A Cooperative Perspective on Sovereign Debt: Past and Present

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Abstract: *Sovereign states often borrow to finance their budgets in the modern era; in the medieval period kings had similar financing needs. Both types of sovereign borrowers share one characteristic: if they failed to fulfill their obligation, legal action is not one of lenders' options. However, as history has shown, some institutions evolved to secure lenders' rights. This paper provides a cooperative perspective of the negotiation between debtors and creditors in a two-period model using the Nash bargaining solution. Our results show that the initial debt and new loans are positively related, while the remaining debt and the interest rate are negatively related. These conclusions match the development of England after the Glorious Revolution.*

I. Introduction

The sovereign debt issue in the nineteenth and the early twentieth centuries was a destabilizing factor in the international financial market. Likewise, the contemporary sovereign debt crisis has affected the world economy critically. On August 12, 1982, the Finance Minister of Mexico announced that Mexico could not continue to service its huge foreign debt, which triggered a decade long international financial crisis. This debt crisis seriously hurt the international financial system and dampened the world economy. There were hundreds of commercial banks and dozens of sovereign states (Buchheit, 1991), engaged in several hundred billion sovereign debts (Wesberry, 1983). For the nine American money-center banks involved, the stake was about 280% of their capital (Cline, 1985). This event drew the attention of numerous researchers because of the voluminous debts and the world's inexperience in coping with such a global crisis.

Due to the large sum of the debt and the lack of coordination among the members of the international community, debtor countries and creditor banks had to negotiate constantly over rescheduling of old debts and refinancing of new loans. Creditor banks, debtor countries, and the international community all neglected that a debt this large cannot be paid off in a short period of time. Researchers, therefore, extend their efforts on the renegotiation processes and the reputation issue (Bulow and Rogoff, 1989a and 1989b).

In most of the literature concerning the debt crisis, researchers have relied on bargaining games in which creditors and debtors can make offers to each other in a non-cooperative setting. The reputation of being a good debtor or a tough creditor plays a role in explaining their results. As debtors' economies worsened, researchers shifted their attention to the effect of debt overhang, the burden of adjustment, and the alternatives of forgiving the debts (Bulow and Rogoff, 1991).

All the issues mentioned above are, of course, very important and well worthy of our efforts. However, we regard it equally important to consider a cooperative approach since efficiency is socially desirable, in particular when such a goal is attainable, as history has shown. In addition, such an approach makes qualitatively clear the relationships among the variables we are interested in, which in turn provides us valuable guidelines in how to achieve efficiency. Our models extend Krugman's (1985) framework by using the Nash bargaining solution, searching for an efficient and cooperative settlement. The cooperative outcome shows that the initial debt and new loans are positively related, while the remaining debt and the interest rate are negatively related.

However, achieving the cooperative outcome is not easy. The long term advantage of committing to repayment is not unfamiliar to sovereign borrowers, but the temptation of reneging is difficult to resist, in particular when fiscal crises are in sight. Nevertheless, we find a cooperative episode in history: the financial evolution of England after the Glorious Revolution. This episode matched the results in our models, the amount of available credit increased dramatically and lending rates dropped, after the king and Parliament reached a cooperative solution. What interests us is not only the financial evolution of England, but also how the financial institutions were formed and how we can achieve the same result in our time. By institutions we mean the rules of a game (North, 1991), in this case, the norms regulating borrowers and lenders. We will pursue a model which can make sovereign debt self-enforcing elsewhere.

In the next section we set up a basic two-period model, delineating a cooperative settlement between debtors and creditors. Then, we consider several variations based on the basic model by taking into account the possibility of the debtor's production, investment opportunities, and the bank's probability of bankruptcy. Our analysis may shed light on certain amendments of the functions of international financial organizations, such as the International Monetary Fund (IMF) and the World Bank.

II. The Basic Model (1): Without Production

The bargaining game between two parties has long been an important framework in economic research. Rubinstein (1982) offers a non-cooperative bargaining framework in which players make offers to each other in alternation. Bulow and Rogoff (1989a and 1989b) adopted Rubinstein's model to discuss the negotiation among the parties involved in the Third World debt crisis. Their conflictive perspective details the negotiation procedures between debtors and creditors, in which both parties try their best to protect their own interests, given the other's strategies.

Nash (1953) offers his insights for a cooperative outcome in a bargaining game. Assuming that both parties can commit to any agreement they jointly reach, he envisaged the intuitive axioms such an outcome should satisfy and showed that there is one and only one such solution, which has come to be called the Nash bargaining solution. A cooperative bargaining game consists of a threat point (also known as *status quo*) and a payoff space that players can achieve if they reach an agreement. The axioms of the Nash bargaining solution include Pareto efficiency and equity, which is what a society desires.[1]

In our two-period model, if an agreement is reached, then the bank will disburse new loans in the first period and the debtor will pay the debt due in both periods. The notations used in this model include the bank's and the debtor's utility function, U^B and U^D of income. The debt due at time t is D_t and Y_t is the debtor's endowment at time t, t = 1, 2.[2] In case there is no agreement, the bank can seize θY_t from the debtor. The debtor's time preference rate is ρ and the bank's time preference rate is δ. The range of θ, ρ, and δ is (0, 1). We assume $\rho > \delta$, that is, the debtor's discount rate is larger than that of the bank. This assumption represents typically the operation of financial markets. All the variables mentioned above are exogenous.

In the bargaining game, the bank and the debtor jointly determine new loans L and the interest rate r. At a Pareto optimal settlement, new loans will be greater than zero and

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[1] It is possible to achieve Nash bargaining solution through non-cooperative means, see Binmore (1993).
[2] The endowment may be considered as debtor country's GNP.

$r \in (\delta, \rho)$. If no agreement is reached, both parties will end up at the threat point. $\overline{U}^B = U(\theta Y_1) + U(\theta Y_2)/(1 + \delta)$ and $\overline{U}^D = V((1 - \theta)Y_1) + V((1 - \theta)Y_2)/(1 + \rho)$ are the utility levels for the bank and the debtor at the threat point, respectively.

The agreement reached by the bank and the debtor takes the following form. In the first period, the bank offers new loans L and the debtor will pay off the initial debt D_1, while in the second period the debtor will disburse the remaining debt D_2 and new loans plus accrued interests $L(1 + r)$. The bank's and the debtor's utilities in the second period are discounted by their respective time preference rate. Thus, given that an agreement (L, r) has been reached, the bank's and the debtor's utilities are $U^B = U(D_1 - L) + U(D_2 + (1 + r)L)/(1 + \delta)$ and $U^D = V(Y_1 - D_1 + L) + V(Y_2 - (1 + r)L - D_2)/(1 + \rho)$, respectively.

To explain the above dependence of the bank's and the debtor's utility on (L, r), note that the bank garners positive benefits from the debtor's payment of D_1, D_2, and L plus the accrued interests at their respective due time, but has to disburse new loans in the first period. In contrast, the debtor acquires advantages from new loans, but is required to pay off D_1, D_2, and L plus accrued interests when the due time comes. We are interested in how new loans and the interest rate, as a cooperative outcome given by the Nash bargaining solution, will be affected by the initial debt, the debt outstanding in the second period, and the portion that the bank can capture from the debtor's wealth. The Nash bargaining solution will be in terms of the pair (L, r). From Nash (1953), such a pair solves

$$\max_{L,\, r} \; (U^B - \overline{U}^B)(U^D - \overline{U}^D)$$

where U^B, $\overline{U}^B$, U^D, and $\overline{U}^D$ are as specified before. To simplify the analysis, we assume that both the debtor and the bank are risk neutral. This means that $U^B = D_1 - L + (D_2 + (1 + r)L)/(1 + \delta)$ and $U^D = (Y_1 - D_1 + L) + (Y_2 - (1 + r)L - D_2)/(1 + \rho)$.

Theorem 1: Let $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ be the values of new loans and the interest rate that correspond to the Nash bargaining solution. Then, $\partial L/\partial\theta < 0$, $\partial r/\partial\theta > 0$, $\partial L/\partial D_1 > 0$, $\partial r/\partial D_1 < 0$, $\partial L/\partial D_2 = 0$, and $\partial r/\partial D_2 < 0$.

Proof: See appendix I.

Under the time preference and risk neutrality assumptions, the result implies that θ and L are negatively related, while θ and r are positively related. This indicates that the more the bank can seize from the debtor in case of default, the higher its bargaining power, in the sense that the bank will lend less and charge a higher interest rate to increase its utility.[3] Cooper and Sachs' (1985) model also shows similar results. As for the relationships of other variables, D_1 and L are positively related, while D_1 and r are negatively related. Suppose the debt due in period one increases, other things being equal, new loans will increase to keep the debtor from default, but the interest rate will be lowered to alleviate the debtor's hardship. On the other hand, if D_2 expands, the interest rate will be lowered in order to lessen the debtor's burden. Interestingly, the debt due in period two has no impact on new loans; this may result from our specific assumptions: risk neutrality of both sides, the relationships between their time preference rates, and D_2 is always paid off, provided an agreement is reached.

III. Model (2): With Production

To capture a more realistic picture of the whole scenario, we consider the

[3] A different model may consider the situation in which the debtor can voluntarily offer collateral.

possibility of the debtor's production in this section. If it is possible for the debtor to produce, then he can use new loans to purchase capital and materials, increasing production and expanding capacity. In other words, Y_t depends positively on L. Let Y_{tL} represent the marginal returns of new loans, t = 1, 2. This setup will shed more light on the impact of new loans on the debtor's economy. Using the same framework as in section II, we have:

Theorem 2: Let $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ be the values of new loans and the interest rate that correspond to the Nash bargaining solution. Then, $\partial L/\partial\theta < 0$, $\partial r/\partial\theta < 0$, $\partial L/\partial D_1 > 0$, $\partial r/\partial D_1 > 0$, $\partial L/\partial D_2 = 0$, and $\partial r/\partial D_2 < 0$.

Proof: See appendix II.

The specification of a production function, instead of endowment, has a great impact on interest rates. As θ increases, other things being equal, new loans will decrease; which is the same as in the basic model, *but the interest rate will be lowered*. Due to the possibility of production, the debtor's future production gain is more valuable to the bank than current high interest revenues. The interest rate will be lowered even though the bank's bargaining power increases. This is a fair and efficient result as conceived under the Nash bargaining solution. If the debt due in period one increases, ceteris paribus, the bank will make more loans to keep the debtor's economy vital, *but the possible future production gain enables the bank to charge a higher interest rate,* which is the opposite of the basic model. The specification of a production function does not change the impact of D_2 on L and r, as described in section II. Interest rates are the return on loans; the debtor's accumulation of capital creates potentially a better prospect in the future, giving the bank distinctive incentives to respond to the debtor's financial needs.

IV. Model (3): With Production and Investment

In the refinancing and rescheduling of the Third World debt crisis, oftentimes debtor countries were required to follow certain guidelines in using the funds. New loans were supposed to be used in the projects specified by the IMF or the World Bank. Other investment opportunities, however, might have existed. Suppose the funds could be used for other investments, the production function will now become a function of L and r. Y_{tL} is as stated before, while Y_{tr} denotes the derivatives of Y_t with respect to r in period t, t = 1, 2. Given more new loans, debtor countries could undertake more investments, while higher interest rates would increase the cost of investments and other disbursement in production. Y_t, therefore, depends positively on L and negatively on r in both periods. In our general setup, new loans represent also the opportunities of acquiring more capital and future productivity. Without loss of generality, the choice of capital is not considered. The rest of the assumptions in the basic model are maintained. We have:

Theorem 3: Let $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ be the values of new loans and the interest rate that correspond to the Nash bargaining solution. Then, (i) if $(1 + r) < \theta Y_{2L}$, there exists no clear relationships of L and r with respect to D_1, D_2, and θ; (ii) if $(1 + r) = \theta Y_{2L}$, $\partial r/\partial\theta > 0$, $\partial L/\partial D_1 > 0$, $\partial r/\partial D_1 = 0$, $\partial L/\partial D_2 < 0$, $\partial r/\partial D_2 < 0$, but $\partial L/\partial\theta$ is indeterminate; (iii) if $(1 + r) > \theta Y_{2L}$, $\partial r/\partial\theta < 0$, $\partial L/\partial D_1 > 0$, $\partial r/\partial D_1 > 0$, $\partial L/\partial D_2 = 0$, $\partial r/\partial D_2 < 0$, but $\partial L/\partial\theta$ is indeterminate.

Proof: See appendix III.

If the investment return is so high that $(1 + r) < \theta Y_{2L}$, then, as many researchers have pointed out (see e.g. Cohen (1991: 33-35)), the debtor could incur infinite debts. In this case, there will be no limit to what the debtor can borrow, and thus, there are no clear relationships of L and r with respect to D_1, D_2, and θ. The difference between the case $(1 + r) = \theta Y_{2L}$ and $(1 + r) > \theta Y_{2L}$ is that, in the former case, D_1 has no impact on interest rates. Since the bank is confident that, after considering the possibility of default, the debtor's marginal return of new loans is still equal to the interest rate, it does not matter how high the interest rate the bank charges. As for the relationships of other variables, they are the same for both $(1 + r) = \theta Y_{2L}$ and $(1 + r) > \theta Y_{2L}$. If D_1 increases, other things being equal, the bank will lend more to maintain the vitality of the debtor's economy. Now D_2 has an impact on L; if D_2 increases, new loans and the interest rate both decrease. The potential growth is not strong enough for the bank to lend more, but the bank will reduce the interest charge to ease the debtor's burden on refinancing. If θ increases, then the interest rate will increase; superior bargaining power still enables the bank to achieve higher utility levels.

V. Model (4): With Risk of Bankruptcy

The bank may potentially go bankrupt, if an agreement is not attained. In this section we consider the impact of the bank's bankruptcy on the choice variables L and r; the rest of the assumptions in the basic model are still made. We assume that the bank would go bankrupt with probability P. With this assumption, $\overline{U}^B = (1 - P)U(\theta Y_1) + (1 - P)U(\theta Y_2)/(1 + \delta)$ and $\overline{U}^D = (1 - P)V((1 - \theta)Y_1) + (1 - P)V((1 - \theta)Y_2)/(1 + \rho)$. If the bank went bankrupt, then only with the probability (1 - P) could both parties keep the utility level under the threat point specified before. The utility functions of the bank and the debtor are the same as in section II, since bankruptcy can possibly occur only when an agreement is not reached. We now have

Theorem 4: Let $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ be the values of new loans and the interest rate that correspond to the Nash bargaining solution. Then, (i) if $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] < \theta$, $\partial L/\partial P < 0$ and $\partial r/\partial P < 0$; (ii) if $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] \geq \theta$, $\partial L/\partial P < 0$ and $\partial r/\partial P > 0$.

Proof: See appendix IV.

Under the Nash bargaining solution, L and P are negatively related; if the risk of bankruptcy increases, ceteris paribus, the bank will lend less to protect itself. The relationship between r and P depends on how much the bank can take from the debtor as a protection against default. If the protection is high enough (i.e., θ is greater than the ratio of the present value of the bank's actual profit margin to the present value of the debtor's perceived cost) then $\partial r/\partial P < 0$. That is, if the imposition θ exceeds the ratio of the bank's perceived profit over the debtor's perceived cost, in present value terms, the bank will charge a lower interest rate, even if the risk of bankruptcy grows. In contrast, without decent protection from bankruptcy, the bank will charge a higher interest rate.

VI. The Cooperative Outcomes

Summarizing the results of the four models, we can see two distinct trends (see the table in appendix V). No matter what the models specify, ceteris paribus, if D_1 increases, then new loans will expand, while the interest rate will decline if D_2 increases. This is a cooperative settlement: if the debtor can commit to loan contracts, naturally, the creditor will offer more new loans and lower the interest rate. This is what we often see in the financial markets: banks will extend credit to persons with good credit history and charge them a lower interest rate. On the contrary, if a person has bad credit, he will have difficulty in getting loans; even if he does, the interest rate will be high. Empirical evidence is available to verify this conclusion. Mexico had been notorious for its debt repudiation; therefore, creditors were not willing to extend credit to Mexico and charged a higher interest rate. However, from 1888 to 1910, Mexico succeeded in financial reforms and economic developments. Consequently, after Mexico recovered creditworthiness, new loans augmented and borrowing rates dropped (Aggarwal, 1996: 176-187).

While debtors' credit is a significant factor, the viability of debtors' economies and the hardships that debtors bear are also important in determining new loans and the interest rate. Cutting the supply of new loans and charging higher interest rates hurt debtors and creditors; which is exactly the aftermath of the Third World debt crisis, a Pareto inefficient outcome. This may explain the reasons why we do not have such results in the cooperative models. Regarding the relationships of other variables, the more the bank can capture from the debtor in case of default, the higher the bank's bargaining power; thus, the bank can lend less and charge a higher interest rate to increase its utility, except in model 2, where the interest rate goes down instead because of future production gains. There are no consistent relationships between r and D_1. As far as D_2 is concerned, only in the model with investment opportunities and production can we see some of its effects on new loans. Under this case, if D_2 increases, then the bank will lend less, in other words, new loans dwindle if the bank expects that the debtor will have a higher future liability.

Comparing the results with what actually happened in the negotiation during the Third World debt crisis, we can see what went wrong. After the Mexican Finance Minister announced Mexico's inability to service its due debts, all banks panicked and would no longer lend. This was supposed to protect bankers' interests, but it turned out that such reactions hurt bankers more; considering already a stake about 280% of their capital, it does not make any difference to lend more. In addition, debtor countries suffered from creditors' over-conservative lending. Similar episodes occurred during the Great Depression; the severity of the crisis was lengthened by banks' unwillingness to lend, which resulted from a high rate of business bankruptcy and the fear of bank runs (Bernanke, 1983). The lack of new loans harmed both debtors and creditors: debtors lost the potential economic growth and investment opportunities, while banks missed the chance to make more profits and to get the debt paid off. Moreover, creditor countries sacrificed huge gains from trade and thousands of jobs. For example, about 150 banks operating in the U.S. declared bankruptcy in 1987, and more than five hundred banks have gone bankrupt in seven years (Huot, 1988). This loss cannot be overemphasized, taking into account especially the difficulties that debtor countries had been experiencing over the past decade.

Furthermore, the economic adjustment programs imposed by the IMF were supposed to help debtor countries accumulate resources and pay off their debts, but most of them were contractionary policies, which worsened their economies in the short run, but did not necessarily give them a better chance to recover in the long run. These adjustment policies were not feasible and seriously hurt debtor countries, creditor banks, and creditor countries, according to the summary of the four models and the results we previously stated. Additionally, banks' over-cautious lending made the supply of new loans dwindle drastically. The New Deal showed a cooperative approach to save an economy from depression and financial crises: increasing government expenditures and extending loans. The role of official organizations, domestic or international, became critical; their supply of new loans made up part of the gap of insufficient private lending. However, their attempts were not aggressive enough because of the worry about banks' moral hazard, which did happen in the 1980s savings and loan crisis (Shoven, Smart, and Waldfogel, 1991).

VII. The English Episode

We now turn to medieval history to look for a cooperative solution to sovereign debt crises. Hicks (1969) delineates a general picture of sovereign finances in the Middle Ages, while Jones (1994) offers a brief outline for the financial development of England in the early modern period. The need to support troops and the expenditures on wars were the fundamental causes of the king's financial difficulties. Since tax revenues were not enough to meet expenditures, the crown could not but resort to borrowing. Loans had to be paid off, principal plus interests, which entailed further burdens on the royal treasury. When the king was not able to pay off the debt, repudiation was a common option. Lending to the State amounted to a risky business, as a result, the king had to pay high interest charges, but the new loans acquired decreased.

Sovereign default and other factors aroused great conflicts between the king and

Parliament in the late seventeenth century. The deposition of James II in the Glorious Revolution signified a new relationship between the crown and Parliament. The new king William III had to make every effort to secure the trust of Parliament. In 1694 the king decided to establish the Bank of England to manage the national debts, showing his commitment to financial obligation. The founding of the Bank made the king's promise to repay credible. If sovereign default occurred, then the national debts, the Bank's major assets, would become worthless and the Bank would go bankrupt, a result fatal to the state (Hicks, 1969). Parliament had neither the intent to abuse the Bank nor to renege the loan contracts, since the consequence would be no different from sovereign default. The crown, therefore, became a trustworthy debtor, and a cooperative outcome was thus achieved. The king's borrowing rate dropped dramatically and new loans increased, which are analogous to the results of our models. Before the 1690s, the interest rate was 14%, while in the late 1690s it was 6-8%, and 3% in the 1730s (North and Weingast, 1989). On the other hand, the amount of loans increased tremendously; the Stuarts rarely had a debt over £2 million, nine years after the Glorious Revolution the sovereign debt reached £17 million, reflecting an increase in debt from 5% to 40% or so of GNP in less than a decade (Weingast, 1997). This English episode verifies the results in our models.

VIII. Conclusion

This paper started from a two-period model using the Nash bargaining solution to outline a cooperative outcome. Moreover, several variations of the basic model were derived, considering the possibility of the debtor's production, investment opportunities, and the bank's risk of bankruptcy. A welfare-enhancing result for debtors and creditors in loan contracts is that: the initial debt and the amount of new loans are positively related, while the remaining debt and the interest charge are negatively related. The result is intuitive: the bank will readily extend credit to persons with good credit history and charge them a lower interest rate, in comparison to those who have poor credit. Our static framework has concisely delineated a cooperative aspect of the bargaining between creditors and debtors.

There is at least one episode in history verifying what this paper has depicted concerning a cooperative solution to a debt crisis: the financial development of England after the Glorious Revolution. The financial evolution which occurred in the seventeenth and eighteenth century England gives us valuable insights into the bargaining game between debtors and creditors. The institutions required for both parties to make their respective commitments credible are straightforward to comprehend, but hard to implement particularly in the international community. To initiate and fortify the institutions, which once existed in the past, governing sovereign financing in the international arena is our final goal. The groundwork in this paper may facilitate the pursuit of self-enforcing sovereign borrowing contracts in the international financial forum.

Appendix I

Denote the solution by $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$. The first order conditions are $F(L, r, D_1, D_2, \theta) = \partial((U^B - \overline{U}^B)(U^D - \overline{U}^D))/\partial L = 0$ and $G(L, r, D_1, D_2, \theta) = \partial((U^B - \overline{U}^B)(U^D - \overline{U}^D))/\partial r = 0$. Pareto optimality requires that $L > 0$ and $\rho > r > \delta$, which guarantee that the Nash bargaining solution is an interior point. By substituting $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ into the first order conditions and then differentiating $F(L, r, D_1, D_2, \theta)$ and $G(L, r, D_1, D_2, \theta)$ with respect to D_1, D_2, and θ, respectively, we can obtain the following systems of linear equations.

$$\begin{pmatrix} F_L & F_r \\ G_L & G_r \end{pmatrix}\begin{pmatrix} \partial L/\partial\theta \\ \partial r/\partial\theta \end{pmatrix}=\begin{pmatrix} -F_\theta \\ -G_\theta \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} F_L & F_r \\ G_L & G_r \end{pmatrix}\begin{pmatrix} \partial L/\partial D_1 \\ \partial r/\partial D_1 \end{pmatrix}=\begin{pmatrix} -F_{D1} \\ -G_{D1} \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} F_L & F_r \\ G_L & G_r \end{pmatrix}\begin{pmatrix} \partial L/\partial D_2 \\ \partial r/\partial D_2 \end{pmatrix}=\begin{pmatrix} -F_{D2} \\ -G_{D2} \end{pmatrix} \quad (3)$$

For the ease of elaboration, let $X^D = U^D - \overline{U}^D$ and $X^B = U^B - \overline{U}^B$, subscript denotes derivatives of respective variables. From the linearity of utility functions and the first order conditions, it follows that $F_LG_r - F_rG_L = -(X^B{}_rX^D{}_L - X^B{}_LX^D{}_r)^2$, which is negative. Similarly, we can determine the sign of the remaining equations.

$F_rG_\theta - F_\theta G_r = (X^D{}_rX^B{}_\theta - X^B{}_rX^D{}_\theta)(X^D{}_rX^B{}_L - X^B{}_rX^D{}_L)$

$= [(Y_1 + Y_2/(1 + \delta))L/(1 + \rho) - (Y_1 + Y_2/(1 + \rho))L/(1 + \delta)]\ [(-1 + (1 + r)/(1 + \delta))(-L)/(1 + \rho) - (1 - (1 + r)/(1 + \rho))L/(1 + \delta)] > 0$

$F_\theta G_L - F_LG_\theta = (X^B{}_LX^D{}_r - X^D{}_LX^B{}_r)(X^B{}_LX^D{}_\theta - X^D{}_LX^B{}_\theta)$

$= (-)\ [(Y_1 + Y_2/(1 + \rho))(-1 + (1 + r)/(1 + \delta)) + (Y_1 + Y_2/(1 + \delta))(1 - (1 + r)/(1 + \rho))] < 0$

$F_rG_{D1} - F_{D1}G_r = (X^D{}_rX^B{}_L - X^B{}_rX^D{}_L)(X^D{}_rX^B{}_{D1} - X^BrX^D{}_{D1})$

$= (-)\ [-L/(1 + \rho) - (-L)/(1 + \delta)] < 0$

$F_{D1}G_L - F_LG_{D1} = (X^D{}_LX^B{}_r - X^B{}_LX^D{}_r)(X^D{}_LX^B{}_{D1} - X^B{}_LX^D{}_{D1})$

$= (+)\ [(1 - (1 + r)/(1 + \rho))1 - (-1 + (1 + r)/(1 + \delta))(-1)] > 0$

$F_r G_{D2} - F_{D2} G_r = (X^B{}_L X^D{}_r - X^B{}_r X^D{}_L)(X^B{}_{D2} X^D{}_r - X^B{}_r X^D{}_{D2})$

$= (-)\ [-L/(1 + \rho)(1 + \delta) - L/(1 + \delta)(-1)(1 + \rho)] = 0$

$F_{D2} G_L - F_L G_{D2} = (X^B{}_L X^D{}_r - X^D{}_L X^B{}_r)(X^B{}_L X^D{}_{D2} - X^D{}_L X^B{}_{D2})$

$= (-)\ [-1/(1 + \rho)(-1 + (1 + r)/(1 + \delta)) - (1 - (1 + r)/(1 + \rho))1/(1 + \delta)] > 0$

Therefore, by (1) $\partial L/\partial \theta = (F_r G_\theta - F_\theta G_r)/(F_L G_r - F_r G_L) < 0$, $\partial r/\partial \theta = (F_\theta G_L - F_L G_\theta)/(F_L G_r - F_r G_L) > 0$; by (2) $\partial L/\partial D_1 = (F_r G_{D1} - F_{D1} G_r)/(F_L G_r - F_r G_L) > 0$, $\partial r/\partial D_1 = (F_{D1} G_L - F_L G_{D1})/(F_L G_r - F_r G_L) < 0$; and by (3) $\partial L/\partial D_2 = (F_r G_{D2} - F_{D2} G_r)/(F_L G_r - F_r G_L) = 0$, $\partial r/\partial D_2 = (F_{D2} G_L - F_L G_{D2})/(F_L G_r - F_r G_L) < 0$.

Appendix II

The same procedures and notations in appendix I apply to the proof here. Thus, we can determine the sign of the following equations.

$F_L G_r - F_r G_L = -\ (X^B{}_r X^D{}_L - X^B{}_L X^D{}_r)^2 = (-) < 0$

$F_r G_\theta - F_\theta G_r = (X^D{}_r X^B{}_\theta - X^B{}_r X^D{}_\theta)(X^D{}_r X^B{}_L - X^B{}_r X^D{}_L)$

$= [(Y_1 + Y_2/(1 + \delta))L/(1 + \rho) - (Y_1 + Y_2/(1 + \rho))L/(1 + \delta)]\ [(-1 + (1 + r)/(1 + \delta) - \theta Y_{1L} - \theta Y_{2L}/(1 + \delta))(-L)/(1 + \rho) - (1 - (1 + r)/(1 + \rho) + \theta Y_{1L} + \theta Y_{2L}/(1 + \rho))L/(1 + \delta)] > 0$

$F_\theta G_L - F_L G_\theta = (X^B{}_L X^D{}_r - X^D{}_L X^B{}_r)(X^B{}_L X^D{}_\theta - X^D{}_L X^B{}_\theta)$

$= (-)\ [(-1 + (1 + r)/(1 + \delta) - \theta Y_{1L} - \theta Y_{2L}/(1 + \delta))(-L/(1 + \rho) - (1 - (1 + r)/(1 + \rho) + \theta Y_{1L} + \theta Y_{2L}/(1 + \rho))L/(1 + \delta))] > 0$

$F_r G_{D1} - F_{D1} G_r = (X^D{}_r X^B{}_L - X^B{}_r X^D{}_L)(X^D{}_r X^B{}_{D1} - X^B r X^D{}_{D1}) = (-)\ [-L/(1 + \rho) - (-L)/(1 + \delta)] < 0$

$F_{D1} G_L - F_L G_{D1} = (X^D{}_L X^B{}_r - X^B{}_L X^D{}_r)(X^D{}_L X^B{}_{D1} - X^B{}_L X^D{}_{D1})$

$= (+) [(1 - (1 + r)/(1 + \rho) + \theta Y_{1L} + \theta Y_{2L}/(1 + \rho))1 - (-1 + (1 + r)/(1 + \delta) - \theta Y_{1L} - \theta Y_{2L}/(1 + \delta))(-1)] < 0$

$F_rG_{D2}-F_{D2}G_r = (X^B{}_LX^D{}_r - X^B{}_rX^D{}_L)(X^B{}_{D2}X^D{}_r - X^B{}_rX^D{}_{D2})$

$= (-) [-L/(1 + \rho)(1 + \delta) - (-L)/(1 + \delta)(1 + \rho)] = 0$

$F_{D2}G_L-F_LG_{D2} = (X^B{}_LX^D{}_r - X^D{}_LX^B{}_r)(X^B{}_LX^D{}_{D2} - X^D{}_LX^B{}_{D2})$

$= (-) [-1/(1 + \rho)(-1 + (1 + r)/(1 + \delta) - \theta Y_{1L} - \theta Y_{2L}/(1 + \delta)) - (1 - (1 + r)/(1 + \rho) + \theta Y_{1L} + \theta Y_{2L}/(1 + \rho))1/(1 + \delta)] > 0$

Therefore, $\partial L/\partial\theta = (F_rG_\theta - F_\theta G_r)/(F_LG_r - F_rG_L) < 0$, $\partial r/\partial\theta = (F_\theta G_L - F_LG_\theta)/(F_LG_r - F_rG_L) < 0$, $\partial L/\partial D_1 = (F_rG_{D1} - F_{D1}G_r)/(F_LG_r - F_rG_L) > 0$, $\partial r/\partial D_1 = (F_{D1}G_L - F_LG_{D1})/(F_LG_r - F_rG_L) > 0$, $\partial L/\partial D_2 = (F_rG_{D2} - F_{D2}G_r)/(F_LG_r - F_rG_L) = 0$, $\partial r/\partial D_2 = (F_{D2}G_L - F_LG_{D2})/(F_LG_r - F_rG_L) < 0$.

Appendix III

The derivation procedures are identical as in appendix I. We have

$F_LG_r-F_rG_L = - (X^B{}_rX^D{}_L - X^B{}_LX^D{}_r)^2 < 0$

$F_rG_\theta-F_\theta G_r = (X^D{}_rX^B{}_\theta - X^B{}_rX^D{}_\theta)(X^D{}_rX^B{}_L - X^B{}_rX^D{}_L)$

$= [(\theta Y_{1r} + \theta Y_{2r}/(1 + \rho)-L/(1 + \rho))(-Y_1 - Y_2/(1 + \delta)) - (-\theta Y_{1r}- \theta Y_{2r}/(1 + \delta) + L/(1 + \delta))(Y_1 + Y_2/(1 + \rho))] [(\theta Y_{1r} + \theta Y_{2r}/(1 + \rho) - L/(1 + \rho))(-\theta Y_{1L}- \theta Y_{2L}/(1 + \rho) - 1 + (1 + r)/(1 + \delta)) - (-\theta Y_{1r} - \theta Y_{2r}/(1 + \delta) + L/(1 + \delta))(\theta Y_{1L} + \theta Y_{2L}/(1 + \rho) + 1 - (1 + r)/(1 + \rho))]$

$= [(\rho - \delta)^2(\theta Y_{2r}Y_1 -Y_1L - \theta Y_{1r}Y_2)] [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{1r}(1 + r - \theta Y_{2L})]/ (1 + \delta)^2(1 + \rho)^2$

The sign of the first term is uncertain, while the second term is negative if $(1 + r) \geq \theta Y_{2L}$, uncertain otherwise. Therefore, the sign of this equation is uncertain.

$F_{\theta}G_L - F_LG_{\theta} = (X^B{}_LX^D{}_r - X^D{}_LX^B{}_r)(X^B{}_LX^D{}_{\theta} - X^D{}_LX^B{}_{\theta})$

$= [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{2r}(1 + r - \theta Y_{2L})][Y_1(1 + r - \theta Y_{2L}) + \theta Y_{1L}Y_2 + Y_2]/(1 + \delta)(1 + \rho)$

The second term is positive. If $(1 + r) \geq \theta Y_{2L,}$ then the first term is negative, hence, the sign of the equation is negative. Otherwise, the sign of this equation is uncertain.

$F_rG_{D1} - F_{D1}G_r = (X^D{}_rX^B{}_L - X^B{}_rX^D{}_L)(X^D{}_rX^B{}_{D1} - X^BrX^D{}_{D1})$

$= [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{2r}(1 + r - \theta Y_{2L})][(\rho - \delta)(L - \theta Y_{2r})]/(1 + \delta)(1 + \rho)$

The second term is positive, the sign of the equation is negative if $(1 + r) \geq \theta Y_{2L}$, uncertain otherwise.

$F_{D1}G_L - F_LG_{D1} = (X^D{}_LX^B{}_r - X^B{}_LX^D{}_r)(X^D{}_LX^B{}_{D1} - X^B{}_LX^D{}_{D1})$

$= [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{2r}(1 + r - \theta Y_{2L})](\rho - \delta)[(1 + r) - \theta Y_{2L}]/(1 + \delta)(1 + \rho)$

The sign of this equation is negative if $(1 + r) > \theta Y_{2L}$, equal to zero when $(1 + r) = \theta Y_{2L}$, uncertain otherwise.

$F_rG_{D2} - F_{D2}G_r = (X^B{}_LX^D{}_r - X^B{}_rX^D{}_L)(X^B{}_{D2}X^D{}_r - X^B{}_rX^D{}_{D2})$

$= [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{2r}(1 + r - \theta Y_{2L})][(\rho - \delta)\theta Y_{1r}]/(1 + \rho)(1 + \delta)$

The second term is negative, the sign of the equation is positive if $(1 + r) \geq \theta Y_{2L}$, uncertain otherwise.

$F_{D2}G_L - F_LG_{D2} = (X^B{}_LX^D{}_r - X^D{}_LX^B{}_r)(X^B{}_LX^D{}_{D2} - X^D{}_LX^B{}_{D2})$

$= [(\theta Y_{2r} - L)(\theta Y_{1L} + 1) + \theta Y_{2r}(1 + r - \theta Y_{2L})][(\rho - \delta)(-1 - \theta Y_{1L})]/(1 + \delta)(1 + \rho)$

The second term is negative; therefore, the equation is positive when $(1+r) \geq \theta Y_{2L}$, uncertain otherwise. Consequently, (i) if $(1 + r) < \theta Y_{2L}$, there exist no clear relationships of L and r with respect to D_1, D_2, and θ; (ii) if $(1 + r) = \theta Y_{2L}$, $\partial r/\partial\theta > 0$, $\partial L/\partial D_1 > 0$,

$\partial r/\partial D_1 = 0$, $\partial L/\partial D_2 < 0$, $\partial r/\partial D_2 < 0$, but $\partial L/\partial\theta$ is indeterminate; (iii) if $(1 + r) > \theta Y_{2L}$, $\partial r/\partial\theta < 0$, $\partial L/\partial D_1 > 0$, $\partial r/\partial D_1 > 0$, $\partial L/\partial D_2 = 0$, $\partial r/\partial D_2 < 0$, but $\partial L/\partial\theta$ is indeterminate.

Appendix IV

Denote the solution by $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$. The first order conditions are $F(L, r, D_1, D_2, \theta) = \partial((U^B - \overline{U}^B)(U^D - \overline{U}^D))/\partial L = 0$ and $G(L, r, D_1, D_2, \theta) = \partial((U^B - \overline{U}^B)(U^D - \overline{U}^D))/\partial r = 0$. By substituting $L(D_1, D_2, \theta)$ and $r(D_1, D_2, \theta)$ into the first order conditions and then differentiating $F(L, r, D_1, D_2, \theta)$ and $G(L, r, D_1, D_2, \theta)$ with respect to P, we obtain the following linear equations.

$$F_L\partial L/\partial P + F_r\partial r/\partial P = -F_P \qquad (4)$$

$$G_L\partial L/\partial P + G_r\partial r/\partial P = -G_P \qquad (5)$$

Applying the same procedures and notations as in appendix I to (4) and (5), we derive a set of relationships: $\partial L/\partial P = (F_rG_P - F_PG_r)/(F_LG_r - F_rG_L)$ and $\partial r/\partial P = (F_PG_L - F_LG_P)/(F_LG_r - F_rG_L)$. Therefore, we have $F_LG_r - F_rG_L = -(X^B{}_rX^D{}_L - X^B{}_LX^D{}_r)^2 = (-) < 0$

$F_rG_P - F_PG_r = (X^D{}_rX^B{}_P - X^B{}_rX^D{}_P)(X^D{}_rX^B{}_L - X^B{}_rX^D{}_L)$

$= (-)\,[(\theta Y_1 + \theta Y_2/(1 + \delta))(-L)/(1 + \rho) - ((1 - \theta)Y_1 + (1 - \theta)Y_2/(1 + \rho))L/(1 + \delta)]$

$= (-)\,L[(\theta\rho - 1 - \rho - \theta\delta)Y_1 - Y_2]/(1 + \rho)(1 + \delta) > 0$

$F_PG_L - F_LG_P = (X^D{}_LX^B{}_r - X^B{}_LX^D{}_r)(X^B{}_LX^D{}_\theta - X^D{}_LX^B{}_\theta)$

$= (+)\,[(1 - (1 + r)/(1 + \rho))(\theta Y_1 + \theta Y_2/(1 + \delta)) - (-1 + (1 + r)/(1 + \delta))((1 - \theta)Y_1 + (1 - \theta)Y_2/(1 + \rho))] = (+)\,[((\rho - \delta)(1 + r)\theta + (1 + \rho)(\delta - r))Y_1 + (\theta(\rho - \delta) + (\delta - r))Y_2]/(1 + \rho)(1 + \delta)$

The sign of $F_PG_L - F_LG_P$ depends on the parameter θ. If $\theta > (1 + \rho)(r - \delta)/(1 + r)(\rho - \delta)$, then the second term is positive, negative otherwise. $(1 + \rho)(r - \delta)/(1 + r)(\rho - \delta)$ can be rearranged as [(r - δ)/(1 + r)]/[(ρ - δ)/(1 + ρ)], which means the ratio of the present value of the bank's actual profit margin to the present value of the debtor's perceived cost. Consequently, (i) if $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] < \theta$, $\partial L/\partial P < 0$ and $\partial r/\partial P < 0$, (ii) if $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] \geq \theta$, $\partial L/\partial P < 0$ and $\partial r/\partial P > 0$.

Appendix V

Results of the Four Models

	$\partial L/\partial\theta$	$\partial r/\partial\theta$	$\partial L/\partial D_1$	$\partial r/\partial D_1$	$\partial L/\partial D_2$	$\partial r/\partial D_2$	$\partial L/\partial P$	$\partial r/\partial P$
model 1	-	+	+	-	0	-		
model 2	-	-	+	+	0	-		
model 3 $(1 + r) > \theta Y_{2L}$	?	+	+	-	-	-		
model 3 $(1 + r) = \theta Y_{2L}$	?	+	+	0	-	-		
model 3 $(1 + r) < \theta Y_{2L}$	?	?	?	?	?	?		
model 4 $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] < \theta$							-	-
model 4 $[(r - \delta)/(1 + r)]/[(\rho - \delta)/(1 + \rho)] \geq \theta$							-	+

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