STICKY PRICES AND COSTLY CREDIT*

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Abstract
We develop a framework where money and credit are alternative payment instruments, use it to endogenize price stickiness, and confront the data. Frictions generate equilibria with price dispersion, where sellers set nominal terms that they may keep fixed when aggregate conditions change. Buyers use cash and credit, with the former (latter) subject to inflation (transaction) costs. We provide strong analytic results and novel closed-form solutions for money demand. Calibrated versions of the model match pricing data well, generating realistic durations, plus large average, many small and many negative changes, while staying consistent with macro and micro data on money and credit. Policy implications are discussed.

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1 Introduction

This paper has two intimately related goals: (i) construct a framework where money and credit serve as alternative payment instruments; (ii) pursue within this setting a theory of endogenously sticky prices that we can take to the data. We build on the analysis of price dispersion in frictional goods markets by Burdett and Judd (1983), integrated with the general equilibrium monetary model of Lagos and Wright (2005). This environment generates monetary equilibria with a distribution of prices, and that means sellers can set prices in nominal terms that they may keep fixed when aggregate conditions change. Buyers may use both cash and credit because the former is subject to the inflation tax while the latter involves fixed or variable transaction costs, making the choice of payment method nontrivial and, we think, realistic. Also, introducing costly credit allows us to avoid an indeterminacy of equilibrium than plagues previous attempts to study similar models, as we now explain.

To begin, consider Diamond (1971), where each seller (firm) of an indivisible good posts a price \( p \), then homogeneous buyers (households) sample one at a time until finding \( p \) below their reservation price \( p^* \). Clearly, for any seller, the profit maximizing strategy is \( p = p^* \). Hence, there is a single price in the market. The Burdett-Judd model makes one change in Diamond’s specification: buyers sometimes sample multiple prices simultaneously, and when they do they obviously choose the lowest. This implies there cannot be a single \( p \), nor even a set of sellers with positive measure charging the same \( p \), since that would leave open a profitable deviation to \( p - \varepsilon \) (see fn. 8). In fact, one can compute explicitly the Burdett-Judd distribution \( F(p) \), where any \( p \) in the support \( \mathcal{F} \) yields the same profit, because lower-price sellers earn less per unit, but make it up on the volume by making sales with a higher probability.

If one embeds Burdett-Judd in a monetary economy, it makes sense for sellers to post prices in dollars. Then, if the money supply \( M \) increases, \( F(p) \) shifts so
that the real distribution stays the same. Some firms can keep $p$ fixed, however, even though they are allowed to adjust whenever they like at no cost, and hence sticky prices can emerge. While real prices fall for sellers that stick to their $p$, profits do not fall because sales increase. This is similar to Head et al. (2012), but that paper has a technical problem: the combination of indivisible goods and posting in monetary economies entails an indeterminacy (multiplicity) of equilibria as discussed in Section 2. Hence, that model uses divisible goods, but then another problem pops up, since it is not obvious just what firms should post. Head et al. (2012) simply assume linear menus – a seller posts $p$ and lets buyers who show up choose any $q$ they like as long as they pay $pq$ – but there is no reason to think that this maximizes profit.

Giving agents a choice between cash and credit eliminates the indeterminacy that obtains with indivisible goods and posting and thus avoids the need for restrictions like linear pricing. Intuitively, consider a fixed cost of using credit. Random matching implies consumers might spend a little or might spend a lot, and they tend to use credit in larger transactions. Holding more cash reduces the probability of needing credit, which delivers a well-behaved money demand function and a unique stationary monetary equilibrium. Thus, we can revert to the original Diamond-Burdett-Judd specification, with indivisible goods, and model endogenous sticky prices with fewer “delicate” assumptions.\(^1\) Importantly, in contrast to theories where stickiness is imposed exogenously, changing $M$ is neutral (does not affect real output). The point is not about whether neutrality holds in reality; it is that price stickiness is neither necessary not sufficient for nonneutrality. However, there are reasons other than making this point to study money and credit: (i) there is a long tradition of trying to build models along these lines; (ii) it is arguably quite relevant for monetary policy analysis; and (iii) it allows us to confront data in novel ways.

\(^1\)We do not take a stand on whether divisible or indivisible goods are more “realistic,” as that depends on the context, but would argue that indivisibility is an assumption on the physical environment and hence less “delicate” than a restriction on pricing strategies.
Head et al. (2012) calibrate their model to match the empirical price-change
distribution, then show it also matches other facts reasonably well, including the
average duration between price changes. We perform a similar but more disci-
plined exercise, by calibrating to observables like average price duration, fractions
of cash and credit purchases in the micro data, and money demand observations
in the macro data, then asking how the model matches the price-change distrib-
ution. One specification involves a fixed-cost model, consistent with some earlier
work. It performs reasonably well in terms of money demand, and can match
key facts in the pricing data, including realistic durations, large average price
changes, many small changes, some negative changes and a decreasing hazard.
However, it cannot do this while matching the shares of money and credit in the
micro data, for reasons explained below. Hence, we also consider a variable-cost
model, which in some respects is more tractable and more flexible. This version
can simultaneously match the pricing, money demand and payment data.

The rest of the paper is organized as follows. Section 2 reviews the literature.
Section 3 describes our environment. Sections 4 and 5 characterize equilibrium
with a fixed and a variable cost of credit, respectively. Section 6 presents the
quantitative analysis. Section 7 discusses an extension and some policy implica-
tions. Section 8 concludes.

2 Literature

Many sticky-price papers follow Taylor (1980) or Calvo (1983) by only letting
sellers adjust $p$ at certain points in time. Others follow Rotemberg (1982) or
Mankiw (1985) by letting them change only at a cost. We allow sellers to change
any time for free. A few papers push imperfect-information or rational-inattention
theories, including Mackowiak and Wiederhold (2009), who provide references to
other work. While we are not against these devices, the concentration here is
on search, mainly because when Burdett and Menzio (2014) combine this with
menu costs, the analysis is more complicated and delivers similar results, and because they find the majority of price dispersion in the data (about 70%) is due to search. Hence we abstract from menu costs and related devices.\footnote{Nonmonetary search models with menu costs, where prices are sticky in unit of account, as in Benabou (1988,1992) or Diamond (1993), are special cases of Burdett and Menzio (2014).}

The literature on Burdett-Judd pricing is large, including many labor-market applications following Burdett and Mortensen (1998). In monetary economics, prior to Liu (2010), Wang (2011) and Head et al. (2012) putting Burdett-Judd in Lagos and Wright (2005), Head and Kumar (2005) and Head et al. (2010) put it in the related model of Shi (1997). Alternatives models of price dispersion include Albrecht and Axel (1984) and Diamond (1987), where buyers differ not in terms of what they observe, but in their intrinsic (e.g., preference) type. A monetary version in Curtis and Wright (2004) generically delivers a two-point $p$ distribution, for any number of types, which is less useful for our purposes. Also, as in Shi (1995) or Trejos and Wright (1995), and diametrically from us, that paper assumes goods are divisible while money is not, which limits the applicability for many policy and empirical issues.

Caplin and Spulber (1987) and Eden (1994) analyze models that are similar in spirit yet also very different. In addition using Burdett-Judd, we build on the foundation for monetary economics in Lagos-Wright, which builds on Kiyotaki and Wright (1989,1993), Kochelakota (1998), Wallace (2001) etc. See Wallace (2010), Williamson and Wright (2010), Nosal and Rocheteau (2011) or Lagos et al. (2014) for surveys of this approach, sometimes called New Monetarist Economics, trying to provide relatively clean descriptions of how agents trade and specifications for specialization, commitment and information that give money and related institutions essential roles. We adopt this approach because we believe it is desirable to analyze monetary phenomena in environments that are explicit about the frictions that money-like institutions are meant to ameliorate.\footnote{If one prefers short-cuts, similar points can also be made in, say, CIA or MUF models, but we find it natural to use monetary theory based on search since search is also the friction that drives price dispersion and stickiness.}
There is much empirical work on price adjustment. Campbell and Eden (2014) report that in grocery-store data the average duration between price changes is 10.3 weeks, but we do not want to focus exclusively on groceries. In BLS data, Bils and Klenow (2005) find at least half of prices last less than 4.3 months, or 5.5 months excluding sales. Klenow and Kryvtsov (2008) report durations from 6.8 to 10.4 months, while Nakamura and Steinsson (2008) report 8 to 11 months excluding substitutions and sales. These papers also find large fractions of small and negative price changes, plus some evidence of a decreasing hazard. Eichenbaum et al. (2011) report a duration of 6 months for “reference” prices, those most often quoted in a quarter (presumably to avoid, e.g., recording Saturday Night Specials as two changes in a week). Cecchetti (1986) finds durations for magazine prices from 1.8 months to 14 years, while Carlton (1986) finds durations for wholesale prices ranging from 5.9 months for household appliances to 19.2 for chemicals, and also finds many small changes, with 2/3 below 2%.⁴

An issue for simple Mankiw-style models is that average price changes are fairly big, suggesting high menu costs, but there are also many small changes, suggesting low menu costs. Midrigan (2011) accounts for this by having firms sell multiple goods, and paying a cost to change one price lets them change the rest for free. This is reasonable, but we think is still worth considering alternatives. Our baseline model can account for realistic durations, large average changes, many small changes and many negative changes. It also implies pricing behavior depends on inflation, which is an obvious problem for standard Calvo models. It also generates price dispersion at low, or even zero, inflation, consistent with evidence (e.g., Campbell and Eden 2014), while dispersion arises in many models

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⁴More empirical work is surveyed by Klenow and Malin (2010). In conversations with people in the area, they agree on the following list of facts: (1) Prices change slowly, although durations vary across studies. (2) The frequency and size of price changes vary across goods. (3) Two sellers changing at the same time do not typically change to the same price. (4) About 1/3 of changes are negative. (5) Hazard rates are slightly declining with duration. (6) There are many small (below 5%), and many big (above 20%) changes. (7) The frequency, size and fraction of negative changes vary with inflation. (8) There is price dispersion even at low inflation.
only due to inflation. Based on all of this, we think our model should be part of the conversation on sticky prices and the implications for policy.

In terms of interpretations and implications, we think it is fair to highlight Ball and Mankiw (1994), who champion the position that stickiness leads to non-neutrality: “We believe that sticky prices provide the most natural explanation of monetary nonneutrality since so many prices are, in fact, sticky.” Moreover: “As a matter of logic, nominal stickiness requires a cost of nominal adjustment.” Golosov and Lucas (2003) similarly say that “menu costs are really there: The fact that many individual goods prices remain fixed for weeks or months in the face of continuously changing demand and supply conditions testifies conclusively to the existence of a fixed cost of repricing.” We agree that prices are in fact sticky, but the other statements are simply incorrect: menu costs may or may not be important, and money may or may not be neutral, but logically sticky prices imply neither menu costs nor nonneutrality.

The paper may also be considered a contribution to pure monetary theory. Our environment delivers nice money demand functions expressing real balances in terms of the nominal interest rate. While the details are different, the economics is closely related to classic results in Baumol (1952), Tobin (1956), Miller and Orr (1966) and Whalen (1966), although those are partial-equilibrium models, or, more accurately, decision-theoretic analyses of how to manage one’s money assuming that it (as opposed to credit, barter or something else) must be used for transactions. While such models are still being applied to good effect (e.g., Alvarez and Lippi 2009), we admit to a preference for theories where the roles of money and credit are not taken for granted, and as argued by Wallace (2013), we think this is relevant for understanding important aspects monetary policy, as we try to explain below.⁵

⁵ Also, incorporating Burdett-Judd pricing with indivisible goods is a further step in understanding the New Monetarist approach. As discussed in the surveys cited above, previous analyses in this context have used Nash, Kalai or strategic bargaining, price posting with random or directed search, competitive price taking, auctions and pure mechanism design.
The above-mentioned multiplicity of monetary equilibria in models with indivisible goods and posting occurs in a series of papers spawned by Green and Zhou (1998). Jean et al. (2010) provide citations and further discussion, but we can give the following intuition: If all sellers post \( p \) then buyers’ best response is to bring \( m = p \) dollars to the market as long as \( p \) is not too big. If all buyers bring \( m \) then sellers’ best response is post \( p = m \) as long as \( m \) is not too small. Hence, any \( p = m \) in some range is an equilibrium, and payoffs depend on the one we select. Sometimes this multiplicity can be refined away, but then the outcome is to be sensitive to details, like who moves first. Head et al. (2012) avoid this because, even if buyers think all sellers post \( p \), they need not bring \( m = p \) when \( q \) is divisible, but again that leads to other issues. Our alternative is instead based on the venerable idea that credit is costly.

Before proceeding, we mention heterogeneity. As is well understood in other applications (e.g., Mortensen and Pissarides 1999), if firms are homogeneous as they are here, theory does not pin down which one charges which \( p \), but only the distribution \( F(p) \). With heterogeneity, however, low cost firms prefer low \( p \) since they are more interested in high volume. Still, for any subset of firms with the same marginal cost, it does not matter which one posts which \( p \). Hence, heterogeneity eliminates neither dispersion nor stickiness within sets of similar sellers. This is especially relevant for retail, where the marginal cost is the wholesale price, because even if a few retailers get better deals – e.g., Kmart has a quantity discount – many others face the same wholesale terms. Irrespective of fixed costs, wages etc., these sellers are homogeneous for our purposes, and our theory of price dispersion and price stickiness applies to them without qualification.

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3 Environment

Each period in discrete time has two subperiods: first there is a decentralized market, called BJ for Burdett-Judd; then there is a frictionless centralized market, called AD for Arrow-Debreu. There is a set of firms interpreted as retailers with measure 1, and a set of households with measure $\bar{b}$. Households consume a divisible good $x_t$ in AD and an indivisible good $y_t$ in BJ, but supply labor $\ell_t$ only in AD. The BJ good is produced at cost $c$ by firms. As agents are anonymous in the BJ market, they cannot use credit, unless they access a technology to authenticate identities and record transactions at a cost. By incurring the cost, households can get BJ goods in exchange for commitments to deliver $d_t$ dollars in the next AD market; otherwise cash is required at point of sale.\(^7\) Below we consider cases where the cost of credit is fixed at $\delta$ and where it is proportional $\tau d_t$. The money supply per capita evolves according to $M_{t+1} = (1 + \pi) M_t$, with changes occurring in AD via lump-sum transfers.

Household utility within a period is $U(x_t) + u \mathbf{1}(y_t) - \ell_t$, where $U'(x_t) > 0 > U''(x_t)$, $u > 0$ is a parameter and $\mathbf{1}(y_t)$ is an indicator function giving 1 iff the indivisible BJ good is consumed. Let $\beta = 1/(1 + r)$ be a discount factor between today’s AD market and tomorrow’s BJ market, with $r > 0$. We impose $\pi > \beta - 1$, where in stationary equilibrium $\pi$ is the inflation rate, and the nominal interest rate is given by the Fisher equation $1 + i = (1 + \pi)(1 + r)$. Notice $i > 0$, and the Friedman rule is the limiting case $i \to 0$. As usual, $1 + i$ is the amount of money agents require in the next AD market to give up a dollar in the current AD market, whether or not such trades occur in equilibrium. Let $x_t$ be AD numeraire, an assume it is produced one-for-one with $\ell_t$, so the wage is 1 (purely for simplicity; the results below do not dependent on this). The AD price of money in numeraire is $\phi_t$, and $1/\phi_t$ is the nominal price level.

\(^7\)While the cost is paid by households, it is not hard to show the allocation is identical if it is instead paid by firms, as in elementary tax-incidence theory. Also, it does not matter if debt due in the frictionless AD market is denominated in dollars or numeraire.
Firms enter the BJ market for free, but households must pay cost $k$, which in general determines participation $b_t \leq \bar{b}$. However, in the baseline model $k = 0$, so $b_t = \bar{b}$. Firms use BJ profits to buy AD goods, over which they have linear utility, but not much changes if instead they disburse profits to households as dividends.

Each firm posts a price taking as given household behavior and the CDF of other firms’ prices, $F_t(p)$, with support $\mathcal{F}_t$. Every household in BJ randomly samples $n$ firms – i.e., sees $n$ independent draws from $F_t(p)$ – with probability $\alpha_n = \alpha_n(b_t)$, generally depending on the buyer-seller ratio, or market tightness, $b_t$. For our purposes it suffices to have $\alpha_1, \alpha_2 > 0$ and $\alpha_n = 0$ $\forall n \geq 3$, but this can be generalized easily enough (see, e.g., Burdett et al. 2014).

### 3.1 Firms

Expected real profit for a firm posting $p$ at date $t$ is

$$
\Pi_t(p) = b_t \{\alpha_1(b_t) + 2\alpha_2(b_t) [1 - F_t(p)]\} (p\phi_t - c). \tag{1}
$$

Thus, net revenue per unit is $p\phi_t - c$, and the number of units is determined as follows: The probability a household contacts this firm and no other is $\alpha_1(b_t)$. Then the firm makes a sale. The probability a household contacts this firm plus another is $2\alpha_2(b_t)$, as this can happen in two ways, this one first and the other second or vice versa. Then the firm makes a sale iff it beats the other’s price, which occurs with probability $1 - F_t(p)$. This is all multiplied by tightness $b_t$ to convert buyer probabilities into seller probabilities. Profit maximization means every $p \in \mathcal{F}_t$ yields the same profit and no $p \notin \mathcal{F}_t$ yields higher profit.

As is standard in this kind of model, there is a unique outcome, and it has $F_t(p)$ continuous with $\mathcal{F}_t$ an interval.\textsuperscript{8} Taking as given for now the upper limit of the interval $\bar{p}_t$, profit from any $p \in \mathcal{F}_t$ must equal the profit from $\bar{p}_t$, which is

\textsuperscript{8}There cannot be a mass of firms posting the same $p$ in equilibrium, because any one of them would have a profitable deviation to $p - \varepsilon$, since he would lose only $\varepsilon$ per unit and make discretely more sales by undercutting others at the mass point. Similarly, if there were a gap between say $p_1$ and $p_2 > p_1$, a firm posting $p_1$ can deviate to $p_1 + \varepsilon$ and earn more per unit without losing sales.
given by

$$\Pi_t(\bar{p}_t) = b_t \alpha_1 (b_t) (\bar{p}_t \phi_t - c). \tag{2}$$

Equating (1) and (2), for every \( p \in \mathcal{F}_t \), we can solve for

$$F_t(p) = 1 - \frac{\alpha_1 (b_t)}{2 \alpha_2 (b_t)} \frac{\phi_t \bar{p}_t - \phi_t p}{\phi_t p - c}. \tag{3}$$

This is the BJ distribution. Notice

$$F_t'(p) = \frac{\alpha_1 (b_t)}{2 \alpha_2 (b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^2} > 0 \tag{4}$$

$$F_t''(p) = -\frac{\alpha_1 (b_t)}{\alpha_2 (b_t)} \frac{\phi_t \bar{p}_t - c}{(\phi_t p - c)^3} < 0. \tag{5}$$

Also, since the lower limit of \( \mathcal{F}_t \) satisfies \( F(\underline{p}_t) = 0 \),

$$\underline{p}_t = \frac{\alpha_1 (b_t) \phi_t \bar{p}_t + 2 \alpha_2 (b_t) c}{\phi_t [\alpha_1 (b_t) + 2 \alpha_2 (b_t)]}. \tag{6}$$

As regards \( \bar{p}_t \), there are different possibilities, discussed below, but for now it is undetermined. To translate from dollars to numeraire, let \( q_t = \phi_t \bar{p}_t \) and write the distribution of real BJ prices as

$$G_t(q) = F_t(\phi_tp) = 1 - \frac{\alpha_1 (b_t)}{2 \alpha_2 (b_t)} \frac{\bar{q}_t - q}{q - c}. \tag{7}$$

### 3.2 Households

We focus on stationary equilibrium, where real variables are constant and nominal variables grow at rate \( \pi \). This makes it convenient to frame the household problem in real terms. The state variable in AD is net worth, \( A = \phi m - \phi d - \chi(d) + \bar{A} \), where \( \phi m \) and \( \phi d \) are real money balances and debt carried over from the previous round of BJ trade, \( \chi(d) \) is the cost of using credit, and \( \bar{A} \) is any other source of purchasing power including government transfers. Since preferences are linear in \( \ell \), we can assume with no loss in generality that all obligations are settled in AD, so households start BJ debt free. The state variable in BJ is real balances carried into that market, \( z \). The AD and BJ value functions are \( W(A) \) and \( V(z) \).
For a household that decides to enter the next BJ market,

\[ W^1(A) = \max_{x, \ell, z} \{ U(x) - \ell + \beta V(z) \} \quad \text{st} \quad x = A - k + \ell - (1 + \pi) z, \]

where \( k \) is the entry cost, and due to inflation the cost of having \( z \) next period in terms of current numeraire is \( (1 + \pi) z \). Eliminating \( \ell \), we get

\[ W^1(A) = A - k + U(x^*) - x^* + \max_z \beta O_i(z), \] (8)

where \( U'(x^*) = 1 \) and the objective function for the choice of \( z \) is \( O_i(z) \equiv V(z) - (1 + i) z \), with \( i \) again given by the Fisher equation. For a household that decides to skip the next BJ market,

\[ W^0(A) = A + U(x^*) - x^* + \beta W(A). \] (9)

Then \( W(A) \equiv \max \{ W^1(A), W^0(A) \} \). As in Lagos and Wright (2005), \( W'(A) = 1 \) and the choice of \( z \) does not depend on \( A \), which keeps things very tractable.

4 Fixed Cost

Consider first a fixed cost \( \delta \), where \( \delta < u - c \). This guarantees that when \( k = 0 \) there exists a nonmonetary equilibrium with \( b = \bar{b} \), where all households participate and all transactions use credit – basically, the original Burdett-Judd equilibrium. Now, to characterize the distribution \( G_t(q) \), we need to determine \( \bar{q} \). There are different possibilities for monetary equilibrium, depending on real balances. First, \( 0 < z < u - \delta \) implies the highest price is \( \bar{q} = u - \delta > z \). In this case buyers use credit for \( q \in (z, \bar{q}] \) and cash otherwise. Second \( u - \delta < z < u \) implies \( \bar{q} = z \). In this case buyers always use cash. Finally, \( u < z \) implies \( \bar{q} = u \), but this never happens since no one carries more money than they ever need. There is also the possibility of nonmonetary equilibrium, where \( z = 0 \) and \( \bar{q} = u - \delta \).

\(^9\)This follows because no buyer would pay more than \( u - \delta \), and if \( \bar{q} < u - \delta \) the highest price firm would have a profitable deviation to \( u - \delta \).
With a fixed cost \( \delta \), net worth in AD is
\[ A = \phi m - \phi d - \delta \mathbf{1}(d) + \bar{A}, \]
where \( \mathbf{1}(d) \) is again an indicator function giving 1 iff \( d > 0 \). This implies\(^{10}\)
\[ V(z) = W(z + \bar{A}) + [\alpha_1(b) + \alpha_2(b)] \{ u - E_Jq - [1 - J(z)] \delta \}, \quad (10) \]
where \( E_Jq = \int_q \bar{q} dJ(q) \) and \( J(q) \) is the CDF of transaction prices,
\[ J(q) = \frac{\alpha_1(b) G(q) + \alpha_2(b) [1 - [1 - G(q)]^2]}{\alpha_1(b) + \alpha_2(b)}. \quad (11) \]
Notice \( J(q) \) differs from \( G(q) \) because buyers seeing multiple prices pick the lowest. From (10) it is clear that the benefit of holding more \( z \) is that it reduces the probability of needing to use credit, \( 1 - J(z) \).

For the BJ entry decision, it is easy to check \( \Phi \equiv (1 + r) [W^1(A) - W^0(A)] \) is independent of \( A \) and satisfies
\[ \Phi = [\alpha_1(b) + \alpha_2(b)] \{ u - E_Jq - [1 - J(z)] \delta \} - \kappa - iz, \quad (12) \]
where \( \kappa = k/\beta \). The first term is the expected return to participating in BJ, while \( \kappa + iz \) is costs. Then the equilibrium entry condition is
\[ b = \bar{b} \implies \Phi \geq 0; \quad b = 0 \implies \Phi \leq 0; \quad b \in (0, \bar{b}) \implies \Phi = 0. \quad (13) \]

### 4.1 Equilibrium

There are three cases: a nonmonetary equilibrium NME, where \( z = 0 \), so only credit is used in BJ; a mixed monetary equilibrium MME, where \( 0 < z < u - \delta \), so

\(^{10}\)To derive (10), first write
\[ V(z) = W(z) + \alpha_1(b) \int_q^z (u - q) dG_1(q) + \alpha_1(b) \int_z^{q} (u - q - \delta) dG_1(q) \]
\[ + \alpha_2(b) \int_q^z (u - q) dG_2(q) + \alpha_2(b) \int_{q}^{z} (u - q - \delta) dG_2(q), \]
where \( G_n(q) = 1 - [1 - G(q)]^n \) is the CDF of the lowest of \( n \) independent draws from \( G(q) \). The first term is the continuation value if a buyer does not trade. The second is the probability of meeting a seller with \( q \leq z \), so cash is used, times the expected surplus. The third is the probability of meeting a seller with \( q > z \), so credit is used, which has an additional cost \( \delta \). The last two terms are similar except the buyer meets two sellers. The rest is algebra.
cash and credit both used in BJ; and a pure monetary equilibrium PME, where \( u - \delta < z \), so only cash is used in BJ. In NME, the BJ prices must be described in terms of numeraire; in the other cases they can equivalently be described in numeraire or dollars. Formally, we have this:

**Definition 1** A stationary equilibrium is a list \( \langle G(q), b, z \rangle \) solving (7), (13) and the money demand problem in (8).

**Definition 2** A NME is one with \( z = 0 \) and \( \bar{q} = u - \delta \). A MME is one with \( u - \delta > z > 0 \) and \( \bar{q} = u - \delta \). A PME is an equilibrium with \( z > u - \delta \) and \( \bar{q} = z \).

These definitions involve only real variables, but in monetary equilibrium the nominal price distribution is \( F_t(p) = G(\phi_t p) \). Other variables can be computed, too, include AD consumption, which is \( x = x^* \), and labor supply \( \ell \), which comes from the budget equation, but we do not need these below. For now \( k = 0 \), so \( b = \bar{b} \) is fixed and we omit the argument from the meeting probabilities \( \alpha_1 \) and \( \alpha_2 \). Now consider these preliminary results:

**Lemma 1** For \( q < z < \bar{q} \), \( V(z) \) is smooth, with \( V'(z) > 0 \) and \( V''(z) < 0 \). For \( z < q \) or \( z > \bar{q} \), \( V'(z) = 1 \).

**Proof:** For \( z \in (q, \bar{q}) \),

\[
V'(z) = 1 + (\alpha_1 + \alpha_2) \delta J'(z) \quad (14)
\]
\[
V''(z) = (\alpha_1 + \alpha_2) \delta J''(z) \quad (15)
\]
by virtue of (10). Using (11), we have

\[
J'(q) = \frac{\alpha_1 G'(q) + 2\alpha_2 [1 - G(q)] G''(q)}{\alpha_1 + \alpha_2} > 0 \quad (16)
\]
\[
J''(q) = \frac{\alpha_1 G''(q) + 2\alpha_2 [1 - G(q)] G'''(q) - 2\alpha_2 G'(q)^2}{\alpha_1 + \alpha_2} < 0 \quad (17)
\]
where \( G \) is smooth by virtue of (7). The rest is obvious. ■
These results deliver a unique stationary MME, in contrast to the above-mentioned indeterminacy of stationary monetary equilibrium in models with indivisible goods, price posting and no credit. Indeed, \( V''(z) < 0 \) implies a unique \( z_i = \arg \max_{z \in [q, \bar{q}]} O_i(z) \) and it satisfies the FOC

\[
(\alpha_1 + \alpha_2) \delta J'(z_i) = i. \tag{18}
\]

However, we need to check \( z_i \in (q, \bar{q}) \) to get a MME. To begin, it is easy to check \( O'_i(z) = -i \forall z > \bar{q} \) and \( \forall z < q \). Then, let \( \hat{z}_i \) be the global maximizer of \( O_i(z) \), and let \( O_i^-(z) \) and \( O_i^+(z) \) be the left and right derivatives. We first check \( O_i^+(q) \).

If \( O_i^+(q) \leq 0 \) then \( \hat{z}_i = 0 \), as in the left panel of Figure 1. If \( O_i^+(q) > 0 \) we check \( O_i^-(\bar{q}) \). If \( O_i^-(\bar{q}) \geq 0 \) then either \( \hat{z}_i = 0 \) or \( \hat{z}_i = \bar{q} \), as in the center panel. If \( O_i^-(\bar{q}) < 0 \) then either \( \hat{z}_i = 0 \) or \( \hat{z}_i = z_i \), as in the right panel. This leads to the following results.

![Figure 1: Possible Types of Equilibria](image)

**Proposition 1** There exists a unique NME.

**Proof:** With fiat currency \( \phi = 0 \) is always self-fulfilling, so we simply set \( G(q) \) according to (7), as in the original Burdett-Judd model. This implies positive payoffs for buyers and sellers given the maintained assumption \( \delta < u - c \). ■
Proposition 2 There exists a unique MME iff

\[ \delta < \tilde{\delta} \equiv u - \frac{2\alpha_2^2 + 2\alpha_1\alpha_2}{2\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2} c \]  

and \( i \in (\hat{i}, \bar{i}) \), where \( \hat{i} = \delta\alpha_1^2/2\alpha_2 (u - \delta - c) \) and \( \bar{i} \in (\hat{i}, \infty) \).

Proof: From Figure 1, necessary and sufficient conditions for MME are: (i) \( O_i^-(\bar{q}) < 0 \); (ii) \( O_i^+(\bar{q}) > 0 \); and (iii) \( O_i(z_i) > O_i(0) \). Condition (i) is equivalent to \((\alpha_1 + \alpha_2) \delta J^-(\bar{q}) < i\). A calculation implies this holds iff \( i > \hat{i} \). Condition (ii) is equivalent to \((\alpha_1 + \alpha_2) \delta J^+(\bar{q}) > i\), which holds iff \( i < \bar{i} \) where

\[ \bar{i} = \frac{\delta (\alpha_1 + 2\alpha_2)^3}{2\alpha_1\alpha_2 (u - \delta - c)} > \hat{i}. \]  

Condition (iii) is equivalent to \((\alpha_1 + \alpha_2) \delta J(z_i) - iz_i > (\alpha_1 + \alpha_2) \delta J(0)\), which holds iff \( \Delta(i) > 0 \) where

\[ \Delta(i) = -ic + \frac{\delta (\alpha_1 + 2\alpha_2)^2}{4\alpha_2} - i^2 \delta^{\frac{3}{2}} \alpha_1^{\frac{3}{2}} \alpha_2^{-\frac{1}{2}} (u - \delta - c)^{\frac{3}{2}} \left( 2^{-\frac{1}{4}} + 2^{-\frac{1}{4}} \right). \]

Notice \( \Delta(0) > 0 > \Delta(\bar{i}) \) and \( \Delta'(i) < 0 \). Hence \( \exists! \bar{i} \) such that \( \Delta(\bar{i}) = 0 \), and \( \Delta(i) > 0 \) iff \( i < \bar{i} \). It remains to verify that \( \bar{i} > \hat{i} \), so that (i) and (iii) are not mutually exclusive. It can be checked that \( \bar{i} > \hat{i} \) iff \( \delta < \tilde{\delta} \). Hence a MME exist under the stated conditions. It is unique because \( \bar{q} = u - \delta \), which pins down \( G(q) \), and then \( \hat{z}_i = \arg \max_{z \in \{q, \bar{q}\}} O_i(z) \).

The simplicity of MME comes from households having a unique best response \( \hat{z}_i \) to \( G(q) \), and we do not have to worry about firms’ best response because \( \bar{q} = u - \delta \) and hence \( G(q) \) are independent of \( \hat{z}_i \) in MME (a big simplification over, e.g., Head et al. 2012). Although the interest below is on MME, for completeness, consider PME. As in the models mentioned above, there is a continuum of PME, but we can still provide sharp conditions for existence.

Proposition 3 PME exists iff either: \( \bar{\delta} < \delta < u - c \) and \( i < \hat{i} = \delta (\alpha_1 + \alpha_2)/(u - \delta) \); or \( \delta < \bar{\delta} \) and \( i < \hat{i} \).
\textbf{Proof}: From Figure 1, necessary and sufficient conditions for NME are: (i) $O_i^-(\bar{q}) > 0$; (ii) $O_i^+(\bar{q}) > 0$; and (iii) $O_i(\bar{q}) > O_i(0)$. Now (i) holds iff $i < \hat{i}$ and (ii) holds iff $i < \breve{i}$. Condition (iii) holds iff $i < \hat{i}$. For $\delta > \breve{\delta}$, it is easily checked that $\hat{i} < \breve{i}$, and $\hat{i} < \breve{i}$ by (20), so the binding condition is $i < \hat{i}$. For $\delta < \breve{\delta}$, it is easily checked that $\hat{i} > \breve{i}$, and $\hat{i} < \breve{i}$, so the binding condition is $i < \hat{i}$.

Figure 2: Different Equilibria in Parameter Space

Figure 2 partitions parameter space into three regions. For intermediate values of $(\delta, i)$, there is a MME where buyers use both money and credit because their costs are moderate. In this case, inserting the BJ distribution into the FOC for $\hat{z}_i$ and rearranging, we get the closed-form solution for money demand

$$\hat{z}_i = c + \left[\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2^2\right]^{1/3} i^{-1/3}.$$

This expresses real balances in terms of the cube-root of $1/i$, reminiscent of the money demand functions in Baumol (1952), Tobin (1956), Whalen (1966) or Miller and Orr (1966), except the first two get a square- rather than cube-root (we get something more like a square-root in Section 5 with a variable cost of credit). Although the models are very different, the results derive from similar economic forces.

In particular, consider the standard story behind Baumol-Tobin, where an agent sequentially incurs expenses requiring money, by assumption, and has a
fixed cost of rebalancing $z$. The decision rule compares $i$, the opportunity cost of holding cash, with the benefit of reducing the number of financial transactions, usually interpreted as trips to the bank. Buyers here make at most one transaction in BJ before rebalancing $z$ in AD, but the size of the transaction is random, and credit (spending beyond $z$) is costly. Still, they compare $i$, again the cost of carrying cash, with the benefit of reducing the likelihood of using costly financial transactions, which can also be interpreted as trips to the bank, although one might say buyers now go there for a loan and not to make a withdrawal.

Figure 3: Changing Nominal Price Density with Inflation

The nominal price distribution $F_t(p)$ is uniquely determined $\forall t$, but individual-firm price dynamics are not. Consider Figure 3, drawn for the calibrated parameters in Section 6. With $\pi > 0$, the density $F'_{t+1}$ is a right shift of $F'_t$. Firms with $p < p_{t+1}$ at $t$ (Region A) must reprice at $t + 1$, because, while $p$ maximized profit at $t$, it no longer does so at $t + 1$. But as long as the supports $F_t$ and at $F_{t+1}$ overlap, there are firms with $p > p_{t+1}$ at $t$ (Region B) that can keep the same $p$ at $t + 1$ without reducing profit, since they earn less per unit but make it up on the volume. Since equilibrium puts weak restrictions on individual price behavior, we consider two additional conditions: (i) we add a payoff-irrelevant tie-breaking rule; and (ii) we focus on symmetric equilibria.
According to (i), firms use strategies of the following class: if \( p_t \notin \mathcal{F}_{t+1} \) then \( p_{t+1}(p_t) = \hat{p} \) where \( \hat{p} \) is a new price; and if \( p_t \in \mathcal{F}_{t+1} \) then
\[
 p_{t+1}(p_t) = \begin{cases} 
 p_t & \text{with prob } \sigma \\
 \hat{p} & \text{with prob } 1 - \sigma 
\end{cases} \tag{22}
\]
so those that are indifferent stick with probability \( \sigma \) to their current \( p \).\(^{11}\) Then (ii) says that all changers pick a new \( \hat{p} \) from the same repricing distribution \( H_{t+1}(\hat{p}) \).

Given \( \sigma \) and \( F_t(p) \), the unique \( H_{t+1}(\hat{p}) \) generating the equilibrium \( F_{t+1}(p) \) is:
\[
 H_{t+1}(p) = \begin{cases} 
 F_t\left(\frac{p}{1+\sigma}\right) - \sigma[F_t(p) - F_t(\hat{p}_{t+1})] & \text{if } p \in [\hat{p}_t, \hat{p}_{t+1}] \\
 \frac{F_t\left(\frac{p}{1+\sigma}\right) - \sigma[F_t(\hat{p}_{t+1})]}{1-\sigma+F_t(\hat{p}_{t+1})} & \text{if } p \in [\hat{p}_t, \hat{p}_{t+1}] 
\end{cases} \tag{23}
\]

Different \( \sigma \) capture a range of price dynamics, and it is routine to compute statistics comparable to those studied in the empirical literature, including duration, the distribution of changes and the hazard.\(^{12}\)

5 Variable Cost

Now consider a variable cost of credit, so that \( \tau d \) replaces the fixed cost \( \delta \). One reason is that it constitutes a basic check on robustness. Second, one interpretation (not the only one) is that \( \tau \) is a proportional tax that can be avoided using cash. Third, a variable cost is in some respects easier. Fourth, it delivers a different but still nice money demand function. And finally, it avoids a technical problem that we waited until now to mention. As is well known from monetary theory, or economic models, more generally, with nonconvexities, it is sometimes desirable to use lotteries. Thus, a seller may want to post: “you can get my good for sure if you pay \( p \) using money or credit; if you pay \( p' < p \) then you get my good with probability \( \gamma = \gamma (p') \).” With a fixed cost, when a buyer with \( m = p - \varepsilon \) meets a seller posting \( p \), the buyer pays \( p - \varepsilon \) in cash, \( \varepsilon \) in credit, and

\(^{11}\)This is quite different from Calvo pricing, where some firms are desperate to change \( p \) but are simply not allowed; here they are indifferent and hence happy to randomize. The calibrations below actually have few indifferent firms changing \( p \).

\(^{12}\)See Head et al. (2012) for derivations of formulae for these statistics and (23).
\( \delta \) as a fixed cost. It can be preferable for the pair to trade only using cash, avoid the fixed cost, and have the seller deliver the good with probability \( \gamma < 1. \)

Now one could try to argue that lotteries are infeasible or unrealistic or otherwise unavailable, but that would be awkward, for us, because ruling out randomized exchange might be uncomfortably close to the linear menu restriction criticized in the Introduction. We imposed a fixed cost of credit above because this has a long history in the literature, including papers cited in fn. 6, but wanted to avoid the complication of introducing lotteries. While that setup still may be interesting, if ignoring lotteries is considered problematic, the variable cost version has no such technical problem.

The price distribution emerging from the firms’ problem is similar to the previous model. For households, as usual they never choose \( z \geq u \), and \( z < u \) now implies the surplus from a transaction at real price \( q > z \) is \( u - q - (q - z)\tau \). This leads to an upper bound \( \bar{q} = (u + z\tau)/(1 + \tau) > z \), and a lower bound

\[
q = \frac{\alpha_1 (u + z\tau) + 2\alpha_2 c (1 + \tau)}{(\alpha_1 + 2\alpha_2) (1 + \tau)}.
\]

Conveniently, the objective function \( O_i (z) \) is now differentiable even at \( \bar{q} \) and \( q \). Indeed, \( O_i (z) \) is smooth, strictly concave in \((q, \bar{q})\), and otherwise linear, with

\[
O_i'(z) = -i \forall z > \bar{q} \quad \text{and} \quad O_i'(z) = (\alpha_1 + \alpha_2)\tau - i \forall z < q.
\]

As shown in Figure 4, this implies there are only two types of equilibria, NME and MME.\(^{14}\)

If \( i > (\alpha_1 + \alpha_2)\tau \) then holding cash is too costly and we get a NME. In order to guarantee positive profits for firms, we need \( \bar{q} = u/(1 + \tau) \geq c \), implying \( \tau \leq u/c - 1 \). In this case, households pay with credit and get positive surplus from BJ trade. However, still focusing on NME, the BJ shuts down if \( \tau > u/c - 1 \). Similarly, if \( i < (\alpha_1 + \alpha_2)\tau \) we get a unique MME where the choice of real balances satisfies the FOC, \( i = \tau (\alpha_1 + \alpha_2)[1 - J(\hat{z}_i)] \). Inserting \( J(z) \), we arrive at the inverse

---

\(^{13}\)See e.g. Berentsen et al. (2002) for an analysis of lotteries in simple monetary models. To be clear, here it is not the indivisibility of the BJ good that creates a role for lotteries, but nonconvexity in the transaction cost.

\(^{14}\)Intuitively, there cannot be a PME because buyers always want to use at least a little credit in this model.
money demand function

\[ i = \frac{\alpha_1^2 \tau (u - \hat{z}_i) [(1 + 2\tau) (\hat{z}_i - c) + u - c]}{4\alpha_2 (1 + \tau)^2 (\hat{z}_i - c)^2}, \]  
(24)

which is different from the result with a fixed cost, but equally nice.

Define the RHS of (24) as \( \Lambda(\hat{z}_i) \), and note that \( \Lambda(u) = 0, \lim_{\hat{z}_i \to c} \Lambda(\hat{z}_i) = \infty \), and \( \Lambda'(\hat{z}_i) < 0 \). So for any \( i \in (0, \infty) \), there is a unique \( \hat{z}_i \) satisfying (24), and \( \partial \hat{z}_i / \partial i < 0 \). We know that \( \hat{z}_i < \hat{q} \) iff \( \hat{z}_i < u \), and still need to guarantee \( \hat{z}_i > \hat{q} \), which one can check holds iff \( i < \tau (\alpha_1 + \alpha_2) \). Instead of the inverse demand curve, one can also rewrite (24) as a quadratic in \( (\hat{z}_i - c) \) and solve for

\[ \hat{z}_i = c + \frac{(u - c) \left[ \tau + (1 + \tau) \sqrt{1 + 4\alpha_2 i / \alpha_1^2 \tau} \right]}{1 + 2\tau + 4\alpha_2 (1 + \tau)^2 i / \alpha_1^2 \tau}. \]  
(25)

This is similar to a classic square-root rule except \( i \) also appears in the denominator.

Finally, to guarantee that households get positive surplus from BJ trade in MME, we need

\[ \Phi = (\alpha_1 + \alpha_2) [u + \tau \hat{z}_i - (1 + \tau) \mathbb{E}_J q] - i \hat{z}_i > 0. \]  
(26)

Substituting \( \mathbb{E}_J q \) and (26) we reduce this to

\[ \Phi_{var} = \alpha_2 [u + \tau \hat{z}_i - c (1 + \tau)] - i \hat{z}_i > 0. \]
Since $i \hat{z}_i$ increases with $i$, $\Phi$ monotonically decreases with $i$. Define $i_v$ as the rate that solves $\Phi = 0$, and note that (26) holds for all $i < i_v$. This proves:

**Proposition 4**  With a variable cost of credit $\tau$, there exists a unique MME iff $0 < i < \min \{\tau (\alpha_1 + \alpha_2), i_{\text{var}}\}$.

![Figure 5: Different Equilibria in Parameter Space: Variable Cost](image)

In Figure 5, for large $(\tau, i)$ the BJ market shuts down. For large values of $i$ but moderate values of $\tau$ there is a nondegenerate NME, and households in BJ use only credit. And a unique MME exists for large values of $\tau$ but moderate values of $i$. When the cost of using money is not too high, households use both money and credit. Notice that credit is always used at high prices, since $\bar{q} > \hat{z}_i \forall i$, and the maximum amount of debt $\bar{q} - \hat{z}_i$ increases with $i$. On the other hand, one can show that $\hat{z}_i$ increases with $\tau$ since $\partial \Lambda (\hat{z}_i) / \partial \tau > 0$, and $\bar{q} - \hat{z}_i$ decreases with $\tau$, so households hold less credit debt as the cost of using credit increases. As $\tau$ gets bigger, $\bar{q}$ converges to $\hat{z}_i$, and households eventually stop using credit.

6 **Quantitative Results**

While one can in principle examine $F_t(p)$ directly, here we study the distribution of changes, $(p_{t+1} - p_t)/p_t$, because that is what is emphasized in the literature.
For preferences, again, we use \( \log(x) + u \mathbf{1}(y) - \ell \) where \( u \) governs the trade off between the BJ and AD goods.\(^{15} \)

Focusing on MME, a key statistic for our purposes is the fraction of monetary transactions, \( J(\hat{z}_i) \). Another is the BJ markup, defined using posted prices by \( \mathbb{E}_G q/c \). Also important is an empirical notion of money demand, which following Lucas (2000) and others, we take to be \( L(i) = M/PY = \hat{z}_i/Y \), with \( Y = x + (\alpha_1 + \alpha_2) \mathbb{E}_J q \) real output.

In MME, with \( \bar{q} = u - \delta \) and \( q = [\alpha_1 (u - \delta) + 2\alpha_2 c] / (\alpha_1 + 2\alpha_2) \), the real posted-price and transaction-price distributions implied by the model are

\[
G(q) = 1 - \frac{\alpha_1}{2\alpha_2} \left( \frac{u - \delta - q}{q - c} \right), \\
J(q) = 1 - \frac{\alpha_2^2 (u - \delta - q) (u - \delta + q - 2c)}{4\alpha_2 (\alpha_1 + \alpha_2) (q - c)^2}.
\]

Then (21) implies the fraction of monetary transactions is

\[
J(\hat{z}_i) = \frac{(\alpha_1/2 + \alpha_2)^2 - [\alpha_1 \alpha_2 (u - \delta - c)]^{2/3} i^{2/3}}{\alpha_2 (\alpha_1 + \alpha_2) (4\delta)^{2/3}}. \quad (27)
\]

The markup is

\[
\frac{\mathbb{E}_G q}{c} = 1 + \frac{\alpha_1 (u - \delta - c) \log (1 + 2\alpha_2/\alpha_1)}{2\alpha_2 c}. \quad (28)
\]

After some simplification, the money demand becomes

\[
L(i) = \frac{c + [\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2]^{1/3} i^{-1/3}}{1 + \alpha_1 (u - \delta) + \alpha_2 c}, \quad (29)
\]

which implies an elasticity

\[
\varepsilon(i) = \frac{-1}{3 + 3c [\alpha_1^2 \delta (u - \delta - c)^2 / 2\alpha_2]^{-1/3} i^{1/3}}. \quad (30)
\]

We try below to match \( L(\mathbb{E}i) \) and \( \varepsilon(\mathbb{E}i) \) in the data.

\(^{15}\)We could put a parameter in front of \( \ell \) and set it to match average hours \( \bar{\ell} \), as in standard business cycle theory, but there is no need, because the results below are independent of \( \ell \), as well as the AD technology and wage.
6.1 Data

We focus on 1988-2004, because we are going to evaluate the theory’s ability to account for the empirical price-change distribution from that period. However, longer series can be used to calibrate some parameters. For money we use the M1J measure in Lucas and Nicolini (2012), which adjusts M1 for money market deposit accounts, related to the way M1S adjusts it for sweeps as discussed in Cynamon et al. (2006). Lucas and Nicolini provide an annual M1J series from 1915 to 2008, and a quarterly series from 1984 to 2013, and make the case that there is a stable relationship between these series and (3-month T-Bill) nominal interest rates. We use the quarterly series, which better corresponds to the price-change sample, and implies an average annualized nominal rate of $E_i = 4.80\%$, implying $L(E_i) = 0.279$ and $\varepsilon(E_i) = -0.149$.\footnote{For the record, in the annual data $E_i = 3.83\%$, $L(E_i) = 0.257$ and $\varepsilon(E_i) = -0.105$. Using this instead does not affect the results much. We also considered truncating the sample in 2004, to better match the price-change sample, and to eliminate the recent financial crisis, but these also did not affect the results much.}

Following several related studies, markup data comes from the U.S. Census Bureau Annual Retail Trade Report 1992-2008 (https://www.census.gov/retail). In these data, the low end, in Warehouse Clubs, Superstores, Automotive Dealers and Gas Stations, gross margins over sales range between 1.17 and 1.21; at the high end, in Specialty Foods, Clothing, Footwear and Furniture, they range between 1.42 and 1.44. Our target is for the gross margin is 1.3, the middle of these data. A gross margin of 1.3 implies a markup 1.39 (see also Bethune et al. 2014). However, the exact value for this target does not matter very much over a reasonable range (similar to the findings in Aruoba et al. 2011, although they use bargaining, not posting).

On the fraction of transactions using money and credit, there are various micro data sources. First, in terms of concept, we follow the literature by interpreting monetary transactions broadly to include cash, check and debit card purchases. As a rationale, first, checks and debit cards use demand deposits, which very much
like currency are liquid and pay basically no interest, and it does not matter for the theory whether the money is in one’s pocket or bank account. Second, for us, a key feature of credit is that it allows one to pay for BJ goods by working in the next AD market, while cash, check and debit purchases require working in the previous AD market, which matters a lot when transaction are random. Third, this notion of money in the micro data is consistent with our use of M1J in the macro data.

Older calibrations of monetary models (Cooley 1995, chapter 7) use a target of 16% for credit, but these days there is much more information. In detailed grocery-store data from 2001, Klee (2008) finds credit cards account for about 12% of purchases. In 2012 Boston Fed data discussed by Bennett et al. (2014) and Schuh and Stavins (2014), credit cards account for 22% of purchases in the survey and 17% in the diary sample, higher than Klee’s numbers mainly because they are not just for groceries. In Bank of Canada data discussed by Arango and Welte (2012), the fraction of credit card purchases is 19%. While there are some differences across studies, these numbers are not especially far apart, and also they also do not vary all that much over time, so given this, we target 20%.¹⁷

Our tie-breaking parameter σ is set to match average price duration. This is different from Head et al. (2012), where they calibrate the comparable parameter jointly with the curvature of u(y) to match the price-change distribution, and then examine what the model predicts for duration.¹⁸

¹⁷ Over time the big evolution in payments has been out of checks and into debit cards, not between monetary and credit transactions as defined here. Also, to be clear, the numbers in the text are for transactions by volume, not value. In Bank of Canada data, the fraction of credit card purchases by value is 40%, double the fraction by volume, because cash tends to be used for smaller transactions, exactly as predicted by the model. In particular, “Cash [accounts] for 76 per cent of all transactions below $15, and for 49 per cent of those in the $15 to $25 dollar range ... whereas credit cards clearly dominate payments above $50” (Arango and Welte 2012). However, in Boston Fed data the fraction by value is 16%, about the same by volume. There is no consensus as to why American and Canadian data differ on transactions by value; in any case, we use volume, where they agree.

¹⁸ In principle, we thought there was more discipline in evaluating model performance by its ability to the match the price-change distribution after calibrating σ to duration, since there is a direct mapping from one moment to the parameter. In practice, however, it makes little difference whether we match duration and check the distribution, or vice versa.
from Klenow and Kryvtsov (2008), where the average duration ranges between 6.8 and 10.4 months, with an average of 8.6. We benchmark 8.6, but also report results with alternatives, since there is variability across studies, e.g., Nakamura and Steinsson (2008) report durations between 8 and 11 months. We also target the average price change in absolute value, 11% in the Klenow-Kryvtsov data, which is above inflation because there are many negative changes. Since the price data are monthly, we let the period in the model be a month, and aggregate model-generated money demand to a quarterly frequency for comparison with the Lucas-Nicolini data.

6.2 Findings

Calibration results for the fixed- and variable-cost models are in Table 1. In choosing parameters, we are constrained to stay in the region where MME exists. In the fixed-cost model, this allows us to hit most of the targets, but not the fraction of trades using credit. In particular, trying to set δ small enough to get 20% credit transaction implies MME does not exist at the observed average i. Therefore, we report the smallest δ that admits MME, which only allows us to get 6.5% credit transactions. In any case, the utility parameter u is much larger than δ, and intuitively comes from matching average real balances. The value of c, about half of u, comes primarily from the markup. The probability of sampling one price is α₁ = 0.18, and the probability of sampling two is α₂ = 0.20, which are low due to the monthly calibration.¹⁹

<table>
<thead>
<tr>
<th></th>
<th>BJ utility</th>
<th>BJ cost</th>
<th>credit cost</th>
<th>pr(n = 1)</th>
<th>pr(n = 2)</th>
<th>tie-break</th>
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<td>0.18</td>
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<td>11.53</td>
<td>0.063</td>
<td>0.12</td>
<td>0.17</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 1: Baseline Calibration

¹⁹As usual, a convenient feature of search models is they can be fit to different frequencies simply by scaling parameters like arrival rates.
With a variable cost of credit, the parameters are calibrated similarly, with \( \tau \) replacing \( \delta \). This model can simultaneously match all of the targets of credit use, including a 20% share of credit transactions, without violating the existence of MME. The values of utility \( u \) and cost \( c \) are smaller than in the fixed-cost model, while the probabilities of sampling one and two prices are of similar magnitudes compared to their fixed-cost counterparts. Both models generate almost the same tie-breaking rule, which implies that sellers who are indifferent to changing prices stick to their incumbent price 90% of the time.

Figure 6: Money Demand for Different Specifications

Figure 6 shows predicted money demand, with the solid curve coming from the fixed-cost mode, and the dashed curve from the variable-cost model. The fit is good in both cases, as is key for evaluating the model from a macro perspective, although note that the variable cost model yields a somewhat lower \( z \) at small \( i \) (which could be important for some issues not studied here, e.g., the welfare cost of inflation). In terms of micro evidence, Figure 7 shows the predicted \( p \) change distribution along with the Klenow-Kryvtsov empirical counterpart. Both models capture the overall shape, although the variable-cost model is perhaps not quite as good. Importantly, the models are consistent with various details deemed important in the literature.
In particular, the average absolute change is 11.1% in the fixed cost model, which we hit by calibration, while in the case of variable cost, the value is 12.3%. The fraction of small changes (below 5%) is 44% in the data, 32.4% in the model with fixed cost, and 30.3% with variable cost, not dramatically different.\textsuperscript{20} The fraction of negative changes is 37% in the data, 42.9% in the fixed cost model, and 43.4% with variable cost, not too far off. As discussed above, these observations are difficult to match simultaneously with simple menu cost or Calvo models, although Midrigan (2011) manages. Our search model does fairly well without complications like heterogeneity, idiosyncratic shocks, etc. A shifting $F_t$ combined with a simple tie-breaking calibrated to duration rule naturally generates large average, many small, and many negative price adjustments.

Figure 7: Distribution of Price Changes

Figure 8 shows the hazard – the probability of changing $p$ as a function of the duration since the last change – over 18 months, from the fixed cost model and from the data in Nakamura and Steinsson (2008).\textsuperscript{21} The model does not\textsuperscript{20}Eichenbaum et al. (2015) find a fraction of $p$ changes below 5% of 5.2% after correcting for measurement error, and suggest other evidence on small price changes is partly due to not taking this into account. Here we take the Klenow-Kryvtsov numbers at face value.\textsuperscript{21}The variable cost model generates an almost identical hazard, which we do not report here to avoid repetition. We also looked at the hazard in the Klenow-Kryvtsov data, but emphasize
generate enough action in the first few months, clearly, but at least the hazard slopes downward, something those authors say cannot happen in other models. Now one should not expect to explain every detail, since there must be a lot going on in the market that is not in the model, e.g., experimentation by sellers trying to learn about demand conditions, which can yield more changes at low durations. Even without such complications, our hazard decreases at least for a while, before eventually increasing, since with continuing inflation any $p$ falls off the support $\mathcal{F}_t$ in the long run. However, even at 10 years, our hazard is only up to 13.6%, which means some firms can stick to a prices for a very long time, even as long Cecchetti’s (1986) mucilaginous magazines mentioned in Section 2.

![Figure 8: Price Change Hazard over 18 Months](image)

Because there is a range of findings in the empirical literature, Figure 9 shows results of matching average durations of 1, 4, and 24 months, as well as choosing $\sigma$ to minimize the sum of squared errors between the model-predicted and empirical distributions, which implies a duration of 15.9 months (this uses a fixed cost, but the variable cost model is similar). The overall shape of the distribution does not change a lot, but the fraction of small changes goes up and the fraction of negative changes goes down, contributing to a somewhat better overall fit, at higher durations. Figure 10 shows results of different inflation rates (this uses Nakamura-Steinsson on this point because they argue that it is important.

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a variable cost, since it is hard to guarantee the existence of MME with higher
inflation in the fixed-cost model). The overall shape of the distribution does not
change too much, but the fraction of small changes goes up with inflation and
the fraction of negative changes goes down. The important point is that pricing
behavior in this model is not invariant to inflation.

Figure 9: Distribution of Price Changes for Different $\sigma$

We conclude from all of this that, while the models miss some details, they
generally perform fairly well in terms of the price change evidence. Certainly it
is hard to say there is anything especially puzzling about the behavior of price
changes in the data – it is pretty much what one should expect from simple
search theory. It would be even harder to argue there is anything informative
about menu costs or Calvo parameters in the data, given the outcomes we gen-
erated with a model without these ingredients. Also, we emphasize that there is
quantitative discipline imposed here by matching both the macro facts on money
demand and micro facts on consumers’ use of cash and credit.
Here we consider an extension of the baseline model where buyers must pay to participate in the BJ market. With $k = 0$ all households enter, $b = \bar{b}$, so the arrival rates $\alpha_n(\bar{b})$ are fixed exogenously. Since $y$ is indivisible, BJ output is fixed, too. Since AD output is fixed by $U'(x) = 1$, changes in the level of $M$, or in inflation and interest rates, $\pi$ and $i$, have no impact on output. Therefore monetary policy is neutral, regardless of the source of stickiness. However, the specification can be extended in various ways to make output endogenous and illustrate how the source of stickiness matters. Here, based some earlier work (Liu et al. 2011), we pursue one way, by setting $k > 0$ and making the measure of BJ buyers endogenous. This determines the arrival rates, which adjust until the marginal participant is indifferent.

First note that if prices were sticky due to Calvo or Mankiw, a one-time unanticipated change in $M$ can have real effects. The reason is that at least

![Figure 10: Distribution of Price Changes for Different $\pi$](image)
some firms could not (with Calvo) or would not (with Mankiw) adjust \( p \), and so the nominal distribution \( F(p) \) may not change enough to keep the same real distribution \( G(q) \). If \( M \) increases, one should expect the real distribution to turn in favor of buyers, increasing \( b \) (a shopping spree) and thus stimulating output. By contrast, with prices endogenously sticky, as modeled here, jumps in \( M \) impact neither \( G(q) \) nor output. hence, in our economy, a policy pundit seeing only a fraction of sellers adjusting \( p \) each period may conclude that changes in \( M \) will have real effects, but that would be a mistake.\(^{22}\)

Figure 11: Real Balances and Free Entry

With endogenous entry, \( M \) is irrelevant but \( \pi \) and \( i \) are not – i.e., money is neutral but not superneutral. To see this in a simple way, suppose BJ buyers attempt to solicit two price quotes, and succeed in each try with probability \( \lambda(b) \), where \( \lambda(0) = 1, \lambda(\bar{b}) = 0, \lambda'(b) < 0 \) and \( \lambda''(b) > 0 \). Then \( \alpha_1(b) = 2\lambda(b)[1 - \lambda(b)] \) and \( \alpha_2(b) = \lambda(b)^2 \). Inserting these into (21), we get

\[
\hat{z}_i = c + (2\delta)^{1/3} \left[ 1 - \lambda(b) \right]^{2/3} \left( u - \delta - c \right)^{2/3} i^{-1/3}.
\]

This defines a relation between \( \hat{z}_i \) and \( b \) called the RB (real balance) curve.

\(^{22}\)In our view, recognizing this as just another example of the Lucas critique does not make it any less relevant.
Similarly, (13) defines a relation called the FE (free entry) curve,

$$\kappa = \lambda(b)^2 (u - \delta - c) - \frac{\delta [1 - \lambda(b)]^2 (u - \delta - c)^2}{(\hat{z}_i - c)^2} + \delta - i\hat{z}_i. \quad (32)$$

As Figure 11 shows, RB is increasing and convex, with $\hat{z}_i = c$ at $b = 0$, while FE is upward sloping to the left and downward sloping to the right of RB, with $b \in (0, \bar{b})$ at $\hat{z}_i = 0$. Hence there is a unique intersection of FE and RB. Now consider an increase in $i$, which ultimately comes from higher inflation and hence higher monetary expansion. This shifts both curves toward the origin, reducing $b$ and output. Monetary policy matters, although this has nothing to do with nominal rigidities – it is in fact due to higher $i$ taxing households’ participation in decentralized exchange. But a one-time unanticipated jump in $M$ is neutral. To be clear, at the risk of repetition, the point is is not that money is neutral in reality; it is that observations on sticky nominal prices do not imply money is nonneutral.

One can calibrate the model with endogenous participation. Now $b$ is endogenous, and hence $\alpha_1$ and $\alpha_2$ are endogenous once one specifies a matching technology. The matching technology may involve a new parameter and there is also the entry cost $\kappa$, so we have two new parameters, but we also lose two parameters, $\alpha_1$ and $\alpha_2$, because they are now endogenous. Basically, given same targets, we pin down the new parameters and these deliver $\alpha_1$ and $\alpha_2$, but that cannot improve over calibrating $\alpha_1$ and $\alpha_2$ directly. Hence, we do not report numerical results for the model with endogenous participation, but it is clearly capable of delivering similar results. The point of including it here is to show how our endogenous

8 Conclusion

One contribution of search theorists generally is to show how some observations that appear anomalous from the perspective of standard theory can be readily understood once frictions are modeled explicitly. A leading example is price
dispersion, with deviations from the law of one price emerging naturally in (some
versions of) search equilibrium. It turns out something similar is true for price
rigidity. It is a huge puzzle for standard theory that some sellers let their real
prices vary in arbitrary ways by not adjusting nominal prices in response to
monetary or other shocks. Yet once one understands how frictions lead to price
dispersion, the puzzle of price stickiness vanishes, because letting real prices vary
in arbitrary ways over some range does not affect profit. This paper exploits
that insight to build a general equilibrium monetary model where sticky nominal
prices emerge endogenously.

Constructing rigorous monetary models is nontrivial – indeed, money is an-
other anomaly for standard theory that can be better understood once frictions
are modeled carefully. Our approach is based on lessons from the literature. We
build on Lagos-Wright because it is easy and flexible, and it has proved useful
in other applications. Into this we inserted the original Burdett-Judd indivisible-
goods specification. Then we added costly credit, because this avoids technical
problems related to indeterminacy, and of course because it is interesting for its
own sake. With either a fixed or variable cost of credit, we get nice money de-
mand functions that fit macro data well. Calibrated versions do a reasonable
job emulating the empirical price-change distribution. In particular, we get large
average, many small, and many negative changes. We get a decreasing hazard
and pricing behavior that depends on inflation. We found that the variable cost
model is better at matching the micro facts on money and credit usage. Finally,
we illustrated how sticky prices do not imply nonneutrality. There is more that
can be done, but we consider this progress.
References


