Problem Set 5

THEORY OF PRODUCTION

1. a. If a firm's production function is given by F(L,K) = $\sqrt[3]{18L + 9K}$, which is larger $AP_L(8, 8)$ or $AP_K(8, 8)$?

2. a. What is the MRTS of the function $F(L,K)= L^2 + K$?
   b. Does this function exhibit diminishing MRTS?
   c. Does this function exhibit increasing returns to scale?

3. Suppose $MP_L (9, 20) =3$ and $AP_L(9,20) = 6$.
   a. Is $AP_L(9,20)$ smaller than, larger than, or equal to $AP_L(10, 20)$?
   b. Is $AP_L(9, 20)$ smaller than, larger than, or equal to $AP_K(10, 20)$?

4. Suppose $AP_L(L, K)$ is always 17K.
   a. What is F(L,K)?
   b. What is $MP_L(L,K)$?
   c. Does F(L,K) exhibit diminishing MRTS?

5. If MRTS(L,K) is always 3/2, draw the isoquants of F(L,K).

6. Let F(L,K) = 5LK. When K rises does MP_L rise or fall? Can you think of a reason why this might be a good description of a real firm's production?

7. My firm digs ditches. Its production inputs are workers and shovels and its output, measured in cubic feet per day, is ditches. Each worker needs a shovel (the ground is too hard to dig with bare hands) and each worker is then able to dig 50 cubic feet in a day. Write down this firms' production function

THEORY OF COST

8. Derive the short-run total cost function STC(Q,w,r, $\bar{K}$) for $F(L,K)=5LK^{1/3}$
9. Derive the short-run **average** variable cost function SAVC(Q,w,r, $\bar{K}$) for $F(L,K)=5LK^{1/3}$

10. Which would have a greater effect on a firm's **short run marginal cost function,** a doubling of the wage rate or a doubling of the rental rate on capital? Or would they have the same effect?

11. If a firm's short run average total cost function and its short run variable cost function satisfy:

    SATC(100, w_0, r_0, $\bar{K}$) = 8 SAVC(100, w_0, r_0, $\bar{K}$) = 6 SAVC(200, w_0, r_0, $\bar{K}$) = 8 ,

    at a specific wage w_0, a specific rental rate r_0 and specific fixed capital level $\bar{K}$, then:

a. What is the firm's short-run fixed cost? (This will be a numerical value).
b. What is the numerical value SATC(200, w_0, r_0, $\bar{K}$) ?

12. Given formulas SATC(Q, w, r, $\bar{K}$)= 5Q + 55/Q + 5 and SVC (Q, w, r, $\bar{K}$) = $5Q^2$ + 5Q, how much is the firm currently spending on capital?