date specifications.
21 The length of 6 quarters is chosen because the three onsets of Mexican sovereign defaults are apart from each other for about 3 years and 6-quarter is the middle point.
22 This section uses the same simulation results as in the previous section.
3.4.1 Macroeconomic Dynamics

Figure 4 shows the model’s macroeconomic dynamic results, i.e., deviations from steady state (or trend in the data) of borrower country 2. In the model, the sharp decline and the slow recovery of GDP value (in creditor country 1’s final goods 1) are consistent with those of the real GDP value data (in USD) for Mexico, although the model generates a deeper recession. In the model, the decline of income upon default comes from two sources: GDP volume decrease and real depreciation. First, Mexico’s GDP volume (seasonally adjusted) does not seem affected much by the default events, whereas in the model GDP volume decreases by about 10 percent on average.

Second, on average, the terms of trade deterioration after a default in the model matches well with the data, about 10 percent. But the data show a faster recovery of terms of trade than the model results. The model’s real exchange rate declines much smaller than terms of trade and the data do. The reason is that even though the borrower country’s terms of trade deteriorate and its aggregate price index lowers, the creditor country also adjusts its consumption towards cheaper imported final goods resulting in a lower aggregate price index as well. Additionally, in this model nominal exchange rate is fixed at one, whereas in reality it usually changes around sovereign default episodes. These limit the decline of real exchange rate in this model. If I eliminate nominal exchange rate in the data, as shown by the dotted line (the ratio of Mexican real exchange rate divided by its nominal exchange rate with USD), the model result matches much better with the data. Overall, the real GDP loss upon default is driven by the deterioration of terms of trade and real exchange rate as well as the decline of production activities in the model, and even more so by the former in the Mexican data.

In the sovereign debt market, this model predicts that the debt-to-GDP ratio is relatively stable over the four quarters prior to a default, then surges as it approaches and reaches the default, and drops afterwards. To make the model’s debt ratio comparable to the data, I follow Mendoza and Yue (2012) and adjust the mean of the post-default debt ratios from the model to be the average of the pre-default debt ratio and the debt ratio chosen once the countries reenter the bond market. However, the adjusted post-default debt ratio in the model still declines faster than in the data. Also, the model’s debt ratio is significantly lower than the data’s. It is an issue common to the previous strategic sovereign default models as well.

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23It is worth noting the fact that little GDP volume declines around the default episodes for Mexico is not uncommon for other countries. The changes to GDP volume growth upon a default are diverse across countries. For instance, Paraguay grew by 4.32 percent in GDP volume during its 2003 default, while Indonesia’s GDP volume declined by 13.13 percent during 2008 default.
Figure 4: Macroeconomic Dynamics of Borrower Country 2 around Default Events

Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1. Except for the interest rate and debt-to-GDP ratio, all other data are HP-filtered. GDP value (in USD), GDP volume, terms of trade, domestic sector labor, hours worked, and intermediate goods export sector (FDI) labor are logged before being detrended. All data are real. For the subplots with a different scale on the right axis, the scale is for the data of the variable. The model results of GDP value are measured in creditor country 1’s final goods 1. The dotted line in Real Exchange Rate plot is the ratio of Mexican real exchange rate divided by its nominal exchange rate with USD. The dotted line in Domestic Sector Labor plot is Mexico’s total hours worked.
Moreover, the model matches well the increase in the real interest rate around default events. I do not show the model interest rate at the default quarter because it does not exist due to exclusion from the bond market. On average, the model is able to support high real interest rates and bond spreads that not only incorporate the default risk, but also compensate the creditor country for its welfare loss and risk aversion.

In addition, the model produces the qualitative feature of the employment decline in the intermediate goods exports sector. The pattern is consistent with the finding of Bergin, Feenstra, and Hanson (2009, 2011). But the model does not match the domestic sector employment. It is mainly due to two reasons. On one hand, the model does not include unemployment. More specifically, the borrower country’s domestic goods production sector has to take whatever amount of labor remains after the intermediate goods sector’s employment. If this limitation were eliminated, the model could potentially have another labor market channel affecting the costs of default for the borrower country, as in Mendoza and Yue (2012).

On the other hand, the employment data here captures poorly about informal sector employment in the early 1980s. Fernandez and Meza (2014) documents that the informal sector employment counts about half of Latin American employment and is strongly countercyclical and negatively correlated with formal employment in Mexico using data available since 1987. If including informal sector employment, in Figure 4 the decline of the employment during the defaults in Mexican data can be milder. Furthermore, if we look at total hours worked (the dotted line), it did not decline as much as employment upon default. In fact, hours worked increased slightly right after default, before it starts to decline. Overall, I simplify the labor market in this paper to highlight the terms-of-trade channel and international trade results.

### 3.4.2 Trade Dynamics

I now analyze the model’s performance of variables related to international trade in Figure 5 and Figure 6. Before going into details in Figure 5, one thing to notice from the data is that there are significant declines in export volume and share of GDP around $t = -3$.

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24 For the time periods immediately after the sovereign default quarter, not all of them have interest rates because the countries can only re-enter the bond market with a certain probability. Hence, I show the statistics of those cases in which the countries do re-enter shortly.

25 Bergin, Feenstra, and Hanson (2009, 2011), using Mexico’s maquiladora sectors, find that the country’s offshoring industries experience employment fluctuations that are much more volatile than those in the U.S.

26 Notice that due to data limitation, here total hours worked data is for manufacturing sector and is not necessarily for domestic production only.
This is due to the rise of Mexico’s real exchange rate from mid-1980 to early 1982, as shown in Figure 4’s real exchange rate data. This rise is exogenous to the components of this model. Disregarding this irregularity in the data, the model captures the qualitative features of different measures of exports in the data, as shown in Figure 5.

In particular, in the model, borrower country 2’s total export value (measured in creditor country 1’s final goods) declines, even though on average export volume declines only slightly and export share of GDP rises. The main cause of their differences is terms of trade deterioration upon default causing export value to decline the most. I then separate the total exports data into intermediate goods and final goods, as in the model. Since there is no intermediate and final goods export volume data available for Mexico, I use the measure in peso (logged) to approximate the volumes. For final goods exports, the model matches the post-default rising value, volume, and share of GDP, although it fails to capture the initial declines of the final goods export value that are due to real exchange rate declines upon default.

For the intermediate goods exports, their value, volume, and percentage of GDP all decline initially upon default in the model as in the data, even though the volume and the percentage of GDP only decline slightly and recovered faster in the data than in the model. It is worth noting that intermediate goods exports performing below trend is not uncommon for other developing countries’ default episodes. As shown in Table 1 during default periods, the average annual growth rate of intermediate goods export value is -2.22 percent. A similar picture is true for the average growth rates of intermediate goods export volume (0.57 percent) and its share of GDP (-1.87 percent), much lower than those at non-default time.

Figure 6 plots the dynamics for imports and trade balances (including intermediate goods exports). The model does well in matching the qualitative patterns of imports upon default. It succeeds especially in being in line with the data of import-to-GDP ratio, given that most of the data path is within the error band of the model result. Adding import value and export value together, bilateral trade value is more than 8 percent below trend upon default, which is consistent with the finding of Rose (2005).

Lastly and importantly, the rise in the trade balances when a default occurs is a result of the increase in the export-to-GDP ratio and the decrease in the import-to-GDP ratio, in both the data and the model. In previous sovereign default models, which is inconsistent with the data. In this respect, Mendoza and Yue (2012) is an exception. Their model introduces default-triggered exogenous capital flows to support the surge of trade balances. This paper differs in that no exogenous element is needed. Here, the
Figure 5: Export Dynamics of Borrower Country 2 around Default Events

Note: One thing to notice from the data on this figure is that there are significant declines in export volume and share of GDP around $t = -3$. It is due to the rise of Mexico’s real exchange rate from mid-1980 to early 1982, as shown in Figure 4’s real exchange rate data. It is exogenous to the components of this model. Disregarding this irregularity in the data, the model captures the qualitative features of different measures of exports in the data. Default events are identified as 1982Q3, 1986Q1, and 1989Q1. All data are real and HP-filtered. The data of export value (in USD), export volume, and final goods exports are logged before being detrended. For the subplots with a different scale on the right axis, the scale is for the data of the variable. The model results of export values are measured in creditor country 1’s final goods 1.
Figure 6: Trade Dynamics of Borrower Country 2 around Default Events

Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1. All data are real and HP-filtered. Import value and volume are logged before being detrended. For the subplots with a different scale on the right axis, the scale is for the data of the variable.

Trade balances are supported by capital flows of not only bond trading but also global vertical integration. They are naturally generated by the trading of two final goods for consumption and intermediate goods for production. However, the modeled post-default trade balance improvement is not as long-lasting as in the data. It is mostly due to an export-to-GDP decline resulting from a persistently low intermediate-goods-export-to-GDP ratio during the post-default periods in the model.

3.4.3 Welfare Dynamics

Since this paper uses a two-country model, I can study the welfare of both creditor and borrower (Figure 7). For the creditor country, on average a default triggers a 1.5 percent decline in welfare; this loss remains negative and withers slowly back to zero during the next 12 quarters and beyond. The magnitude of the damage to the creditor country is much smaller than that to the borrower country (an 8 percent decline in welfare), but the effect on the creditor is much longer lasting than that on the borrower.

This is mainly because the creditor country’s budget constraint is tightened by the defaulted debt repayment, and additional income losses from the inefficiency in imported intermediate goods imports and the reallocation of capital. However, favorable terms of
Note: Default events are identified as 1982Q3, 1986Q1, and 1989Q1.

trade reduce the pain levied on the creditor country. Yet, by no means we should take the welfare numbers reported here literally. I acknowledge that in practice the impact of a sovereign default to a creditor country’s welfare depends on many other factors. For instance, it hinges on the substitutability between imports from the defaulting country and other goods for consumption and production in the creditor country.

For the borrower country, a default delivers, on average, an 8 percent drop in welfare. However, this loss can be recovered in fewer than 12 quarters, as the country restores its productivity and reenters the bond market. The defaulting country gains by forgoing the debt repayment and loses through financial autarky and endogenous income declines. The welfare pattern before and after a default is more symmetric than it is for the creditor country. The source for its quick recovery mainly comes from the labor market shifting to domestic goods production that supports post-default domestic final goods consumption and exports. Additionally, in the model the world welfare worsens upon default as well.

### 3.5 Results for the Sample Period

The model can also replicate the time series of Mexico output and bond spreads for the sample period of 1981Q1-2012Q4. I feed the corresponding productivity shocks into the model and compare borrower country 2’s results with the data. Figure 8 plots the HP-filtered output value and volume, along with the simulated results. The model matches the data well in terms of the two output measures. The grey areas in the figures show the model-predicted default occurrences in 1985Q1 and 1995Q3. Even though 1995Q3 is not officially documented as a sovereign default, Mexico would have defaulted following its 1994 crisis without the aid it received from foreign countries (mainly the US).
model result of 1985Q1 default comes from a productivity crash from a prior boom. The productivity shocks fed into the model show a big spike for the output boom in Mexico right before 1985.\footnote{The spike appears using HP filter or BP filter.}

Figure 8: Mexican Output and Bond Spreads in the Data and Borrower Country 2 in the Model (1981Q1-2012Q4)

![Figure 8: Mexican Output and Bond Spreads in the Data and Borrower Country 2 in the Model (1981Q1-2012Q4)](image)

Note: All data are real, and output measures are HP-filtered.

At the bottom of Figure\textsuperscript{8} the model is shown to also match the bond spreads data well for the period of 1986Q1-2001Q4, but less so for the periods before and after. Overall, the model results indicate that Mexico faces countercyclical bond spreads, and it defaults on sovereign debts when the output is low and the interest rate is high.

### 3.6 Sensitivity Analysis

In this section I conduct a sensitivity analysis to evaluate the robustness of the model’s quantitative results regarding terms of trade, the real exchange rate, bond spreads, and trade flows. The model results are robust to changes in data filter, the borrower coun-
try’s shock persistence ($\rho$), patience ($\beta_2$), post-default efficiency loss in intermediate goods export sector ($\epsilon$), both countries’ degrees of home bias ($\rho_1, \rho_2$), and elasticities of substitution ($\frac{1}{1-\theta_1}, \frac{1}{1-\theta_2}$). The results are summarized in Table 4 and Table 5. These tables show the main business cycle statistical moments as in Table 3 for each alternative scenario, plus default frequency and several around-default statistics for variables of interest.

3.6.1 BP Filter

In the first scenario, I re-calibrate the entire model with BP-filtered data. BP filter may extract the desired frequency from the quarterly data more precisely than HP filter. In fact, it generates a much higher persistence of productivity shocks for Mexico (0.8876) than the baseline using HP filter (0.4162), and a lower volatility (0.0143) than the baseline (0.0377). Again, to calibrate the model’s default frequency so that it is consistent with the data (0.01), I adjust $\beta_2$ to 0.982 (baseline is 0.9697) and $\epsilon$ to 0.908 (baseline is 0.833).

The results are close to the baseline and qualitatively robust to different filters. Yet three differences stand out. First, the average debt-to-GDP ratio is lower than that of the baseline. This may be due to the higher shock persistence that supports a lower debt level. Second, the variation of bond spreads, terms of trade, and real exchange rate all decline, since the shock volatility is smaller than that of the baseline. Third, the defaulter registers a much larger welfare loss upon default than baseline. Switching the filter makes it difficult to disentangle the exact causes for those changes, because multiple parameters have been changed at the same time (since the entire model is re-calibrated). Most of the following scenarios have only one parameter’s value changed at a time unless otherwise mentioned.

3.6.2 Shock Persistence

The next two columns of Table 4 report the results for shocks with lower persistence ($\rho = 0.2$) and higher persistence ($\rho = 0.6$) than the baseline ($\rho = 0.4162$). Comparing the three scenarios, I find that most variables have small changes that are monotonic with the change of shock persistence. Some more significant changes occur to the average debt-to-GDP ratio, and the defaulter’s welfare losses.

First, lower shock persistence allows a country to get back on track faster after an

\footnote{I also generated around-default dynamics plots comparable to Figure 4, 5, and 6. The quantitative differences are small and qualitative patterns are the same, so I do not put them in the paper due to space limitation. But several around-default values for variables of interest are reported in Table 4 and Table 5.}
Table 4: Sensitivity Analysis

<table>
<thead>
<tr>
<th>Statistics</th>
<th>HP Data</th>
<th>HP Baseline</th>
<th>HP Model</th>
<th>BP Data</th>
<th>BP Model</th>
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<tbody>
<tr>
<td>Default frequency</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Ave. debt/GDP ratio (%)</td>
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<td>20.53</td>
<td>74.94</td>
<td>11.70</td>
<td>21.11</td>
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<tr>
<td>Ave. spreads (%)</td>
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<td>4.66</td>
<td>4.35</td>
<td>3.44</td>
<td>4.64</td>
</tr>
<tr>
<td>Spreads std. dev. (%)</td>
<td>4.71</td>
<td>0.79</td>
<td>4.71</td>
<td>0.43</td>
<td>0.88</td>
</tr>
<tr>
<td>REXR std. dev. (%)</td>
<td>17.30</td>
<td>1.29</td>
<td>17.30</td>
<td>0.55</td>
<td>1.40</td>
</tr>
<tr>
<td>Terms of trade std. dev. (%)</td>
<td>6.21</td>
<td>5.45</td>
<td>5.32</td>
<td>2.59</td>
<td>5.89</td>
</tr>
<tr>
<td>Dom. prod. cons. std/GDP std.</td>
<td>1.23</td>
<td>0.70</td>
<td>1.15</td>
<td>0.74</td>
<td>0.68</td>
</tr>
<tr>
<td>Total cons. std./GDP std.</td>
<td>1.12</td>
<td>1.07</td>
<td>1.10</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>Trade balance std. (%)</td>
<td>2.08</td>
<td>1.14</td>
<td>1.90</td>
<td>0.76</td>
<td>1.22</td>
</tr>
<tr>
<td>Final goods exp. std./GDP std.</td>
<td>0.82</td>
<td>0.05</td>
<td>0.73</td>
<td>0.05</td>
<td>0.06</td>
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<td>1.13</td>
<td>0.37</td>
<td>0.45</td>
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<tr>
<td>Total imp. std./GDP std.</td>
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<td>0.44</td>
<td>0.99</td>
<td>0.35</td>
<td>0.47</td>
</tr>
</tbody>
</table>

**Correlation with GDP**

| Bond spreads | -0.39 | -0.67 | -0.23 | -0.64 | -0.72 | -0.68 | -0.78 | -0.62 | -0.76 | -0.60 |
| Real exchange rate | 0.53 | 0.39 | 0.44 | 0.13 | 0.49 | 0.37 | 0.56 | 0.28 | 0.56 | 0.24 |
| Terms of Trade | 0.25 | 0.38 | 0.31 | 0.11 | 0.48 | 0.36 | 0.55 | 0.25 | 0.55 | 0.21 |
| Trade balance/GDP | 0.65 | 0.15 | -0.24 | -0.25 | -0.24 | -0.26 | -0.22 | -0.26 | -0.22 | -0.60 |
| GDP volume | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 |

**Correlation with bond spreads**

| Real exchange rate | -0.76 | -0.85 | -0.76 | -0.66 | -0.66 | -0.82 | -0.87 | -0.81 | -0.89 | -0.80 |
| Terms of Trade | -0.13 | -0.85 | -0.11 | -0.65 | -0.88 | -0.82 | -0.87 | -0.81 | -0.89 | -0.79 |
| Trade balance/GDP | 0.30 | 0.49 | 0.16 | 0.78 | 0.43 | 0.58 | 0.62 | 0.39 | 0.51 | 0.49 |
| Intermediate goods exports | -0.08 | -0.05 | -0.17 | -0.18 | -0.60 | -0.55 | -0.31 | -0.77 | -0.49 | -0.70 |
| GDP volume | -0.19 | -0.16 | -0.13 | -0.19 | -0.34 | -0.13 | -0.20 | -0.18 | -0.15 | -0.17 | -0.15 |
| Default occurrence | 0.18 | 0.29 | 0.18 | 0.29 | 0.29 | 0.29 | 0.28 | 0.27 | 0.31 | 0.28 |
| Default duration | 0.56 | 0.94 | 0.56 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |

**Upon default**

| Ave. TOT deviation in a year | -0.06 | -0.09 | -0.06 | -0.02 | -0.10 | -0.07 | -0.09 | -0.08 | -0.11 | -0.06 |
| Ave. creditor’s welfare deviation | n.a. | -0.01 | n.a. | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| Ave. defaulter’s welfare deviation | n.a. | -0.08 | n.a. | -0.20 | -0.05 | -0.11 | -0.07 | -0.08 | -0.07 | -0.08 |
| Ave. world welfare deviation | n.a. | -0.09 | n.a. | -0.21 | -0.07 | -0.13 | -0.08 | -0.10 | -0.09 | -0.09 |

Note: Except for bond spreads, debt-to-GDP ratio, and default occurrence and duration, all other data in the table are HP-filtered.

All data are in real terms and at quarterly frequency. For BP filter model results, $\beta_2 = 0.982$, $\epsilon = 0.908$, shock process $\rho = 0.8876$ and $\sigma = 0.0143$. 
adverse productivity shock and thus to maintain a higher average debt level. Second, lower shock persistence also leads to a smaller welfare loss for the defaulting country. These changes are reversed in the case of higher shock persistence.

### 3.6.3 Patience

The discount factor $\beta_2 = 0.9697$ in the baseline is much higher than the value commonly used in the sovereign default literature. For instance, the discount factors in Aguiar and Gopinath (2006), Arellano (2008), Yue (2010), and Mendoza and Yue (2012) are all below 0.953. However, it is lower than the typical value in real business cycle literature. Here I conduct some analysis to see how the results vary with the value of the discount factor. A lower discount factor of 0.9597 and a higher one of 0.9797 are used to compare the results with the baseline’s 0.9697. All the variables change monotonically with the discount factor.

Intuitively, less care about the future brings more frequent defaults, a lower average debt-to-GDP ratio, and a higher average bond spread. Meanwhile, less patience also generates more volatility in total consumption and trade along business cycles, moving the model results closer to the data. The resulting terms of trade, real exchange rates, and trade balances also fluctuate more in correlation with GDP and bond spreads. Additionally, the borrower registers a smaller post-default welfare loss than in the baseline, since it is not as concerned about their sovereign default losses in the near future.

### 3.6.4 Post-default Efficiency Loss in Intermediate Goods Export Sector

Next, I report results with a lower value of $\epsilon = 0.813$ (i.e., a greater efficiency loss in intermediate goods export sector upon default) and a higher $\epsilon = 0.853$ than that in the baseline $\epsilon = 0.833$. It is important to experiment with different values of $\epsilon$ because it governs the magnitude of post-default losses in the defaulter’s intermediate goods exports, and thus also losses in its income, terms of trade, real exchange rates, and trade flows. Even though the results change slightly under different values of $\epsilon$, the signs of all the statistics remain consistent with the data across the two scenarios.

With a lower value of $\epsilon = 0.813$, the borrower suffers from a larger efficiency penalty upon default, which induces two main effects. First, it helps the borrower to maintain a higher level of average debt-to-GDP ratio. Second, together with home bias preference, the larger penalty and tighter budget make the borrower’s trade flows, terms of trade, and real exchange rates more responsive to a default crisis. Hence, the model delivers a larger terms of trade deterioration of 11 percent upon default. Meanwhile, the volatility
of the real exchange rates and trade balances are higher, and their correlations with GDP and bond spreads are also higher.

As for welfare, a lower value of \( \epsilon = 0.813 \) does not alter the countries’ welfare losses very much from the baseline. Given a higher penalty upon default, one would expect to cost the borrower more if it defaults and the borrower to default less frequently. However, with a higher average debt-to-GDP ratio, the borrower also benefits more from forgoing a larger debt repayment if it defaults, so its welfare does not lose as much and it defaults slightly more frequently, than in the baseline. Meanwhile, the creditor loses more from a larger debt default. Hence, the world’s total welfare loss is similar to that of baseline. This result implies that the default penalty may not always be an effective tool to reduce the number of sovereign debt crises.

### 3.6.5 Home Bias

Table 5 reports sensitivity check of the model performance regarding home bias and Armington elasticity of substitution in consumer preference. The third column of the table displays the model result if both countries’ utility function is \( U(c_{i1t}, c_{i2t}) = c_{i1t}^{\theta_i} + c_{i2t}^{\theta_i} \) \((i = 1, 2)\). In this case, neither country exhibits home bias in consumption. The results show that without home bias, for the borrower country: (1) terms of trade are strongly negatively associated with GDP value, (2) trade balances are not negatively correlated with output, (3) real exchange rate is positively associated with bond spreads, and (4) terms of trade improve upon default. These contradict the data. This is why home bias preference plays a crucial role in this model to generate data-consistent results.

Next two columns present the model results with a lower consumer preference towards domestic final goods \((\rho_2 = 0.6)\) and a stronger home bias \((\rho_2 = 0.76)\) for the borrower country than that of the baseline \((\theta_2 = 0.68)\). Intuitively, with a lower degree of home bias, the borrower would become more concerned about its imported consumption once it defaults, leading it to default less frequently, which is indeed what happens in the model. It also allows the borrower to sustain a higher average debt-to-GDP ratio.

Moreover, with a higher consumption preference for imports, it is optimal for the borrower country to maintain lower terms of trade volatility, and thus the correlations of terms of trade and real exchange rate with GDP and bond spreads both decline. Upon default, terms of trade’s drop (-0.05) is about half of that in the baseline (-0.09). In terms of welfare, having more stable terms of trade does allow the borrower to have less welfare loss during a sovereign default crisis. Worldwide, welfare also declines less upon default.

\[29\] Alternatively, one can use international trade cost to generate the same impact of home bias.
Table 5: Sensitivity Analysis: Preference

<table>
<thead>
<tr>
<th>Statistics</th>
<th>HP Data</th>
<th>Home Bias</th>
<th>Substitution</th>
<th>0.5</th>
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<tr>
<td>Default frequency</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>Ave. debt/GDP ratio (%)</td>
<td>74.94</td>
<td>20.53</td>
<td>11.85</td>
<td>21.73</td>
</tr>
<tr>
<td>Ave. spreads (%)</td>
<td>4.35</td>
<td>4.66</td>
<td>4.47</td>
<td>4.68</td>
</tr>
<tr>
<td>Spreads std. dev. (%)</td>
<td>4.71</td>
<td>0.79</td>
<td>0.51</td>
<td>0.76</td>
</tr>
<tr>
<td>REXR std. dev. (%)</td>
<td>17.30</td>
<td>5.45</td>
<td>7.47</td>
<td>2.97</td>
</tr>
<tr>
<td>Terms of trade std. dev. (%)</td>
<td>6.21</td>
<td>5.45</td>
<td>7.47</td>
<td>2.97</td>
</tr>
<tr>
<td>Dom. prod. cons. std/GDP std.</td>
<td>1.23</td>
<td>1.07</td>
<td>1.06</td>
<td>1.07</td>
</tr>
<tr>
<td>Total cons. std./GDP std.</td>
<td>1.12</td>
<td>1.07</td>
<td>1.06</td>
<td>1.07</td>
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<tr>
<td>Trade balance std. (%)</td>
<td>2.08</td>
<td>1.14</td>
<td>0.82</td>
<td>1.09</td>
</tr>
<tr>
<td>Final goods exp. std./GDP std.</td>
<td>0.82</td>
<td>0.05</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>Total imp. std./GDP std.</td>
<td>1.13</td>
<td>0.42</td>
<td>0.80</td>
<td>0.44</td>
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<tr>
<td>Bond spreads</td>
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<td>-0.67</td>
<td>-0.33</td>
<td>-0.62</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>0.53</td>
<td>0.39</td>
<td>0.83</td>
<td>0.25</td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>0.25</td>
<td>-0.84</td>
<td>-0.23</td>
<td>-0.51</td>
</tr>
<tr>
<td>Trade balance/GDP</td>
<td>-0.65</td>
<td>-0.15</td>
<td>0.00</td>
<td>-0.16</td>
</tr>
<tr>
<td>Total exports</td>
<td>0.21</td>
<td>0.82</td>
<td>0.99</td>
<td>0.83</td>
</tr>
<tr>
<td>Intermediate goods exports</td>
<td>0.18</td>
<td>0.83</td>
<td>0.96</td>
<td>0.83</td>
</tr>
<tr>
<td>Total import</td>
<td>0.75</td>
<td>0.90</td>
<td>0.93</td>
<td>0.90</td>
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<tr>
<td>GDP volume</td>
<td>0.65</td>
<td>0.79</td>
<td>0.96</td>
<td>0.82</td>
</tr>
<tr>
<td>Default occurrence</td>
<td>-0.14</td>
<td>-0.24</td>
<td>-0.24</td>
<td>-0.23</td>
</tr>
<tr>
<td>Default duration</td>
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<td>-0.62</td>
<td>-0.26</td>
<td>-0.57</td>
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<tr>
<td>Real exchange rate</td>
<td>0.76</td>
<td>-0.85</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>Terms of Trade</td>
<td>-0.13</td>
<td>-0.85</td>
<td>-0.07</td>
<td>-0.35</td>
</tr>
<tr>
<td>Trade balance/GDP</td>
<td>0.30</td>
<td>0.49</td>
<td>0.35</td>
<td>0.49</td>
</tr>
<tr>
<td>Total exports</td>
<td>-0.02</td>
<td>-0.50</td>
<td>-0.90</td>
<td>-0.60</td>
</tr>
<tr>
<td>Intermediate goods exports</td>
<td>-0.08</td>
<td>-0.63</td>
<td>-0.97</td>
<td>-0.73</td>
</tr>
<tr>
<td>Total import</td>
<td>-0.28</td>
<td>-0.82</td>
<td>-0.35</td>
<td>-0.78</td>
</tr>
<tr>
<td>GDP volume</td>
<td>-0.19</td>
<td>-0.16</td>
<td>-0.14</td>
<td>-0.15</td>
</tr>
<tr>
<td>Default occurrence</td>
<td>0.18</td>
<td>0.29</td>
<td>0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>Default duration</td>
<td>0.56</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Ave. DEV in a year</td>
<td>-0.06</td>
<td>-0.09</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Ave. creditor's welfare deviation</td>
<td>n.a.</td>
<td>-0.01</td>
<td>-0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>Ave. defaulter's welfare deviation</td>
<td>n.a.</td>
<td>-0.08</td>
<td>-0.15</td>
<td>-0.07</td>
</tr>
<tr>
<td>Ave. world welfare deviation</td>
<td>n.a.</td>
<td>-0.09</td>
<td>-0.17</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

Note: Except for bond spreads, debt-to-GDP ratio, and default occurrence and duration, all other data in the table are HP-filtered.
All data are in real terms and at quarterly frequency. For the case of No $\rho_1, \rho_2$, I re-calibrate the model to have $\beta_2 = 0.972, \epsilon = 0.96$.
For $\theta_1 = \theta_2 = 0.5$, I re-calibrate the model to have $\rho_2 = 0.615, \beta_2 = 0.967, \epsilon = 0.79$. 40
This result implies that increasing trade openness may help limit the damage to the world during a default crisis. Additionally, next two columns show that varying the creditor country’s degree of home bias does not affect the model’s quantitative performance very much either, as long as the majority of consumption is still home goods.

3.6.6 Armington Elasticities of Substitution

Finally, I check the robustness of the elasticities of substitution between home and imported final goods for both countries. When the substitutability in the borrower country decreases ($\theta_2 = 0.45$), imports are less replaceable by domestic final goods in a downturn when terms of trade deteriorate. Therefore, the borrower country defaults slightly less frequently to avoid terms of trade deterioration. Its imports become not as volatile as in the baseline, and exports and trade balances become more responsive to business cycles. Meanwhile, a larger terms of trade deterioration in the borrower country is required upon sovereign default to decrease the share of imports in consumption due to home bias. In the last column, when I change the two countries’ elasticities of substitution both to 2 (Backus, Kehoe, and Kydland, 1994) and recalibrate the entire model, it generates consistent results as before.

Summing up, this sensitivity analysis shows that although the model’s statistical moments vary somewhat as I change key parameters, the main quantitative and qualitative findings are robust to these changes. The model produces a sharp decline in terms of trade upon default, a high average bond spread, a negative correlation between trade balances and GDP, and other data-consistent correlations between various trade flows and GDP or bond spreads. This analysis also indicates that increasing trade openness (i.e., reducing consumption home bias) can be an effective policy in reducing terms of trade volatility, default frequency, and default-triggered welfare losses.

4 Conclusion

This paper proposes a two-country open-economy model of sovereign default, including both production and risk-averse agents. Its quantitative predictions are consistent with observed empirical regularities around Mexico’s sovereign defaults and along its business cycles. The model contributes to the sovereign default theory with its endogenous trade flows, as well as bilateral trade declines and trade balance reversal upon default. The paper also contributes to the literature by endogenizing terms of trade and real exchange rate that interact with default risk and that deteriorate reducing income and trade upon
default.

The model features a terms-of-trade amplification channel that links sovereign default (risk) with trade and income. As a country borrows more and more, its default risk and interest rate increases, income declines, and terms of trade deteriorates, which in turn reduces the borrower country’s income and raises its default risk. Once the borrower does default, its income further declines, trade balances reverse to be positive, and terms of trade and real exchange rate deteriorate sharply. This real depreciation then again takes another toll on the defaulter’s income and trade values. This term-of-trade channel produces a novel feedback loop among the borrower’s sovereign default (risk and occurrences), income, and trade.

The model results are consistent with three important stylized facts about emerging markets’ business cycles and sovereign defaults. First, it delivers countercyclical trade balances and procyclical bilateral trade flows over business cycles. Second, it produces countercyclical bond spreads with a data-consistent average. Third, this model accounts for sharp deterioration of terms of trade and real exchange rate, trade balance improvements, and bilateral trade declines upon default. Moreover, the model does not need an exogenous output loss following a sovereign default, but endogenously generates GDP losses, partially from real depreciation, partially from production activity decline, as in the data.

This model also predicts long-lasting welfare losses for the creditor country, but relatively short-lived welfare losses for the borrower country during and after a sovereign default. Furthermore, this paper offers interesting perspectives on how default penalty ($\epsilon$) and consumers’ taste of foreign imports ($\rho_2$, $\theta_2$) may interact with default incentives. Surprisingly, default frequency is only slightly affected by these factors. The most important element impacting default frequency lies in the borrower’s degree of patience ($\beta_2$). However, a higher tendency to consume foreign imports (lower $\rho_2$, lower $\theta_2$) or a higher default penalty through intermediate goods exports (lower $\epsilon$) does allow the borrower country to sustain a higher average debt-to-GDP ratio. But the former also reduces the creditor and borrower’s total welfare losses upon default and slightly limits default frequency, while the latter does the opposite. This implies that increasing trade openness could be an effective policy to deal with sovereign defaults.

It is worth noting that the story behind this model has the borrower country exporting intermediate goods. This allows sovereign defaults to interact with vertical integration. However, exporting intermediate goods is not necessarily the only story that can be told by this model. The model setup here is sufficiently versatile to be compatible with other
stories that are also consistent with empirical observations. For instance, instead of borrower country 2 exporting intermediate goods, it could receive FDI $k^*_1$ from creditor country 1 to produce final goods 1 and exports them back to country 1. When the borrower country defaults, the FDI declines, triggering changes in trade and the real exchange rate. Or, as in Mendoza and Yue (2012), borrower country 2 could instead import intermediate (capital) goods from creditor country 1; and a default causes a decline in such imports. The simple model adjustments needed for those alternative stories are accounting for GDP, trade balances, and trade flows.

This line of research into the connections between default, income, trade, and exchange rate is far from complete. For instance, it would be interesting to study the case when both countries suffer from productivity shocks. Valid questions to ask include: how are the shocks transmitted across countries and how are the risk shared in a sovereign default model with trade and terms of trade? In particular, this model, with risk-averse investors, has the potential to explain why risk sharing worsens for emerging markets after financial integration (Bai and Zhang, 2012). Moreover, introducing more labor market dynamics and exchange rate regimes (see Na, Schmitt-Grohe, Uribe, and Yue, 2014) are also promising subjects for future research.

References


Fuentes and Saravia (2010) find that a default event can reduce FDI inflows by 72 percent.


Appendix: Data Sources

GDP value (in USD) and volume (in index), trade value (in USD) and volume (in index), trade as a share of GDP, terms of trade, real exchange rate, and consumption are from the IMF and the World Bank. Real Effective Exchange Rates for Mexico come from FRED maintained by the Federal Reserve Bank of St. Louis. Mexico’s domestic capital stock is calculated by the author from combined data from FRED and the IMF.

I use Mexico’s treasury bill rates from the IMF as its sovereign bond interest rates, and calculate the bond spreads with the U.S. government bond interest rates (FRED). Mexico’s external public debt is from the combined sources of Mexico’s Secretary of Finance and Public Credit and the IMF.

The frequency of defaults over the long term is calculated with information from Reinhart (2010). Sample default episodes are based on Reinhart (2010), the treatment dates from the Paris Club, and Mendoza and Yue (2012). Slight date adjustments according to GDP fluctuations have been made with regard to the Paris Club dates to reflect delayed treatments after defaults. The results are not sensitive to the default date specifications.

The annual data for the intermediate goods exports of multiple countries (see Table 6) come from the World Bank (WITS). The quarterly data for Mexico’s intermediate goods exports is from Mexico’s National Institute of Statistics and Geography (INEGI). It is also cross-checked with Mexico’s annual intermediate goods exports data from the World Bank.

Mexico’s total hours worked (quarterly) comes from FRED, and its total employment comes from the combined sources of the World Bank (WDI), the IMF, and Mexico’s INEGI. Total FDI stock in Mexico is calculated using data from the OECD, U.S. Bureau of Economic Analysis (BEA), and the IMF. Furthermore, according to the UN’s 2013 report on Foreign Direct Investment in Latin America and Caribbean, about 4.4 jobs are created in Mexico for every 1 million USD invested from abroad during 2003-2013. Using this number, I compute Mexico’s FDI related employment according to its FDI stock data. Hence, Mexico’s domestic sector employment is its total employment minus its FDI employment. In calibration, the data for labor share in production is from the OECD. Canadian employment is from Statistics Canada.
<table>
<thead>
<tr>
<th>Country</th>
<th>Event</th>
<th>Available Data</th>
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<tbody>
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<td>Brazil</td>
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<td>Iceland</td>
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<td>1998</td>
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<td>Uruguay</td>
<td>1990</td>
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</tbody>
</table>

Note: Intermediate goods export value, volume, and share of GDP are also available for Brazil and Peru since 1988, after their sovereign defaults. The dating of external debt defaults mainly comes from Mendoza and Yue (2012) and Reinhart (2010). Other dating sources for cross-checking include Laeven and Valencia (2008), Paris Club data.