

Econ 100A - Problem Set 1

I. Math Review

1. $y = 5x^{2/3} + 7x + 2$. What is the expression for $\frac{dy}{dx}$?
2. $y = zx^{3/4} + z \ln d$. What is the expression for $\frac{\partial y}{\partial x}$?
3. $y = 7k \ln(x)$. What is the expression for $\frac{\partial y}{\partial x}$?
4. $E_{y,x} = \frac{dy}{dx} \cdot \frac{x}{y}$. Let $y = 2x^2$. What is the expression for $E_{y,x}$?

II. Preferences and Utility

5. Fred likes apples. He doesn't really care how many bananas he consumes (they add nothing to his utility.) Draw his indifference curves.
6. I need a minimum of 7 apples/week to make one apple pie per week. I don't like to eat apples unless they are in a pie. (They'll just sit on the shelf and get rotten in a week if I get less than 7.) I do, however, like bananas, even if they are not in a pie. Draw my indifference curves.
7. My preferences are monotonic.
 - a. Do I prefer 3 apples to 2 bananas? Why or why not?
 - b. Write down a monotonic utility function in apples and bananas.
 - c. Write down a utility function showing I prefer 2 apples to 3 bananas.
8. My utility function is $-3x-7y$, what's my optimal consumption bundle? Name some commodities these might be.
9. Utility is $U(x,y) = x^2 + y$
 - a. Derive the formula for the indifference curve that gives me a utility of exactly 8. Draw that indifference curve.
 - b. Derive marginal utility functions $mu_x(x,y)$ and $mu_y(x,y)$. Derive $MRS(x,y)$.
 - c. Does this utility function satisfy the hypothesis of diminishing MRS? (Explain.)
 - d. Name some commodities these might be.
10. Define Lance's utility to be $u(X_f, X_w)$ where X_f are bicycle frames and X_w bicycle wheels. Assume Lance derives utility from bicycles, but not from their parts alone. What do his indifference curves look like? Write down a specific function u that represents Lance's preferences.
11. My utility function $U(x_1, x_2)$ depends on the two commodities x_1, x_2 . If my preferences are monotonic and satisfy the Hypothesis of Strictly Diminishing Marginal Rate of Substitution, and if $MRS(5,5) = -2$, what can you say about how I would rank

commodity bundles (4,6) and (6,5)? Draw a graph to illustrate. (Hint: How would each of these bundles compare with the bundle (5,5)?)

12. What is the MRS for the utility function $U(x_1, x_2) = \sqrt{(x_1 x_2 + 6.8)^{53} + 11}$?

13. My utility function is $U(x_1, x_2) = x_1^{1/2} x_2$

a. Holding my consumption level of x_1 constant, would a rise in my consumption level of x_2 raise or lower my marginal utility of x_1 ? Give an algebraic proof.