Capital in Transition: 
Housing and Sectoral Reallocation in the Long Run

Xintong Yang *

November 17, 2015

Job Market Paper

Abstract

This paper studies the sectoral allocation of capital between housing and non-residential sectors using a two-sector general equilibrium model in a neoclassical growth environment. Calibrated to both the United States and China, the model can account for both the positive correlation between the share of housing capital and the consumption-output ratio in the United States and the negative correlation between the share of housing capital and the consumption-output ratio in China. The calibration to the Chinese economy implies that the rapid increase in the share of housing capital and the simultaneous decrease in the consumption-output ratio observed in China can be rationalized by a combination of three factors: a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform. This paper provides a tractable framework to understand the sectoral allocation of capital between housing and non-residential sectors across countries.

JEL Classification: E13, E21, E22, O41.  
Keywords: share of housing capital, consumption-output ratio, transitional dynamics, Chinese economy

*University of California, Santa Barbara xintongyang@umail.ucsb.edu I am indebted to my advisors Peter Rupert, Javier Birchenall, Marek Kapička, and Cheng-Zhong Qin for their continued guidance and support. I am also grateful to Steve LeRoy, Christine Braun, Ben Grifly, Daniel Moncayo, and Brian Thomas for their valuable feedback. All errors are my own. This is the link to the current version of the paper.
1 Introduction

Developed economies have experienced a sectoral reallocation of capital over the course of their long-run economic development. The share of housing capital has grown annually by approximately 0.2 percent for the United Kingdom, France, Germany, Canada, and the United States since 1700. While the sectoral reallocation of capital happens gradually in these five developed economies, it has been rapid in China. Since the housing market reform in the 1980s, the share of housing capital in China has increased by 1.5 percent annually.

The question I address in this paper is whether standard neoclassical growth theory can qualitatively and quantitatively explain the features of the sectoral reallocation of capital not only for developed economies but also for China. I begin by documenting the empirical regularities regarding the share of housing capital across developed economies and China. Alongside the different annual growth rate of the share of housing capital, another salient difference between the two types of economies is the correlation between the share of housing capital and the consumption-output ratio. On the one hand, the share of housing capital is positively correlated with the consumption-output ratio across developed economies, but on the other hand the correlation between the two variables has been negative in China since 1987. While there are many institutional differences between developed economies and China, I start with a simple framework to investigate the factors that determine the observed differences. I find that in a standard two-sector neoclassical growth framework, the distinction between two key parameters, the elasticity of substitution between the two sectors and the capital intensity of production of the housing sector, can explain the differences observed in the two types of economies.

I build a two-sector general equilibrium model with housing and non-residential sectors. The model features preferences with a constant elasticity of substitution between the two sectors, and Cobb-Douglas production technologies within each sector. The two types of capital are treated symmetrically and are endowed with dual functions: each unit of capital can be used as a factor input of production and a capital good that generates a rental return. In a frictionless environment, the equilibrium allocation features a balanced growth path with a constant sectoral allocation of capital between the two sectors. During the transitional dynamics, the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors.

Calibrating the model to the United States and China, I examine whether the dy-

---

1 The decomposition of domestic capital follows Piketty and Zucman (2014). Domestic capital is broken down into three categories: agricultural land, housing (including residential structure and land value), and nonresidential capital (including non-residential structure, equipment and machinery, and intellectual property products). From here on, the share of housing capital is defined as the share of value of housing capital out of the sum of value of non-residential and housing capital.

2 See Section for a detailed discussion of the empirical regularities of sectoral allocation of capital.
namics proposed by the model are consistent with empirical observations. With plausible parameters, the model generates reallocations that are consistent with the experiences of the United States and China. Moreover, the model can account for (on the one hand) the positive correlation between the share of housing capital and the consumption-output ratio in the United States, and (on the other hand) the negative correlation between the share of housing capital and the consumption-output ratio in China. In particular, the rapid increase in the share of housing capital and the decrease in the consumption-output ratio observed in China can be explained by a combination of three factors: a high elasticity of substitution between housing and non-residential sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform.

To the extent that the model can reproduce the key features of the data in China, I apply the model to quantify the effect of the initial share of housing capital before the housing market reform on the Chinese economy. Given that there was no market for housing before the reform in the mid-1980s, the initial sectoral allocation when the market mechanism starts to work is a key factor to study the Chinese economy. When changing the Chinese sectoral capital allocation in 1987 to the US level of a comparable development stage, the comparative study suggests that the initial low share of housing capital before the housing market reform has led to an over-investment in housing, and an under-investment in non-residential capital since 1987 in China.

This paper makes four main contributions to the literature on structural change and housing. First, this paper highlights the role of the share of capital in the process of structural change, complementing the existing structural change literature that focuses on labor reallocation (Kongsamut et al. (2001), Ngai and Pissarides (2004) and Cao and Birchenall (2013)). The model of Acemoglu and Guerrieri (2008) features both capital and labor reallocation between sectors with differentiated capital intensities of production, but does not consider the housing service consumption in the utility function. Building on the theory of Acemoglu and Guerrieri (2008), this paper stresses a novel mechanism to explain the different patterns of the correlation between the share of housing capital and the consumption-output ratio.

Second, this paper characterizes the transitional dynamics of sectoral allocation of capital. The model inherits the features of home production models where housing service consumption enters the preferences as an object of interest (Benhabib et al. (1991), Greenwood and Hercowitz (1991) and Davis and Heathcote (2005)). While most of the literature focuses on the business cycle properties of housing, few discuss long-term trends

---

3 Kongsamut et al. (2001) study the structural change in production through labor reallocation using a nonhomothetic preference that features different income elasticities of demand among different sectors; Ngai and Pissarides (2004) use a preference with constant elasticity of substitution and defines the structural change as a change in labor share; Cao and Birchenall (2013) investigate the role of agricultural productivity on the economic growth and sectoral allocation of labor and production for China’s post-reform economy, using a two-sector model with nonhomothetic preferences and Cobb-Douglas production functions.
and the transitional dynamics of capital allocation along with the impact they have on economic development. A growing body of literature studies the effect of financial market liberalization on the acceleration of capital reallocation to the housing sector (Favilukis et al. (2010), Kiyotaki et al. (2011), and Sommer et al. (2013)). This literature features a life-cycle model with heterogeneous agents under an incomplete market environment. These models omit the supply side of housing capital. Also, the numerical solution procedure might obscure certain economic mechanisms. This paper complements the literature by studying the long-term trend of capital allocation with transitional dynamics and their macroeconomic implications.

Third, this paper provides a theoretical framework to study the correlation between the share of housing capital and the consumption-output ratio. The spillover effect of housing wealth on consumption growth is well studied by the empirical literature (Case et al. (2005), Gan (2010), and Iacoviello (2011)). It is empirically shown that changes in housing wealth have a larger impact than changes in other financial assets in influencing household consumption. While data confirm the higher contemporary correlation between housing wealth and consumption, the causal relationship between housing wealth accumulation and consumption/saving motive is not clear. This paper proposes using the share of housing capital to study the interaction between the housing wealth and household consumption.

Last, this paper develops a unified theory to explain different empirical patterns across developed economies and China within the neoclassical growth framework. In the literature, the models applied to study developed economies and China are separated given the different empirical observations (Garriga et al. (2014) and Chen and Wen (2014)). This paper calibrates a standard two-sector neoclassical growth model to both the United States and China, and showcases the model’s ability to reproduce the features that are consistent with both the United States and China. By providing a comparative perspective of the study of China with that of developed economies, this paper justifies the applicability of neoclassical growth theory.

The rest of the paper is organized as follows. Section 2 documents the empirical regularities regarding the sectoral allocation of capital across developed economies and China. Section 3 describes the model environment. Section 4 presents the model calibration to the United States. Section 5 presents the calibration to China. Section 6 draws conclusions.

2 Empirical Regularities of Sectoral Allocation of Capital

In this section, I document three major features regarding the share of housing capital across countries. First, I show that there is a consistent decline over time in the value of agricultural land, which is accompanied by a rise in the value of housing and non-residential capital. Second, I demonstrate a positive correlation between GDP per capita and the share of housing capital in the United States and China. Finally, I show that
the correlation between the share of housing capital and the consumption-output ratio is positive across developed economies, but is negative in recent decades in China.

2.1 Structural Transformation of Capital and Share of Housing Capital

Figure 2.1 shows that in the United Kingdom, France, Germany, Canada, and the United States, the total value of capital, measured as a fraction of national income, has not changed much over time, but that the capital structure has been transformed: the value of land has gradually been replaced by the value of non-residential and housing capital. In contrast, China experienced the same structural transformation of capital within a short period of 30 years.

Table 2.1: Share of Housing Capital out of Total Value of Capital

<table>
<thead>
<tr>
<th></th>
<th>1700</th>
<th>1800</th>
<th>1900</th>
<th>2000</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>0.333</td>
<td>0.287</td>
<td>0.386</td>
<td>0.404</td>
<td>0.555</td>
</tr>
<tr>
<td>FRA</td>
<td>0.437</td>
<td>0.416</td>
<td>0.412</td>
<td>0.568</td>
<td>0.610</td>
</tr>
<tr>
<td>GER</td>
<td>0.227</td>
<td>0.345</td>
<td>0.581</td>
<td>0.621</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.340</td>
<td>0.291</td>
<td>0.415</td>
<td>0.400</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>0.276</td>
<td>0.361</td>
<td>0.568</td>
<td>0.610</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>0.303</td>
<td>0.220</td>
<td>0.314</td>
<td>0.340</td>
<td>0.458</td>
</tr>
</tbody>
</table>

Data Sources:
Computed using non-residential capital and housing from Piketty and Zucman (2014) for the five developed economies; from Table B.3 for China.

Table 2.1 summaries the evolution of the share of housing capital across the five developed economies since 1700, and in China between 1987 and 2013. As shown in the table, China has experienced a rapid increase in the share of housing capital since 1987. The percentage increase of the share of housing capital within 30 years in China is at the same level with that of the United States over a hundred years during the 20th century.

2.2 GDP Per Capita vs. Share of Housing Capital

Figures 2.2 and 2.3 show that the share of housing capital is positively correlated with GDP per capita. This is true for China during its 30-year transition and the United States throughout its longer-term transition. The correlation is insignificant for the US economy from 1950 to 2011, consistent with the notion that the postwar US economy is on a “balanced growth path”.

4More precisely, the capital-output ratio presents a U-shaped pattern for the United Kingdom, France, Germany, and Canada due to WWII. For the United States, the U-Shaped pattern is less strong.
Figure 2.1: Allocation of Domestic National Capital

Data Sources:
1. Piketty and Zucman (2014) for five developed economies.
2. Constructed for China. (See Appendix B for detailed construction approaches)

2.3 Share of Housing Capital vs. Consumption-Output Ratio

Figure 2.4 documents an unconditional correlation between the average share of housing capital and the average consumption-output ratio between 1995 and 2013 for OECD countries. There is a significant positive correlation between the share of housing capital and
Figure 2.2: GDP Per Capita vs. Share of Housing Capital

Data Sources:
1. GDP per capita: Bolt and van Zanden (2014) for the United States; Penn World Table 8.1 for China. Both are in 2005 PPP-adjusted USD.
2. Share of housing capital: Computed using non-residential capital and housing (real) from Piketty and Zucman (2014) for the United States; from Table B.3 for China.

Figure 2.3: GDP Per Capita vs. Share of Housing Capital

Data Sources:
1. GDP per capita: Penn World Table 8.1 for both the United States and China. Both are in 2005 PPP-adjusted USD.
2. Share of housing capital: Computed using non-residential capital and housing (real) from Fixed Assets Table for the United States; from Table B.3 for China.

the consumption-output ratio across OECD countries. When looking at the correlation between the two variables for the postwar US economy as shown in Figure 2.5 (b), the positive correlation remains.

However, during the 30-year transition in China, the correlation between the share of housing capital and the consumption-output ratio becomes negative. As shown in Figure 2.5 (a), the time-series plot of the unconditional correlation between the share of housing capital and the consumption-output ratio present a significant negative correlation.
Figure 2.4: Share of Housing Capital vs. Consumption-Output Ratio

Data Sources: OECD Statistics
1. Average share of housing capital: Computed using non-residential capital and housing (real) from the Balance Sheets for Non-financial Assets.
2. Average consumption-output ratio: Computed using consumption and output (real) from the Final Consumption Expenditure of Households Table.

Figure 2.5: Share of Housing Capital vs. Consumption-Output Ratio

Data Sources:
1. Share of housing capital: Computed using non-residential capital and housing (real) from Fixed Assets Table for the United States; from Table B.3 for China.

The three empirical regularities regarding the share of housing capital point to three facts. First, the share of housing capital positively correlates with the income of an economy in transition. Second, the growth rate at which the share of housing capital increases has been dramatically different between developed economies and China. Lastly, the consumption-output ratio is negatively correlated with the share of housing capital in China, which contrasts with the positive relationship seen in the developed economies.
3 The Model

In this section, I present the neoclassical growth model environment, a two-sector model with exogenous technological progress. Capital and labor (re)allocation on both the balanced growth path and the transitional dynamics are characterized.

3.1 The Environment

The model economy is infinite horizon. Time is discrete. There is a representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$$ \hspace{1cm} (3.1)

where $c$ is the consumption of non-residential goods and services, $s$ is the consumption of housing services, and $\beta$ is the utility discount factor. Labor is supplied inelastically and normalized to one. The instantaneous utility function combines the two types of consumption with a constant elasticity of substitution $\epsilon \in [0, \infty)$:

$$u(c, s) = \frac{\left[(\eta c^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)s^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} - 1\right]}{1-\sigma}$$

where $\eta \in (0, 1)$ indicates the preference weight between the two types of consumption; and $\frac{1}{\sigma} \in [0, \infty)$ denotes the intertemporal elasticity of substitution.$^5$

Final output in the two sectors are produced with the following production functions:

$$y_t = k_t^{\alpha_k} (z_t (1-l_t))^{1-\alpha_k}, \quad s_t = h_t^{\alpha_h} (z_t l_t)^{1-\alpha_h}$$ \hspace{1cm} (3.2)

where $y$ and $s$ denote the production of the non-residential sector and the housing sector respectively; $k$ and $h$ denote the factor inputs of capital in the two sectors respectively; 1 is the household’s normalized time endowment, and $l$ denotes the share of labor allocated to the housing sector. $\alpha_k \neq \alpha_h$ denote the capital intensities of production of the two sectors, and $z$ represents the labor-augmenting technological progress, which evolves according to $z_t = z_0 \cdot A^t$, for $A > 1$ and $z_0 \geq 1$.

Non-residential capital and housing evolve as follows:

$$k_{t+1} = k_t (1-\delta_k) + i_{kt}, \quad 0 < \delta_k < 1$$ \hspace{1cm} (3.3)

and

$$h_{t+1} = h_t (1-\delta_h) + i_{ht}, \quad 0 < \delta_h < 1$$ \hspace{1cm} (3.4)

where $i_k$ and $i_h$ are the non-residential and housing investment, $\delta_k$ and $\delta_h$ denote the depreciation rate for non-residential capital and housing respectively. Denote $a = k + h$

\hspace{1cm}$^5$Note that the CES instantaneous utility function is a homothetic preference, which implicitly assumes that the income elasticity of both types of consumption equals to one. In this paper, different income elasticities of demand between sectors are not considered.
as the aggregate capital stock. Assume that $\delta_k = \delta_h$. The aggregate resource constraint is:

$$c_t + a_{t+1} \leq y_t + (1 - \delta)a_t \quad (3.5)$$

which requires consumption and investment to be less than output of the non-residential sector.

### 3.2 The Competitive Equilibrium and the Social Planner’s Problem

Normalize the price of the consumption of non-residential goods and services to one. Denote the rental price of capital and the wage rate by $R$ and $w$, and the interest rate by $r$. Let $q$ denote the relative price of housing services. Define the share of housing capital as $\kappa = \frac{h}{a}$, and the share of labor allocated to the housing sector as $l$. A competitive equilibrium is defined as the paths of prices $(R_t, w_t, r_t, q_t)_{t \geq 0}$, the factor allocations $(l_t, \kappa_t)_{t \geq 0}$, and the consumption and stock holding decisions $(c_t, s_t, a_{t+1})_{t \geq 0}$ such that:

(a) Given the aggregate state $(a_t, z_t)_{t \geq 0}$ and the paths of prices $(R_t, w_t, q_t)_{t \geq 0}$, firms choose the factor allocations $(l_{kt}, \kappa_{kt})_{t \geq 0}$, for $i \in \{k, h\}$, to maximize profits at each period $t$:

$$\max_{l_{kt}, \kappa_{kt}} \left\{ (\kappa_{kt}a_t)^{\alpha_k} (z_t l_{kt})^{1 - \alpha_k} - R_t \cdot (\kappa_{kt}a_t) - w_t \cdot l_{kt} \right\}$$

and

$$\max_{l_{ht}, \kappa_{ht}} \left\{ q_t \cdot (\kappa_{ht} a_t)^{\alpha_h} (z_t l_{ht})^{1 - \alpha_h} - R_t \cdot (\kappa_{ht} a_t) - w_t \cdot l_{ht} \right\}$$

(b) Given the initial endowment of capital stock $a_0$ and the paths of prices $(r_t, w_t, q_t)_{t \geq 0}$, the household makes the consumption and saving decision $(c_t, s_t, a_{t+1})_{t \geq 0}$ to maximize the lifetime utility of

$$\max_{\{c_t, s_t, a_{t+1}\}_{t \geq 0}} \sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$$

s.t.

$$c_t + q_t s_t + a_{t+1} \leq (1 + r_t)a_t + w_t$$

(c) All the markets clear s.t.

$$c_t + a_{t+1} = (\kappa_{kt} a_t)^{\alpha_k} (z_t l_{kt})^{1 - \alpha_k} + (1 - \delta)a_t$$

$$s_t = (\kappa_{ht} a_t)^{\alpha_h} (z_t l_{ht})^{1 - \alpha_h}$$

$$l_{kt} + l_{ht} = 1$$

$$\kappa_{kt} + \kappa_{ht} = 1$$

Since markets are complete and competitive, the Second Welfare Theorem can be applied. The competitive equilibrium can be characterized by solving a social planner’s

---

Note that without aggregate uncertainty, the rate of return on both types of capital are the same. Hence, there is only one effective capital asset market that pools both types of capital together with a single rate of return.
problem: Given the state variables \( \{z, k, h\} \), the factor allocations \( \{l, k', h'\} \) are chosen to solve the following dynamic programming problem:

\[
v(k, h, z) = \max_{l, k', h'} u(c, s) + \beta v(k', h', z')
\]

s.t.

\[
c + k' + h' = k^\alpha_k (z(1 - l))^{1 - \alpha_k} + (1 - \delta)(k + h) \\
s = h^\alpha_h (zl)^{1 - \alpha_h}
\]

Once the solution is characterized, the competitive factor prices \((R, w, r)\) and the factor allocations \((l, \kappa)\) can be backed out. In particular, the relative price for housing services can be derived as:

\[
q = \frac{u_s}{u_c} = \frac{1 - \eta}{\eta} \cdot \left(\frac{c}{s}\right)^{1/\epsilon} \tag{3.6}
\]

### 3.3 The Balanced Growth Path

Detrend the real variables by the growth rate of the economy, \(A\). Denote \(\hat{x}_t = \frac{x_t}{A_t}\), for \(x_t = \{y_t, c_t, a_t, s_t\}\). In equilibrium, the equalization of the marginal product of capital and labor in both sectors within a period implies:

\[
\frac{\dot{k}}{\dot{h}} = \frac{\alpha_k (1 - \alpha_h)}{\alpha_h (1 - \alpha_k)} \cdot \frac{1 - l}{l} \tag{3.7}
\]

Denote the rate of return on non-residential capital and housing as \(r_k = \alpha_k \cdot \frac{\dot{y}}{\dot{k}} - \delta\) and \(r_h = \alpha_h \cdot \frac{\dot{y}}{\dot{h}} - \delta\), respectively. The no-arbitrage condition \(r_k = r_h = r\) implies \(\frac{\dot{k}}{\dot{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{\dot{y}}{\dot{s}}\), which indicates that the capital allocation depends on the relative value of final outputs in both sectors. Substituting \(q = \frac{u_s}{u_c}\) in (3.6),

\[
\frac{\dot{k}}{\dot{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1 - \eta} \cdot \left(\frac{\dot{y}}{\dot{s}}\right)^{1 - 1/\epsilon} \cdot \left(\frac{\dot{y}}{\dot{c}}\right)^{1/\epsilon} \tag{3.8}
\]

Combining (3.7) and (3.8),

\[
\kappa = \left\{1 + \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1 - \eta} \cdot \left(\frac{\dot{y}}{\dot{s}}\right)^{1 - 1/\epsilon} \cdot \left(\frac{\dot{y}}{\dot{c}}\right)^{1/\epsilon}\right\}^{-1} \tag{3.9}
\]

and

\[
l = \left\{1 + \frac{\alpha_h}{\alpha_k} \cdot \frac{1 - \alpha_k}{1 - \alpha_h} \cdot \frac{1 - \kappa}{\kappa}\right\}^{-1} \tag{3.10}
\]

Equations (3.9) and (3.10) imply the share of housing capital and the share of labor allocated to the housing sector in equilibrium. In particular, (3.10) shows that at each period, the share of labor allocated to the housing sector is monotonically increasing in
the share of housing capital. In other words, labor and capital are always reallocated towards the same sector in equilibrium.

Dynamics of the economy are determined by the capital accumulation and the Euler equation. The capital accumulation implies:

\[
\left( \frac{\hat{a}'}{\hat{y}'} \right) \cdot A = (1 - \delta) \cdot \frac{\hat{a}}{\hat{y}} + 1 - \frac{\hat{c}}{\hat{y}}
\]

(3.11)

The Euler equation implies:

\[
\frac{u_c}{u_c'} = \beta A^{-\sigma} \left[ \frac{\alpha_k}{1 - \kappa'} \cdot \left( \frac{\hat{a}'}{\hat{y}'} \right)^{-1} + 1 - \delta \right]
\]

(3.12)

where \( u_c = \eta \cdot [\eta \hat{c}^{\gamma \gamma} + (1 - \eta) \hat{s}^{\gamma \gamma} \cdot \hat{c}^{-\gamma \gamma} \cdot \hat{c}^- \gamma \gamma] \).

A balanced growth path of the economy is defined as an equilibrium trajectory, along which the share of housing capital and the share of labor allocated to the housing sector stay constant, and all the real variables \( \{y_t, c_t, s_t, a_t\} \) grow at the same rate.

**Proposition 1:** Assume that \( A^\sigma > \beta \left[ \alpha_k A + (1 - \alpha_k)(1 - \delta) \right] \). There exists a unique balanced growth path, along which the steady-state capital per capita is:

\[
\hat{a}^* = z_0 \cdot \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{1 - \alpha_k} \cdot \frac{\alpha_h(1 - \alpha_k)}{\alpha_h(1 - \alpha_k)(1 - \kappa^*) + \alpha_k(1 - \alpha_h)\kappa^*}
\]

(3.13)

and the steady-state share of housing capital is:

\[
\kappa^* = \frac{1 - n(\Theta)}{1 + m(\Theta)}
\]

(3.14)

where \( \Theta = \{\alpha_k, \alpha_h, A, \delta, \eta, \epsilon, \beta\} \) is a set of fundamental parameters, \( n(\cdot) \) and \( m(\cdot) \) are both functions of the fundamental parameters of the economy. Along the balanced growth path, all the real variables \( \{y_t, c_t, s_t, a_t\} \) grow at the growth rate of technological progress, \( A \).

Proposition 1 implies that the steady-state share of housing capital depends only on the fundamental parameters of the economy, whereas the steady-state capital per capita is jointly determined by the labor-augmented technological progress and the capital allocation of the economy. The next two propositions demonstrate how the fundamental parameters and technological progress impact the steady-state share of housing capital and capital per capita.

\[\text{[Footnote 7]}\] See Appendix A.1 for an alternative derivation of the equilibrium conditions (3.7), (3.8) and (3.12) from FOCs of the dynamic programming problem. For all the propositions, see Appendix A.2 for detailed proofs.
Proposition 2: The steady-state share of housing capital, $\kappa^*$, is:
1. increasing in the preference weight of the housing service consumption, i.e., $\frac{d\kappa^*}{d(1-\eta)} > 0$;
2. increasing in the inverse of intertemporal elasticity of substitution, i.e., $\frac{d\kappa^*}{d\sigma} > 0$;
3. increasing in the elasticity of substitution between the two types of consumption when the preference weight of the housing service consumption is sufficiently large, i.e., $\frac{d\kappa^*}{d\epsilon} > 0$, for $\eta \in \left(0, \frac{1}{1+\omega(\Theta)}\right)$, where $\omega(\cdot)$ is a function of fundamental parameters of the economy.

Proposition 3: The steady-state capital per capita, $\hat{a}^*$, is:
1. increasing in the level of labor-augmented technological progress, i.e., $\frac{d\hat{a}^*}{dz} > 0$;
2. decreasing in the growth rate of labor-augmented technological progress, i.e., $\frac{d\hat{a}^*}{dA} < 0$.
3. decreasing in the steady-state share of housing capital if the production of the housing sector is labor intensive, i.e., $\frac{d\hat{a}^*}{d\kappa^*} < 0$ if $\alpha_k > \alpha_h$.
4. increasing in the steady-state share of housing capital if the production of the housing sector is capital intensive, i.e., $\frac{d\hat{a}^*}{d\kappa^*} > 0$ if $\alpha_k < \alpha_h$.

While the first two comparative statics in Proposition 3 are consistent with the one-sector neoclassical growth model, the last two statements have not been discussed by the one-sector model. It implies that the sectoral allocation of capital and the capital intensities of production of the two sectors determine the capital accumulation of an economy. In particular, in an economy where the production of the housing sector is labor intensive, a higher steady-state share of housing capital implies a lower investment-output ratio, i.e., a higher consumption-output ratio at steady state.

3.4 The Transitional Dynamics

During the transitional dynamics, the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors.

Proposition 4: During the transitional dynamics, with capital deepening, if the production of the housing sector is labor (capital) intensive, i.e., $\alpha_k > \alpha_h$ ($\alpha_k < \alpha_h$), the share of housing capital increases, as the consumption-output ratio increases (decreases).

The mechanism behind the correlation between the share of housing capital and the...
consumption-output ratio lies in the return on housing capital. The no-arbitrage condition \(3.8\) implies that the rate of return on housing capital is always equal to that on non-residential capital. Consider the case when the production of the housing sector is labor intensive. As capital deepens, the relative output of the housing sector decreases. An increase in the return on housing capital induces a simultaneous increase in the share of housing capital and the consumption-output ratio. Otherwise, an opportunity to arbitrage can emerge.

4 Calibration: US (1948-2005)

In this section, I calibrate the model to the US postwar economy, and examine whether the dynamics generated by the model are consistent with the US data. Further, I investigate the effect of a lower initial share of housing capital on the economy. The benchmark calibration captures the key features of the US economy between 1948 and 2005, and the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to different values of parameters and the lower initial share of housing capital. But, the levels of the share of housing capital and the consumption-output ratio are sensitive to the elasticity of substitution between the two sectors, and the numerical exercise suggests that the initial share of housing capital has a large impact on the speed of sectoral reallocation and the investment structure of the economy.

4.1 Data

The measure of the flow variables of the model is from NIPA. In particular, the output of the non-residential sector includes the consumption of nondurable goods and services, the non-residential investment and the housing investment. The output of the housing sector is from the household expenditure on housing and utilities. Non-residential and housing capital are from the Fixed Assets Table. Labor in the non-residential sector is computed as the total hours worked by the full-time and part-time workers in the private sector, divided by the total numbers of workers and hours in a year for normalization. In addition, I refer to the current-price data as value, and the chain-type fixed-price quantity indices as quantity.

4.2 Calibration

The frequency of the model is annual. The model economy is fully characterized by 8 parameters, \( \beta, \delta, \alpha_k, \alpha_h, A, \epsilon, \eta, \sigma \), and three initial values, \( z_0, a_0 \) and \( \kappa_0 \). Table

\( \text{Data from Davis and Heathcote (2005) are used for result comparison, in which the market value for housing includes both the value for land and residential structure, whereas the data from Fixed Assets Table only includes the value for residential structure. The results remain robust using data from Davis and Heathcote (2005).} \)

\( \text{Labors are taken from the private sector because the output does not include the government expenditures.} \)
Table 4.1: Summary of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
<td>Utility discount factor</td>
<td>Standard value</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.00*</td>
<td>Inverse of intertemporal elasticity of substitution</td>
<td>Standard value</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.06</td>
<td>Depreciation rate of capital</td>
<td>NIPA Table 1.5 and Fixed Assets Table 2.1</td>
</tr>
<tr>
<td>( \alpha_k )</td>
<td>0.30</td>
<td>Capital intensity of production of the non-residential sector</td>
<td>NIPA Table 6.2</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>1.81*</td>
<td>Elasticity of substitution between the two types of consumption</td>
<td>OLS regression in (4.1)</td>
</tr>
<tr>
<td>( A )</td>
<td>1.0149</td>
<td>Growth rate of the labor-augmented technological progress</td>
<td>Growth accounting</td>
</tr>
<tr>
<td>( \alpha_h )</td>
<td>0.10</td>
<td>Capital intensity of production of the housing sector</td>
<td>Ratio of hours labor and leisure in 1948</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.28</td>
<td>Preference weight of the consumption of nondurable goods and services</td>
<td>Ratio of non-residential and housing capital in 1948 ((k/h))</td>
</tr>
<tr>
<td>( z_0 )</td>
<td>10</td>
<td>Initial value of the labor-augmented technological progress</td>
<td>Relative output of non-residential sector in 1948 ((y/s))</td>
</tr>
</tbody>
</table>

Calibrated in the model

| \( \alpha_k \) | 0.10  | Capital intensity of production of the non-residential sector | Ratio of hours labor and leisure in 1948 |
| \( \eta \) | 0.28  | Preference weight of the consumption of nondurable goods and services | Ratio of non-residential and housing capital in 1948 \((k/h)\) |
| \( z_0 \) | 10    | Initial value of the labor-augmented technological progress | Relative output of non-residential sector in 1948 \((y/s)\) |

Note: parameters with \* will be varied for the robustness check.

4.1 summarizes the model parameters. First, I adopt the standard parameter values for \( \beta = 0.96 \) and \( \sigma = 1 \). Then, I estimate \( \delta, \alpha_k, A \) and \( \epsilon \) as follows:

\[
\delta_k = \delta_h = 0.06.
\]

Using the capital accumulation equation (3.3) in the model and data for the real non-residential capital, I back out a series of the implied depreciation rates of non-residential capital \( 1 - (k' - i_k)/k \). The value reported is an average over the sample. The depreciation rate on housing capital is calculated in a similar way.

\( \alpha_k = 0.30 \). The labor income share of GNP net of housing services is about 70 percent during the sample period between 1948 and 2005. Hence, the capital intensity of production of the non-residential sector is chosen to be 0.30.

\( A = 1.0149 \). The labor-augmented technological progress is estimated through the growth accounting equation below for the sample period between 1948 and 2005.

\[
\log z = \frac{1}{1-\alpha_k} \log y - \frac{\alpha_k}{1-\alpha_k} \log k - \log l
\]

The average growth rate of the technological progress is estimated to be 1.0149.

\( \epsilon = 1.81 \). Equation (3.6) suggests a way to evaluate the elasticity of substitution between the two types of consumption:

\[
\log \frac{c_{value}}{s_{value}} = \log \frac{\eta}{1-\eta} + \frac{\epsilon - 1}{\epsilon} \log \frac{c_{quantity}}{s_{quantity}} \tag{4.1}
\]

Hence, the coefficient \( \frac{\epsilon - 1}{\epsilon} \) can be estimated by regressing the log ratio of the nominal expenditure value between the two sectors on the log ratio of the quantities between the two sectors. The regression yields an estimate of \( \epsilon = 1.81 \), with a two standard error coefficient interval of \([1.64, 2]\).

The remaining parameters that need to be assigned values are \( \alpha_h \) and \( \eta \). Equation
(3.7) is referred to evaluate the capital intensity of production of the housing sector, using the ratio of hours for labor and leisure and the ratio of non-residential and housing capital in 1948 ($t = 0$ in the model). Further, (3.8) is referred to evaluate the preference weight between the two types of consumption to match the ratio of non-residential and housing capital, the consumption-output ratio, and the relative output ratio between the two sectors in 1948. For the initial values, I set $\kappa_0 = 0.4817$, which corresponds to the share of housing capital in 1948. $\hat{a}_0 = 10$ and $z_0 = 10$ are jointly chosen to match the range of the relative output ratio between the two sectors.

Table 4.2: Data and Benchmark Calibration, 1948 - 2005

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>US Data</th>
<th>1948</th>
<th>2005</th>
<th>1948</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.4817</td>
<td>0.4949</td>
<td>0.4817</td>
<td>0.5008</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>0.7740</td>
<td>0.8063</td>
<td>0.7819</td>
<td>0.7946</td>
<td></td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.6710</td>
<td>0.7041</td>
<td>0.5092</td>
<td>0.6159</td>
<td></td>
</tr>
<tr>
<td>$s/y$</td>
<td>0.1506</td>
<td>0.2226</td>
<td>0.1579</td>
<td>0.1653</td>
<td></td>
</tr>
</tbody>
</table>

Note: US Data from NIPA. Calibration described in the text.

Table 4.2 presents the comparison between the US data and the benchmark calibration. The first two rows show that the benchmark calibration is consistent with the allocation of capital and labor between the two sectors. In particular, the calibration matches the following feature of the data: capital is evenly allocated between the two sectors, and there is a slight reallocation of both capital and labor towards the housing sector between 1948 and 2005. The last two rows show that the benchmark calibration generates the increase in the consumption-output ratio and the relative output of the housing sector. Although the levels are slightly different, the increasing trend in both variables are captured by the benchmark calibration.

Table 4.3: Data and Model Calibration, 1948-2005 (Robustness)

<table>
<thead>
<tr>
<th>Model</th>
<th>US Data</th>
<th>$\epsilon = 1.64$</th>
<th>Model</th>
<th>$\epsilon = 2.52$</th>
<th>Model</th>
<th>$\epsilon = 3.70$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.4817</td>
<td>0.4949</td>
<td>0.4817</td>
<td>0.5088</td>
<td>0.4817</td>
<td>0.7101</td>
</tr>
<tr>
<td>$l$</td>
<td>0.7740</td>
<td>0.8063</td>
<td>0.7819</td>
<td>0.8520</td>
<td>0.7819</td>
<td>0.9043</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.6710</td>
<td>0.7041</td>
<td>0.5333</td>
<td>0.3555</td>
<td>0.5222</td>
<td>0.2044</td>
</tr>
<tr>
<td>$s/y$</td>
<td>0.1506</td>
<td>0.2226</td>
<td>0.1684</td>
<td>0.1488</td>
<td>0.1684</td>
<td>0.3377</td>
</tr>
</tbody>
</table>

Note: US Data from NIPA. Calibration described in the text.

For the robustness check, I consider different values in the intertemporal elasticity of substitution, the growth rate of the economy, and the elasticity of substitution between capital and leisure.

\footnote{In fact, the choice of $\hat{a}_0$ and $z_0$ provides degree of freedom of the calibration. The dynamics is sensitive to the choice of the two initial conditions, and $\hat{a}_0 = 10$ and $z_0 = 10$ is the pair that better matches the US data between 1948 and 2005, among all the attempted trials.}
the two sectors. While results with different $\sigma$ and $A$ are similar to the benchmark calibration, the one with different $\epsilon$ in Table 4.3 shows that while the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to different elasticities of substitution, a higher elasticity of substitution between the two sectors leads to a higher share of housing capital and a lower consumption-output ratio.

To summarize, the calibration indicates that the model can generate dynamics that are consistent with the US data. It matches the allocation of capital and labor, as well as the positive correlation between the share of housing capital and the consumption-output ratio. The robustness check shows that the positive correlation between the share of housing capital and the consumption-output ratio is robust, but the elasticity of substitution between the two sectors impacts the levels of the two variables.

### 4.3 Initial Conditions

To the extent that the model can reproduce the US data, I investigate the effect of the initial sectoral capital allocation on the economy. In particular, I consider the counterfactual experiment of starting the calibrated US economy with half of the initial share of housing capital, and examine whether it can explain the negative correlation of the share of housing capital and the consumption-output ratio observed in China.

Table 4.4 shows that the positive correlation between the share of housing capital and the consumption-output ratio remains. But the lower initial share of housing capital decreases the relative output of the housing sector during the transition. Moreover, the lower initial share of housing capital has a large impact on the speed of the sectoral reallocation, the relative price of housing services and the investment structure as shown in Figure 4.1

<table>
<thead>
<tr>
<th></th>
<th>US Data</th>
<th>Benchmark Calibration</th>
<th>Counterfactual Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.4817</td>
<td>0.4949</td>
<td>0.4817</td>
</tr>
<tr>
<td>$l$</td>
<td>0.7740</td>
<td>0.8063</td>
<td>0.7819</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.6710</td>
<td>0.7041</td>
<td>0.5092</td>
</tr>
<tr>
<td>$s/y$</td>
<td>0.1506</td>
<td>0.2226</td>
<td>0.1579</td>
</tr>
</tbody>
</table>

Note: US Data from NIPA. Calibration and counterfactual experiment described in the text.

When the economy starts at half of the initial share of housing capital of the benchmark calibration, Figure 4.1 shows that compared to the benchmark case, the share of

---

12 See Appendix A.3 for a detailed discussion on the robustness check.
13 This is because among all the institutional differences between the United States and China, an important one is that when the market mechanism starts to work in China, the share of housing capital in China is low as shown in B.3.
housing capital experiences a rapid increase during the transitional dynamics. It increases to a level that is higher than the steady-state value first, and then gradually falls back to the steady-state value. Further, the relative price of housing services jumps to a high level, and gradually fall down during the transitional dynamics. Compared to the benchmark, the economy that starts at half of the initial share of housing capital present a different investment structure: non-residential investment-output ratio experiences a large drop, while the housing investment-output ratio experiences a large leap.

In summary, the result from this numerical exercise shows that the positive correlation between the share of housing capital and the consumption-output ratio is robust with respect to a lower initial share of housing capital. But, the initial share of housing capital has a large impact on the speed of sectoral reallocation of capital, the relative price of housing services and the investment structure of the economy. Compared to the benchmark case, the economy that starts at half of the initial share of housing capital experiences a rapid increase in the share of housing capital, a higher level of relative price of housing services and a different investment structure during the transitional dynamics.

5 Calibration: China (1987 - 2013)

In section 4, I showed that the elasticity of substitution between the two sectors and the initial share of housing capital have significant impacts on the dynamics of the economy,
but neither different values in the elasticity of substitution nor the initial share of housing capital can explain the negative correlation between the share of housing capital and the consumption-output ratio observed in China. In this section, I calibrate the model to the Chinese economy between 1987 and 2013. I find that to match the negative correlation between the share of housing capital and the consumption-output ratio observed in the Chinese data, it requires a capital intensive production of the housing sector ($\alpha_k < \alpha_h$). Further, I quantify the effect of initial share of housing capital before the housing market reform on the Chinese economy during its 30-year transition.

5.1 Background Introduction and Data

In order to conduct a similar calibration exercise for China as the one for the US, I construct a dataset for capital stock in China. In particular, I choose the year 1987 as the initial period of the dataset. Based on the strand of literature that studies the Chinese housing market, I summarize the facts related to the housing market reform with respect to the following time frame, which justifies the reason for choosing 1987 as the initial period.

**Pre-1978:** Government established public ownership over all new housing stock. Housing was not a commodity, and urban households had little choices in housing consumption, which was provided by the government for a highly subsidized rental charge. In particular, housing investment was seriously neglected during the economic and political turmoil of the 1960s and 1970s. By 1978, per capita housing floor space was only 3.6 square meters.\(^{14}\)

**1979-1987:** Reforms of the housing system started in 1979. The aim of the reform is to decentralize the housing investment. The decentralization led the housing investment as a proportion of GNP to rise from an average of 1.5 percent before reform to over 7 percent during the 1980s. As a result, per capita housing floor space rose to 5.2 square meters by 1985.\(^{15}\)

**Post-1987:** In 1988, the Chinese government endorsed private property rights in urban land, and long-term land leases were granted for private real estate development. The housing construction has grown rapidly since then. In 1996, housing accounted for 86 percent of the building floor area sold. Since 1997, the housing investment has been high compared with that in other countries. Until 2012, per capita housing floor space has improved to 32.7 square meters and the housing investment accounts for about 15 percent of the total fixed asset investment in China.\(^{16}\)

Since the large scale of housing construction started after the Chinese government policy at the end of 1987, I choose the year 1987 as the initial period for the dataset. Data for the consumption of nondurable goods and services, the non-residential investment and the housing investment are from China Statistical Yearbook (CSY). A drawback of


\(^{15}\)See Tolley et al. (1991), Dorcey et al. (1992) and Yearbook (1997).

CSY is the lack of the categorized consumption data for the total population, with only the categorized consumption for per urban capita. To get a consistent measure, all the variables are in the unit of per urban capita, computed through dividing all the variables for urban population by the size of urban population.\footnote{CSY Data for the urban investment of total fixed assets starts from 1995. For the missing years between 1987 and 1994, data are estimated by 78\% (1995 fraction) of the investment of total fixed assets; CSY Data for the urban investment for residential buildings starts from 1995. For the missing years between 1987 and 1994, data are estimated by 20\% (1995 fraction) of the urban investment of total fixed assets; CSY Data for the urban consumption per capita miss the years 1991-1994, 1996-1999, and 2001-2009. For the missing years, data are estimated by interpolation.}

### 5.2 Calibration

Table 5.1 summarizes the parameters. For the calibration exercise for China, I take the parameter values $\beta = 0.96$, $\sigma = 1$ and $\delta = 0.08$. Since there is no available hours worked for labor in China, the growth accounting for the growth rate of the labor-augmented technological progress cannot be conducted for China. For the benchmark calibration in China, I choose $A = 1.04$, leaving other possible values for $A$ for the robustness check.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (China)</th>
<th>Value (US)</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>0.96</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.00</td>
<td>1.00</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.08</td>
<td>0.06</td>
<td>Blanchard and Cooper (2006)</td>
</tr>
<tr>
<td>$A$</td>
<td>1.0400*</td>
<td>1.0149</td>
<td>By assumption</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.28</td>
<td>0.28</td>
<td>By assumption</td>
</tr>
<tr>
<td>$\alpha_k$</td>
<td>0.30</td>
<td>0.30</td>
<td>By assumption</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>3.70*</td>
<td>1.81</td>
<td>OLS regression in (4.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (China)</th>
<th>Value (US)</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_h$</td>
<td>0.50*</td>
<td>0.10</td>
<td>Consumption-output ratio</td>
</tr>
<tr>
<td>$z_0$</td>
<td>5</td>
<td>10</td>
<td>Relative output of housing sector</td>
</tr>
</tbody>
</table>

Note: The growth rate $A$ is a parameter left for robustness check.

To estimate the elasticity of substitution between the two sectors, I follow the same approach for the US, by regressing the log ratio of nominal expenditure value between the two sectors on the log ratio of the quantities between the two sectors.\footnote{The depreciation rate is from Blanchard and Cooper (2006), implying the useful lives of the capital stock is about 12 years.} The estimation for the elasticity of substitution between the two sectors is 3.70, with a two standard error coefficient interval \([2.86, 5.26]\). Compared with $\epsilon = 1.81$ in the US (with a two standard error coefficient interval \([1.64, 2]\)), the estimation indicates a more substitutable preference between the two sector for Chinese households.

Given that there is no labor-capital composition of national income in CSY as in NIPA, the calibration strategy is different for the capital intensities of production of both sectors and the preference weight between the two sectors, i.e., $\alpha_k$, $\alpha_h$ and $\eta$. For the benchmark

\footnote{The nominal values are obtained from CSY. For quantities, I divide the nominal values by the price indices by categories in CSY, of which the available years are from 2001 to 2013.}
calibration, I assume that the preference weight between the two types of consumption for Chinese households is the same with that for US households, i.e. $\eta = 0.28$. To match the negative correlation between the share of housing capital and the consumption-output ratio, the capital intensity of production of the housing sector is set to be $\alpha_h = 0.50$ if assuming the capital intensity of production of the non-residential sector is the same with that of the US at $\alpha_k = 0.30$.

<table>
<thead>
<tr>
<th></th>
<th>China Data</th>
<th>Benchmark Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.3035 0.4580</td>
<td>0.3035 0.8703</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.3489 0.1985</td>
<td>0.5479 0.3230</td>
</tr>
<tr>
<td>$s/y$</td>
<td>0.0505 0.0390</td>
<td>0.0582 0.8411</td>
</tr>
</tbody>
</table>

Note: China Data from CSY(2014). Calibration described in the text.

Table 5.2 presents the benchmark calibration of China. It shows that the benchmark calibration generates the negative correlation between the share of housing capital and the consumption-output ratio in the data, although it does not match the level of both variables, given the potential issue of missing data. Meanwhile, the benchmark calibration captures the rapid increase in the share of housing capital as shown in Figure 5.1.

To summarize, I present a benchmark calibration for China between 1987 and 2013. The benchmark calibration assumes that the preference weight between the two types of consumption for Chinese households and the capital intensity of production of the non-residential sector are the same with the US economy. With a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector and a low initial share of housing capital, the calibration accounts for the fast-growing share of housing capital, and the negative correlation between the share of housing capital and the consumption-output ratio between 1987 and 2013 in China.
5.3 Initial Sectoral Allocation

To the extent that the calibration can reproduce the key feature of the Chinese data, I apply the model to quantify the effect of the initial share of housing capital in 1987 on the Chinese economy. The approach is to compare two calibrated economies, one started with the share of housing capital at the China level in 1987, and the other started with the share of housing capital at the US level of a comparable development stage.\(^{20}\)

In order to find an appropriate development stage in the US economy to refer to, I resample the US data between 1930 and 2013, using a moving block bootstrap approach proposed by Politis and Romano (1994). I find that China in 1987 is most close to the United States in 1956, when using the joint correlation of the consumption-output ratio, the non-residential capital-output ratio, and the housing capital-output ratio as a measure of resemblance.\(^{21}\)

Consider the two calibrated economies, one started with the share of housing capital at the US level in 1956 (\(\kappa_0 = 0.47\)), and the other started with the share of housing capital at the China level in 1987 (\(\kappa_0 = 0.30\)). Figure 5.3 shows the effect of raising the initial share of housing capital from the China level in 1987 to the US level in 1956 on the economy. The transitional dynamics for the non-residential investment-output ratio,

\[^{20}\]To be specific, both economies are calibrated to China.
\[^{21}\]See the Appendix C for a statistical comparison between the US economy and China.
the housing investment-output ratio, the relative output of the housing sector, and the relative price of housing services are compared. It is notable that the investment structure is strongly affected by the initial sectoral allocation of capital as shown in panel (a) and (b). This is because the low initial share of housing capital at the China level in 1987 raises the marginal return on the housing investment. An implication of Figure 5.3 is that the resources that should have been allocated to the non-residential sector are induced to housing during the 30-year transition in China.

Table 5.3: China Benchmark vs. the US allocation

<table>
<thead>
<tr>
<th></th>
<th>s/y</th>
<th>t_k/y</th>
<th>t_h/y</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>China benchmark</td>
<td>0.1331</td>
<td>0.2493</td>
<td>0.3113</td>
<td>5.0176</td>
</tr>
<tr>
<td>US allocation</td>
<td>0.1482</td>
<td>0.3063</td>
<td>0.2611</td>
<td>4.6506</td>
</tr>
</tbody>
</table>

Table 5.3 quantifies the impact of the initial share of housing capital at the China level in 1987 on the economy. It shows that when changing the initial share of housing capital from the China level in 1987 to the US level in 1956, the mean relative output of the housing sector and the mean non-residential investment-output ratio increase by 11.3 percent and 22.9 percent respectively. The mean housing investment-output ratio and the mean relative price of housing services decrease by 16.1 percent and 10 percent respectively. These numbers suggest that the initial low share of housing capital might lead to an over-investment in housing, an under-investment in non-residential capital, and a higher relative price of housing services during its 30-year transition in China.

6 Conclusion

I propose a two-sector general equilibrium model to study the sectoral allocation of capital between housing and nonresidential sector in a neoclassical growth environment. The model features preferences with a constant elasticity of substitution between the two sectors, and Cobb-Douglas production technologies within each sector. The equilibrium dynamics of the model imply that the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors hold the key to understanding the correlation between the sectoral capital allocation and the consumption-output ratio.

Calibrated to the United States and China, the model can account for the positive correlation between the share of housing capital and the consumption-output ratio in the United States on the one hand, and the negative correlation between the share of housing capital and the consumption-output ratio in China on the other hand. In particular, the calibration to China implies that the rapid increase in the share of housing capital and the simultaneous decrease in the consumption-output ratio observed in the Chinese data can be explained by a combination of three factors: a high elasticity of substitution between the two sectors, a high capital intensity of production of the housing sector, and a low initial share of housing capital before the Chinese housing market reform.
The initial low share of housing capital before the reform has a large impact on the Chinese economy. In a counterfactual experiment, when changing the initial share of housing capital at the China level in 1987 to the US level of a comparable development stage, the mean relative output of the housing sector and the mean non-residential investment-output ratio increase by 11.3 percent and 22.9 percent respectively. Meanwhile, the mean housing investment-output ratio and the mean relative prices of housing services decrease by 16.1 percent and 10 percent respectively. The comparative study suggests that the initial low share of housing capital in China has led to an over-investment in housing, an under-investment in non-residential capital and a higher relative price of housing services during its 30-year transition.
References


A  Appendix

A.1 Derivation of equilibrium conditions

Given the state variables \{\hat{k}, \hat{h}\}, i.e. the capital stock in the two sectors. \{l, \hat{k}', \hat{h}'\} are chosen to maximize the following value function:

\[
v(\hat{k}, \hat{h}) = \max_{l, \hat{k}', \hat{h}'} \left\{ u(\hat{c}, \hat{s}) + \beta v(\hat{k}', \hat{h}') \right\}
\]

s.t.:

\[
\hat{c} = \hat{k}^{\alpha_k} (1 - l)^{1-\alpha_k} + (1 - \delta)(\hat{k} + \hat{h}) - A \cdot (\hat{k}' + \hat{h}')
\]

\[
\hat{s} = \hat{h}^{\alpha_h} l^{1-\alpha_h}
\]

Substituting \(\hat{c}\) and \(\hat{s}\) into the value function, one obtains the dynamic programming problem as follows:

\[
v(\hat{k}, \hat{h}) = \max_{l, \hat{k}', \hat{h}'} u(\hat{k}^{\alpha_k} (1 - l)^{1-\alpha_k} + (1 - \delta)(\hat{k} + \hat{h}) - A \cdot (\hat{k}' + \hat{h}'), \hat{h}^{\alpha_h} l^{1-\alpha_h}) + \beta v(\hat{k}', \hat{h}')
\]

FOC w.r.t \([l]\\):

\[
\frac{u_c}{u_s} = \frac{(1 - \alpha_h) \hat{h}^{\alpha_h} l^{1-\alpha_h}}{(1 - \alpha_k) \hat{k}^{\alpha_k} (1 - l)^{1-\alpha_k}} \quad (A.1)
\]

FOC w.r.t \([\hat{k}']\\):

\[
A \cdot u_c = \beta u_k(\hat{k}', \hat{h}') = \beta u_c' \cdot \left( \alpha_k (\hat{k}')^{\alpha_k - 1} (1 - l')^{1-\alpha_k} + 1 - \delta \right) \quad (A.2)
\]

FOC w.r.t \([\hat{h}']\\):

\[
A \cdot u_c = \beta u_h(\hat{k}', \hat{h}') = \beta \left[ u_c' \cdot (1 - \delta) + u_s' \cdot \alpha_h (\hat{h}')^{\alpha_h - 1} (l')^{1-\alpha_h} \right] \quad (A.3)
\]

Combining equation \(A.2\) and \(A.3\):

\[
\frac{u_c}{u_s} = \frac{\alpha_h \hat{h}^{\alpha_h - 1} l^{1-\alpha_h}}{\alpha_k \hat{k}^{\alpha_k - 1} (1 - l)^{1-\alpha_k}} \quad (A.4)
\]

Combining equation \(A.1\) and \(A.4\):

\[
\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k (1 - \alpha_h)}{\alpha_h (1 - \alpha_k)} \cdot \frac{1 - l}{l} \quad (A.5)
\]

Equation \(A.5\) is identical to equation 3.7 from firm profit optimization. Further, rearranging equation \(A.4\) gives:

\[
\frac{\hat{k}}{\hat{h}} = \frac{\alpha_k}{\alpha_h} \cdot \frac{u_c}{u_s} \cdot \frac{\hat{y}}{\hat{s}} \quad (A.6)
\]
Equation A.6 is identical to the non-arbitrage condition 3.8.

Further, equation A.2 can be rewritten into Euler Equation as in 3.12:

\[
\frac{u_c}{u'_c} = \beta \left[ \alpha_k \cdot \frac{\hat{y}' \cdot \hat{k}' + 1 - \delta}{K} \right] \tag{A.7}
\]

Given a CES utility function, the marginal utility of \( c \) is given by:

\[
u_c = \eta \cdot \left[ \eta \hat{c}^{\frac{\epsilon - 1}{\epsilon}} + (1 - \eta) \hat{s}^{\frac{1 - \sigma}{1 - \epsilon}} \cdot \hat{c}^{-\frac{1}{\epsilon}} \right].\] Therefore, one obtains the same Euler equation with 3.12.

A.2 Proof

1. Proof for Proposition 1:

First, show that all economic variables grow at the same rate \( A \). The aggregate resource constraint 3.5 implies that \( y, c, a \) all have to grow at the same rate, denoted by \( G \) along a balanced growth path. \( a = k + h \) implies that \( k \) and \( h \) also grows at \( G \). Further, production function in the nonresidential sector implies \( G = G^{\alpha_k} A^{1 - \alpha_k} \). Also, production function in the housing sector implies that growth rate in the housing sector is \( G = A^{\alpha_h} A^{1 - \alpha_h} = A \). Thus, along the balanced growth path, variable \( y, c, s, a \) all grow at \( G = A \).

Next, compute the steady-state share of capital in the housing sector \( \kappa^* \) and the steady-state capital per capita \( \hat{a}^* \). Euler equation 3.12 implies that along the BGP,

\[
\frac{\hat{a}^*}{z_0 \cdot (1 - l^*)} = \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{1 - \alpha_k} \frac{1}{(1 - \kappa^*)^{-1}} \tag{A.8}
\]

Dynamics of capital accumulation 3.11 implies that along the BGP,

\[
\left( \frac{\hat{c}}{\hat{y}} \right)^* = 1 - (A + \delta - 1) \left( \frac{\hat{a}}{\hat{y}} \right)^* = 1 - (A + \delta - 1) \left[ \frac{\hat{a}^*}{z_0 \cdot (1 - l^*)} \right]^{1 - \alpha_k} \frac{1}{(1 - \kappa^*)^{-1}} \tag{A.9}
\]

Combined with A.8 implies that along the BGP,

\[
\left( \frac{\hat{c}}{\hat{y}} \right)^* = 1 - (A + \delta - 1) \cdot \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \cdot (1 - \kappa^*)^{-1} \tag{A.9}
\]

Production functions of both sectors 3.2 imply that along the BGP,

\[
\hat{y}^* = (1 - \kappa^*)^{\alpha_k} \cdot \left[ \frac{\hat{a}^*}{z_0 (1 - l^*)} \right]^{\alpha_k} \cdot [z_0 (1 - l^*)] \\
\hat{s}^* = (\kappa^*)^{\alpha_h} \cdot \left[ \frac{\hat{a}^*}{z_0 (1 - l^*)} \right]^{\alpha_h} \cdot \left( \frac{1 - l^*}{l^*} \right)^{\alpha_h} \cdot (z_0 l^*)
\]

27
Substituting equation (3.10) and (A.8) into the expressions above implies:

\[
\hat{y}^* = \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{\frac{\alpha_h}{1-\alpha_k}} \cdot [z_0(1 - l^*)]
\]

\[
\hat{s}^* = \left[ \frac{\alpha_h(1 - \alpha_k)}{\alpha_k(1 - \alpha_h)} \right]^{\alpha_h} \cdot \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{\frac{\alpha_h}{1-\alpha_k}} \cdot (z_0 l^*)
\]

Hence,

\[
\left( \frac{\hat{y}}{\hat{s}} \right)^* = \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{1-\alpha_h} \cdot \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{\frac{\alpha_h}{1-\alpha_k}} \cdot \frac{1 - \kappa^*}{\kappa^*}
\]  

(A.10)

Substituting equations (A.9) and (A.10) into equation (3.9) one obtains:

\[
\kappa^* = \frac{1 - n}{1 + m}
\]

where \( m = \left[ \frac{\alpha_h}{\alpha_k} \cdot \eta \right]^{\frac{\alpha_h(1 - \alpha_k)(\alpha_h - 1)}{\alpha_k(1 - \alpha_h)}} \cdot \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{\frac{\alpha_h}{1-\alpha_k}} \cdot \frac{1}{\kappa^*} \)

Substituting \( \kappa^* \) into (A.8) one obtains:

\[
\hat{a}^* = z_0 \cdot \left[ \frac{\alpha_k}{A^\sigma / \beta - (1 - \delta)} \right]^{1-\alpha_k} \cdot \frac{\alpha_h(1 - \alpha_k)}{\alpha_k(1 - \alpha_h)(1 - \kappa^*) + \alpha_k(1 - \alpha_h)\kappa^*}
\]

For capital per effective labor in equation (A.8) to be nonnegative, it needs:

\[
\frac{A^\sigma}{\beta} > 1 - \delta
\]  

(A.11)

Further, for \( \kappa^* \) to take a plausible value within \((0, 1)\), \( n \) needs to be within \((0, 1)\), which implies:

\[
\frac{A^\sigma}{\beta} > \alpha_k A + (1 - \alpha_k)(1 - \delta) \quad \& \quad A > 1 - \delta
\]  

(A.12)

The two inequalities (A.11) and (A.12) implies the following parameter ranges:

\[
\frac{A^\sigma}{\beta} > \alpha_k A + (1 - \alpha_k)(1 - \delta) \quad \& \quad A > 1 - \delta
\]  

(A.13)

Q.E.D.

2. **Proof for Proposition 2:**

\[
\frac{d\kappa^*}{d(1 - \eta)} = \frac{d\kappa^*}{n} = \frac{(1 - n)m}{(1 + m)^2} \cdot \frac{d\log m}{d\eta}
\]

Note that along BGP, the intuition for \( \frac{A^\sigma}{\beta} = 1 + r^* \), i.e. another interpretation of the parameter range is \( r^* \in (\alpha_k(A + \delta - 1) - \delta, \infty) \).

22
Since \( n < 1 \) and \( \frac{d \log m}{d \eta} = \frac{\epsilon}{(1-\eta)\eta} \), \( \frac{d \kappa^*}{d \eta} > 0 \).

\[
\frac{d \ln \kappa^*}{d \sigma} = -\frac{n}{1-n} \cdot \frac{d \ln n}{d \sigma} - \frac{m}{1+m} \cdot \frac{d \ln m}{d \sigma} = \frac{A^\sigma \cdot \ln A}{A^\sigma - \beta(1-\delta)} \left[ \frac{n}{1-n} + \frac{m}{1+m} \cdot \frac{(\alpha_k - \alpha_h)(\epsilon - 1)}{1 - \alpha_k} \right]
\]

Therefore, \( \frac{d \kappa^*}{d \sigma} > 0 \), if \( \epsilon > 1 - \frac{n}{1-n} \cdot \frac{1+\alpha_k}{\alpha_k - \alpha_h} \). Given the parameter ranges of the model, \( \frac{n}{1-n} \cdot \frac{1+\alpha_k}{\alpha_k - \alpha_h} > 1 \). Therefore, \( \frac{d \kappa^*}{d \sigma} > 0 \) for \( \forall \epsilon > 0 \).

\[
\frac{d \ln \kappa^*}{d \epsilon} = -\frac{m}{1+m} \cdot \frac{\partial \ln m}{\partial \epsilon}
\]

Therefore, \( \frac{d \kappa^*}{d \epsilon} > 0 \), if \( \eta \in \left( 0, \frac{1}{1+\omega} \right) \), where \( \omega = \left( \frac{\alpha_k}{\alpha_h} \right)^{\alpha_h} \cdot \left( \frac{1-\alpha_k}{1-\alpha_h} \right)^{\alpha_h-1} \cdot \left( \frac{A^\sigma}{A^\sigma - \beta - (1-\delta)} \right)^{\alpha_k-\alpha_h} \).

Q.E.D.

3. Proof for Proposition 3:

\[
d\hat{a}^* = \left[ \frac{\alpha_k}{A^\sigma - \beta - (1-\delta)} \right]^{\frac{1}{1-\alpha_k}} \cdot \frac{\alpha_h(1-\alpha_k)}{\alpha_h(1-\alpha_k) + \alpha_k(1-\alpha_h)\kappa^*} > 0
\]

\[
d\hat{a}_0^* = \frac{d \ln \hat{a}^*}{d \hat{a}_0^*} \cdot \hat{a}_0^* = -\frac{1}{1-\alpha_k} \cdot \frac{A^\sigma - \beta(1-\delta)}{A^\sigma} < 0
\]

\[
d\hat{a}^* = \frac{d \ln \hat{a}^*}{d \hat{a}^*} \cdot \hat{a}^* = -\frac{(\alpha_k - \alpha_h)\hat{a}^*}{\alpha_h(1-\alpha_k)(1-\kappa^*) + \alpha_k(1-\alpha_h)\kappa^*}
\]

In the last expression, if \( \alpha_k > \alpha_h \), \( \frac{d \hat{a}^*}{d \epsilon} < 0 \); if \( \alpha_k < \alpha_h \), \( \frac{d \hat{a}^*}{d \epsilon} > 0 \)

Q.E.D.

4. Proof for Proposition 4:

Equation 3.9 implies:

\[
\ln \kappa = -\ln \left( 1 + \frac{\alpha_k}{\alpha_h} \cdot \frac{\eta}{1-\eta} \cdot \left( \frac{\hat{y} - \hat{y}}{\hat{y}} \right)^{1-1/\epsilon} \cdot \left( \frac{\hat{c}}{\hat{y}} \right)^{-1/\epsilon} \right)
\]
It follows:

\[
\frac{d \ln \kappa}{d \hat{a}} = -\frac{1}{\kappa} \frac{\alpha_k}{\alpha_h} \eta \cdot \frac{d \left( \frac{\hat{y}}{\hat{s}} \right)^{1-1/\epsilon} \cdot \left( \frac{\hat{c}}{\hat{y}} \right)^{-1/\epsilon}}{d \hat{a}} \\
\approx - \left[ \left( \frac{\hat{y}}{\hat{s}} \right)^{1-1/\epsilon} \cdot \frac{d (\hat{c}/\hat{y})^{-1/\epsilon}}{d \hat{a}} + \left( \frac{\hat{c}}{\hat{y}} \right)^{-1/\epsilon} \cdot \frac{d (\hat{y}/\hat{s})^{1-1/\epsilon}}{d \hat{a}} \right] \\
= - \left[ \frac{1}{\epsilon} \left( \frac{\hat{c}}{\hat{s}} \right)^{1-1/\epsilon} \cdot \left( \frac{\hat{c}}{\hat{y}} \right)^{-1} \cdot \frac{d \ln (\hat{c}/\hat{y})}{d \ln \hat{a}} + \left( \frac{\hat{c}}{\hat{y}} \right)^{-1/\epsilon} \cdot \frac{d (\epsilon - 1)(\alpha_k - \alpha_h)}{\epsilon} \cdot \left( \frac{\hat{y}}{\hat{s}} \right) + \frac{1}{\hat{a}} \right] \\
\propto \frac{d \ln (\hat{c}/\hat{y})}{d \ln \hat{a}} - (\epsilon - 1)(\alpha_k - \alpha_h)
\]

This is because \( \frac{\hat{y}}{\hat{s}} = \frac{(1-\kappa)\alpha_k (1-\eta)\epsilon}{\kappa^{\alpha_h}} \cdot \hat{a}^{\alpha_k - \alpha_h} \) and therefore:

\[
\frac{d \ln \kappa}{d \ln \hat{a}} \approx \left\{ \frac{d \ln (\hat{c}/\hat{y})}{d \ln \hat{a}} - (\epsilon - 1)(\alpha_k - \alpha_h) \right\}
\]

Hence \( \frac{d \ln \kappa}{d \ln \hat{a}} > 0 \iff \frac{d \ln (\hat{c}/\hat{y})}{d \ln \hat{a}} > (\epsilon - 1)(\alpha_k - \alpha_h) \).

The expression above shows that the correlation between the share of housing capital and the consumption-output ratio is determined by the interplay of the elasticity of substitution between the two sectors and the capital intensities of production of the two sectors. To demonstrate the implications and the mechanism of this proposition, I use the following numerical example for illustration. Suppose that a model economy has already reached its balanced growth path.\(^{23}\) Three key parameters of the model are set as: \( \beta = 0.96, \eta = 0.28, \sigma = 1, \delta = 0.06, A = 1.04, z_0 = 10. \)

In particular, for case 1, \( \alpha_k = 0.3 \) and \( \alpha_h = 0.1 \); for case 2, \( \alpha_k = 0.3 \) and \( \alpha_h = 0.5 \). \(^{24}\) For scenario (a), \( \epsilon = 1.81 \); for scenario (b), \( \epsilon = 0.76 \).

**Case 1: When the production of housing sector is labor intensive \( (\alpha_k > \alpha_h) \)**

Suppose that there is a permanent unexpected shock to the growth rate \( A \) such that the economy enters transitional dynamics with capital deepening. Figure A.1 shows the comovement of the share of housing capital and consumption-output ratio for both scenarios when

---

\(^{23}\) Three key parameters of the model \( \epsilon, \alpha_k \) and \( \alpha_h \) are varied for illustration. The other parameters of the model economy are set as: \( \beta = 0.96, \eta = 0.28, \sigma = 1, \delta = 0.06, A = 1.04, z_0 = 10. \)

\(^{24}\) In particular, for case 1, \( \alpha_k = 0.3 \) and \( \alpha_h = 0.1 \); for case 2, \( \alpha_k = 0.3 \) and \( \alpha_h = 0.5 \).
Case 2: When the production of housing sector is capital intensive ($\alpha_k < \alpha_h$)

Suppose that there is a permanent shock to the growth rate $A$ such that the economy enters transitional dynamics with capital deepening. Figure A.2 shows the countermovement of the share of housing capital and consumption-output ratio for both scenarios when $\epsilon > 1$ and $\epsilon < 1$.

Mechanism

Consider the case when the production of housing sector is labor intensive, and the
two sectors are more substitutable ($\alpha_k > \alpha_h$ and $\epsilon > 1$). By Proposition 4, the share of housing capital comoves with the consumption-output ratio. Figure A.3 (a) present the arbitrage opportunities if Proposition 4 is violated, and Figure A.3 (b) shows the case otherwise.

![Figure A.3: Return to the housing capital $R_h$](image)

Q.E.D.

### A.3 Robustness Check

Tables A.1 and A.2 show alternative calibrations of the model economy, in which I consider different values for the intertemporal elasticity of substitution and the growth rate of the economy. The results in Table A.1 are similar to those of the benchmark calibration. The implication of capital and labor reallocation are basically identical to the benchmark calibration. Also, the relative output between the non-residential and the housing sector increases in the cases for all three $\sigma$, implying the increase of the relative prices of housing services. Meanwhile, the general patterns implied by the different values of $A$ in Table A.2 are also similar to the results of the benchmark calibration.

#### Table A.1:
Data and Model Calibration, 1948-2005 (Robustness I)

<table>
<thead>
<tr>
<th>US Data</th>
<th>Model $\sigma = 0.5$</th>
<th>Model $\sigma = 2$</th>
<th>Model $\sigma = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
</tr>
<tr>
<td>1948</td>
<td>0.4817</td>
<td>0.4817</td>
<td>0.4817</td>
</tr>
<tr>
<td>2005</td>
<td>0.4949</td>
<td>0.5008</td>
<td>0.5009</td>
</tr>
<tr>
<td>$\ell$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
</tr>
<tr>
<td>1948</td>
<td>0.7740</td>
<td>0.7819</td>
<td>0.7819</td>
</tr>
<tr>
<td>2005</td>
<td>0.8063</td>
<td>0.7947</td>
<td>0.7947</td>
</tr>
<tr>
<td>$c/y$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
</tr>
<tr>
<td>1948</td>
<td>0.6710</td>
<td>0.4766</td>
<td>0.6159</td>
</tr>
<tr>
<td>2005</td>
<td>0.7041</td>
<td>0.6159</td>
<td>0.6159</td>
</tr>
<tr>
<td>$s/y$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
<td>$\epsilon = 1.81$, $A = 1.0149$</td>
</tr>
<tr>
<td>1948</td>
<td>0.1506</td>
<td>0.1653</td>
<td>0.1653</td>
</tr>
<tr>
<td>2005</td>
<td>0.2226</td>
<td>0.1653</td>
<td>0.1653</td>
</tr>
</tbody>
</table>
Table A.2:
Data and Model Calibration, 1948 - 2005 (Robustness II)

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>κ = 1.81, σ = 1</td>
<td>ε = 1.81, σ = 1</td>
<td>ε = 1.81, σ = 1</td>
</tr>
<tr>
<td>κ</td>
<td>0.4817</td>
<td>0.4949</td>
<td>0.4817</td>
</tr>
<tr>
<td>l</td>
<td>0.7740</td>
<td>0.8063</td>
<td>0.7819</td>
</tr>
<tr>
<td>c/y</td>
<td>0.6710</td>
<td>0.7041</td>
<td>0.5378</td>
</tr>
<tr>
<td>s/y</td>
<td>0.1506</td>
<td>0.2226</td>
<td>0.1370</td>
</tr>
</tbody>
</table>

B  Data Constructions for China: 1987-2013

In [Piketty and Zucman (2014)], data for the developed economies are from their national capital accounts, including consistent annual balance sheets. In contrast, China does not have such a national account for capital stock, giving rise to challenges in measuring capital allocations in China. In this paper, I construct a dataset of capital stock in China that is similar to the measure of capital stock for the five developed economies as in [Piketty and Zucman (2014)]. In the following, I describe the approaches to construct this dataset.

B.1 Agricultural Land Value in China

In order to measure the value of agricultural land in China, data for both the land price and the land area are needed. The data for the land price index in China is not available until 2000. To obtain a consistent time series data for the land price in China, I take the 30 percent of the average sales price for commercial residential housing in China as a proxy for the missing data between 1987 and 2000. It is not the most accurate measure, but is still reasonable given a high correlation between the land price index and the average sales price for commercial residential housing after 2000 in China.

To obtain the time series data for the area of agricultural land between 1987 and 2012, I interpolate the time series from the data points obtained from various sources as shown in Table B.1. Figure B.1 shows that the total area of China’s agricultural land has been steadily decreasing. Table B.2 shows the dataset constructed for the agricultural land value to output ratio in China between 1987-2013, which shows that while the unit land

---

26 Following new international guidelines, the balance sheets report on the market value of all the non-financial and financial assets and liabilities held by each sector of the economy (households, government and corporations) and by the rest of the world.

27 Land price index data after 2000 can be found from Ministry of Land and Resource of China.

28 This is because the availability of the average sales price for commercial residential housing from Chow and Niu (2015). In Davis and Heathcote (2007), the land value is approximately 30% of the housing sales value.

29 Smil (1999) made the effort in collecting data for the agricultural land use from various sources, and I found two more recent data points from China Statistical Yearbook (2014) and the Land and Resource Ministry of China
Table B.1: Data Sources for Areas of Agricultural Land in China

<table>
<thead>
<tr>
<th>Year</th>
<th>Mha</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>137.3</td>
<td>Institute of Applied Remote Sensing CAS</td>
</tr>
<tr>
<td>1994</td>
<td>136.4</td>
<td>Chuanjun and Huancheng (1994)</td>
</tr>
<tr>
<td>1995</td>
<td>131.1</td>
<td>State Land Administration Bureau</td>
</tr>
<tr>
<td>1996</td>
<td>133.3</td>
<td>State Land Administration Bureau</td>
</tr>
<tr>
<td>2008</td>
<td>96.0</td>
<td>China Statistical Yearbook (2014)</td>
</tr>
<tr>
<td>2012</td>
<td>65.0</td>
<td>Land and Resource Ministry of China</td>
</tr>
</tbody>
</table>

The price (yuan per sq.m) has been increasing, the area of agricultural land keeps decreasing, and the agricultural land value to output ratio is also decreasing.

![Figure B.1: Areas of Agricultural Land in China: 1987-2012](image)

B.2 Non-residential and Housing Capital in China

To measure the value of non-residential and housing capital in China, I take the standard perpetual inventory approach. I initialize the housing capital in 1994 as the ratio of the housing investment in 1995\(^\text{30}\) to the sum of the average growth rate of the housing investment between 1995 and 2000 and the depreciation rate of capital. Further, I estimate the housing investment for the missing years between 1987 and 1994 by 40 percent of the investment in construction and installment\(^\text{31}\) and then back out the housing capital.

\(^\text{30}\)the first year when the investment data in housing are available
\(^\text{31}\)In China Statistical Yearbook, data for the investment in construction and installment started earlier from 1981. The available data show that the housing investment is about 40 percent of the investment in
Table B.2: Agricultural Land Value to Output Ratio

<table>
<thead>
<tr>
<th>Year</th>
<th>Mha (10^10 sq.m)</th>
<th>Land price (yuan/sq.m)</th>
<th>Land value to output Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>137.3*</td>
<td>122.5</td>
<td>139.52</td>
</tr>
<tr>
<td>1988</td>
<td>137.2</td>
<td>150.9</td>
<td>137.65</td>
</tr>
<tr>
<td>1989</td>
<td>137.1</td>
<td>172.1</td>
<td>138.72</td>
</tr>
<tr>
<td>1990</td>
<td>137.0</td>
<td>210.9</td>
<td>154.28</td>
</tr>
<tr>
<td>1991</td>
<td>136.8</td>
<td>226.9</td>
<td>142.25</td>
</tr>
<tr>
<td>1992</td>
<td>136.7</td>
<td>298.9</td>
<td>151.74</td>
</tr>
<tr>
<td>1993</td>
<td>136.6</td>
<td>362.5</td>
<td>140.45</td>
</tr>
<tr>
<td>1994</td>
<td>136.4*</td>
<td>358.2</td>
<td>101.56</td>
</tr>
<tr>
<td>1995</td>
<td>131.1*</td>
<td>452.7</td>
<td>99.22</td>
</tr>
<tr>
<td>1996</td>
<td>133.3*</td>
<td>481.4</td>
<td>91.48</td>
</tr>
<tr>
<td>1997</td>
<td>130.0</td>
<td>536.9</td>
<td>89.40</td>
</tr>
<tr>
<td>1998</td>
<td>126.7</td>
<td>556.1</td>
<td>84.88</td>
</tr>
<tr>
<td>1999</td>
<td>123.6</td>
<td>557.1</td>
<td>77.81</td>
</tr>
<tr>
<td>2000</td>
<td>120.5</td>
<td>584.5</td>
<td>71.87</td>
</tr>
<tr>
<td>2001</td>
<td>117.5</td>
<td>605.0</td>
<td>65.78</td>
</tr>
<tr>
<td>2002</td>
<td>114.6</td>
<td>627.5</td>
<td>60.36</td>
</tr>
<tr>
<td>2003</td>
<td>111.7</td>
<td>659.2</td>
<td>54.55</td>
</tr>
<tr>
<td>2004</td>
<td>108.9</td>
<td>764.6</td>
<td>52.23</td>
</tr>
<tr>
<td>2005</td>
<td>106.2</td>
<td>881.1</td>
<td>50.96</td>
</tr>
<tr>
<td>2006</td>
<td>103.6</td>
<td>935.8</td>
<td>44.88</td>
</tr>
<tr>
<td>2007</td>
<td>101.0</td>
<td>1093.6</td>
<td>41.45</td>
</tr>
<tr>
<td>2008</td>
<td>96.0*</td>
<td>1072.7</td>
<td>32.58</td>
</tr>
<tr>
<td>2009</td>
<td>88.8</td>
<td>1337.8</td>
<td>34.91</td>
</tr>
<tr>
<td>2010</td>
<td>82.1</td>
<td>1417.5</td>
<td>29.12</td>
</tr>
<tr>
<td>2011</td>
<td>76.0</td>
<td>1498.0</td>
<td>24.29</td>
</tr>
<tr>
<td>2012</td>
<td>65.0*</td>
<td>1629.0</td>
<td>20.43</td>
</tr>
</tbody>
</table>

between 1987 and 1993 following the same perpetual inventory approach. The values of nonresidential capital are estimated using the same approach. In particular, the depreciation, construction and installment.
ation rate is assumed to be 8% for housing, and 24% for machinery and equipment. To account for the price effect on the housing capital in China, I construct the time series of housing price indices, using the price indices of the investment in fixed assets between 1987 and 2003, and the real residential land price indices from [Wu et al. (2012)] between 2004-2013. Table [B.3] shows the capital allocation in China between 1987 and 2013.

### C A Statistical Comparison between the United States and China

The aim is to find an appropriate development stage in the US economy for comparison in the counterfactual experiment. CSY started to categorize the investment of total fixed assets into non-residential and housing from 1995 in Table 10.4, which allows a 19-year time series for analyzing the investment behavior in China. In order to find an appropriate episode in the US economic history to compare with, I resample the US data between 1930 and 2013, using a moving block bootstrap approach proposed by [Politis and Romano (1994)](https://example.com). In particular, I fix the length of the block to be 19 years, and obtain 66 blocks of 19 years between 1930 and 2013 in the US economy. Then, I calculate the joint correlations of the consumption-output ratio, the non-residential capital-output ratio and the housing capital-output ratio for the 66 blocks.

Figure [C.1](#) shows a comparison between the dynamics of the sectoral allocation of capital in the US and China. Table [C.1](#) shows the joint correlation of the three targeted variables between the US economy and China. The highlighted row in Table [C.1](#) shows the highest joint correlation among the 66 sampled blocks. Hence, I choose the episode of the US economy between 1956 and 1982 as a reference for the study of China between 1987 and 2013.

---

32 In [Blanchard and Cooper (2006)](https://example.com), estimates of the useful lives of structures and buildings is 38 years, and of machinery and equipment is 12 years.
<table>
<thead>
<tr>
<th>Year</th>
<th>Non-residential (%)</th>
<th>Capital-output ratio (%)</th>
<th>Agricultural Land (%)</th>
<th>Aggregate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>117.55</td>
<td>51.22</td>
<td>139.52</td>
<td>308.29</td>
</tr>
<tr>
<td>1988</td>
<td>105.95</td>
<td>46.01</td>
<td>137.65</td>
<td>289.60</td>
</tr>
<tr>
<td>1989</td>
<td>100.92</td>
<td>44.48</td>
<td>138.72</td>
<td>284.12</td>
</tr>
<tr>
<td>1990</td>
<td>97.84</td>
<td>43.60</td>
<td>154.28</td>
<td>295.72</td>
</tr>
<tr>
<td>1991</td>
<td>92.23</td>
<td>41.08</td>
<td>142.25</td>
<td>275.56</td>
</tr>
<tr>
<td>1992</td>
<td>87.64</td>
<td>38.29</td>
<td>151.74</td>
<td>277.66</td>
</tr>
<tr>
<td>1993</td>
<td>86.30</td>
<td>36.22</td>
<td>140.45</td>
<td>262.96</td>
</tr>
<tr>
<td>1994</td>
<td>145.84</td>
<td>60.83</td>
<td>101.56</td>
<td>308.24</td>
</tr>
<tr>
<td>1995</td>
<td>127.25</td>
<td>52.94</td>
<td>99.22</td>
<td>279.40</td>
</tr>
<tr>
<td>1996</td>
<td>118.63</td>
<td>48.94</td>
<td>91.48</td>
<td>259.05</td>
</tr>
<tr>
<td>1997</td>
<td>116.96</td>
<td>47.34</td>
<td>89.40</td>
<td>253.70</td>
</tr>
<tr>
<td>1998</td>
<td>121.08</td>
<td>48.65</td>
<td>84.88</td>
<td>254.61</td>
</tr>
<tr>
<td>1999</td>
<td>123.46</td>
<td>49.97</td>
<td>77.81</td>
<td>251.24</td>
</tr>
<tr>
<td>2000</td>
<td>121.67</td>
<td>49.26</td>
<td>71.87</td>
<td>242.80</td>
</tr>
<tr>
<td>2001</td>
<td>121.78</td>
<td>48.81</td>
<td>65.78</td>
<td>236.37</td>
</tr>
<tr>
<td>2002</td>
<td>123.94</td>
<td>48.65</td>
<td>60.36</td>
<td>232.95</td>
</tr>
<tr>
<td>2003</td>
<td>127.48</td>
<td>47.49</td>
<td>54.55</td>
<td>229.52</td>
</tr>
<tr>
<td>2004</td>
<td>128.80</td>
<td>45.42</td>
<td>52.23</td>
<td>226.45</td>
</tr>
<tr>
<td>2005</td>
<td>136.58</td>
<td>53.63</td>
<td>50.96</td>
<td>241.18</td>
</tr>
<tr>
<td>2006</td>
<td>142.34</td>
<td>61.49</td>
<td>44.88</td>
<td>248.72</td>
</tr>
<tr>
<td>2007</td>
<td>141.75</td>
<td>84.27</td>
<td>41.45</td>
<td>267.46</td>
</tr>
<tr>
<td>2008</td>
<td>148.51</td>
<td>76.41</td>
<td>32.58</td>
<td>257.50</td>
</tr>
<tr>
<td>2009</td>
<td>174.94</td>
<td>112.73</td>
<td>34.91</td>
<td>322.58</td>
</tr>
<tr>
<td>2010</td>
<td>181.23</td>
<td>148.96</td>
<td>29.12</td>
<td>359.31</td>
</tr>
<tr>
<td>2011</td>
<td>188.90</td>
<td>165.19</td>
<td>24.29</td>
<td>378.37</td>
</tr>
<tr>
<td>2012</td>
<td>209.10</td>
<td>173.02</td>
<td>20.43</td>
<td>402.56</td>
</tr>
<tr>
<td>2013</td>
<td>257.01</td>
<td>217.41</td>
<td>18.08</td>
<td>492.50</td>
</tr>
</tbody>
</table>
Table C.1: Joint Correlation of \( c/y \), \( k/y \) and \( h/y \)

<table>
<thead>
<tr>
<th></th>
<th>( c/y )</th>
<th>( h/y )</th>
<th>( k/y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963-81</td>
<td>0.9422</td>
<td>0.5407</td>
<td>0.7401</td>
</tr>
<tr>
<td>1964-82</td>
<td>0.9421</td>
<td>0.6663</td>
<td>0.9003</td>
</tr>
<tr>
<td>1965-83</td>
<td>0.8958</td>
<td>0.5695</td>
<td>0.9479</td>
</tr>
<tr>
<td>1966-84</td>
<td>0.8371</td>
<td>0.4348</td>
<td>0.9376</td>
</tr>
<tr>
<td>1967-85</td>
<td>0.7376</td>
<td>0.2732</td>
<td>0.9099</td>
</tr>
<tr>
<td>1968-86</td>
<td>0.6073</td>
<td>0.0702</td>
<td>0.8879</td>
</tr>
</tbody>
</table>

D Numerical Solution Algorithm

The social planner’s problem in this paper is an extension of the standard dynamic programming problem, with a two-dimensional state vector of nonresidential and housing capital stock, \((k, h)\). In particular, the detrended SP problem of interest can be summarized by the following Bellman equation:

\[
v(\hat{k}, \hat{h}) = \max_{\hat{k}, \hat{h}'} \left[ \left( \frac{c}{\hat{k}} \right)^{\frac{1}{\alpha_k}} + \left( 1 - \eta \right) \left( \frac{\hat{s}}{\hat{h}} \right)^{\frac{1}{1-\alpha_h}} \right]^{1-\sigma} - 1 + \beta v(\hat{k}', \hat{h}')
\]

subject to

\[
\dot{c} = \hat{k}^{\alpha_k}(1 - l)^{1-\alpha_k} + (1 - \delta)(\hat{k} + \hat{h}) - A \cdot (\hat{k}' + \hat{h}')
\]

\[
\dot{\hat{s}} = \hat{h}^{\alpha_h} l^{1-\alpha_h}
\]

\[
l = \left\{ 1 + \frac{\alpha_h(1 - \alpha_k)}{\alpha_k(1 - \alpha_h)} \left( \frac{\hat{k}}{\hat{h}} \right) \right\}^{-1}
\]

The solution algorithm follows:

1. Compute the steady-state value of \( \kappa^* \) and \( \hat{a}^* \) according to the assigned parameters.

2. Discretize the state space \((\hat{k}, \hat{h})\) on the gridded domain, where \( \hat{k} \in [\hat{k}_{\text{min}}, \hat{k}_{\text{max}}] \) and \( \hat{h} \in [\hat{h}_{\text{min}}, \hat{h}_{\text{max}}] \). \([\hat{k}_{\text{min}}, \hat{k}_{\text{max}}]\) and \([\hat{h}_{\text{min}}, \hat{h}_{\text{max}}]\) are chosen according to the steady-state value of \( \kappa^* \) and \( \hat{a}^* \).

3. Taking \( \hat{h}' \) as given, find the optimal \( \hat{k}'(\hat{k}, \hat{h}, \hat{h}') \) by iterating on the following value function until convergence:

\[
v^{j+1}(\hat{k}, \hat{h}) = \max_{k' \in [\hat{k}_{\text{lb}}, \hat{k}_{\text{ub}}]} u(c(\hat{k}, \hat{h}, \hat{k}', \hat{h}')) + \beta v^j(\hat{k}', \hat{h}')
\]

where the lower and upper bound for \( \hat{k}' \) are \( \hat{k}_{\text{lb}} = \max \left\{ \frac{(1-\delta)\hat{k}}{A}, \hat{k}_{\text{min}} \right\} \) and \( \hat{k}_{\text{ub}} = \min \left\{ \frac{\hat{k}^{\alpha_k}(1-\delta)\hat{k} - (\hat{h}' - (1-\delta)\hat{h})}{A}, \hat{k}_{\text{max}} \right\} \). Note that the upper bound is chosen
as in the extreme case when the consumption of the nondurable goods and services is zero.

4. Taking the function \( \hat{k}'(\hat{k}, \hat{\hat{h}}, \hat{h}') \) as given, find the optimal \( \hat{h}'(\hat{k}, \hat{\hat{h}}) \) by iterating on the following value function until convergence:

\[
v^{j+1}(\hat{k}, \hat{\hat{h}}) = \max_{\hat{h}' \in [\hat{h}_{lb}, \hat{h}_{ub}]} u(\hat{c}(\hat{k}, \hat{\hat{h}}, \hat{k}'(\hat{k}, \hat{\hat{h}}, \hat{h}'), \hat{h}')) + \beta v^{j}(\hat{k}', \hat{\hat{h}}, \hat{h}')
\]

where the lower and upper bound for \( \hat{h}' \) are \( \hat{h}_{lb} = \max \left\{ \frac{(1-\delta)\hat{h}}{A}, \hat{h}_{min} \right\} \) and \( \hat{h}_{ub} = \min \left\{ \frac{\hat{k}^{\alpha k(1-l)^{\alpha k}+(1-\delta)\hat{h}}}{A}, \hat{h}_{max} \right\} \). Note that the upper bound is chosen as in the extreme case when both the consumption of the nondurable goods and services and the nonresidential investment is zero, as if all the resources were used for the housing investment.

With the policy functions \( \hat{k}'(\hat{k}, \hat{\hat{h}}) \) and \( \hat{h}'(\hat{k}, \hat{\hat{h}}) \), one can obtain a time-series simulation for the policy functions \( \hat{k}(t) \) and \( \hat{h}(t) \) by feeding in the initial values \( (\hat{k}_0, \hat{h}_0) \).