

## Econ 100A - Problem Set 2

### Utility Maximization and Demand

1. George's utility is given by  $U(x_1, x_2) = 6x_1 * x_2^2$ . Use calculus to work through the utility maximization problem and derive his demand functions  $x_1(p_1, p_2, I)$  and  $x_2(p_1, p_2, I)$ .
2. Ted's demand function for goods 1 and 2 is given by  $U(x_1, x_2) = 3\ln x_1 + 2x_2$ . Derive his demand function for good 1.
3. When Lisa cooks a chicken she only eats the chicken wings. Her utility function over whole chickens and chicken-wings is  $U(X_C, X_W) = 2X_C + X_W$ , where  $X_C$  and  $X_W$  are chickens and chicken-wings, respectively. What is her demand function  $X_C^*(p_c, p_w, I)$  for chickens?
4. If  $x_1(p_1, p_2, I)$ ,  $x_2(p_1, p_2, I)$  are my demand functions for commodities 1 and 2, and  $MRS(x_1, x_2)$  denotes my marginal rate of substitution at the commodity bundle  $(x_1, x_2)$ , what is the **numerical value** of the expression

$$MRS(x_1(1,3,12), x_2(1,3,12))?$$

(Assume  $x_1$  and  $x_2$  are positive.)

5. My utility function is  $U(x_1, x_2) = 2x_1^2 x_2$  and my utility-maximizing bundle (at existing prices and my income) consists of 3 units of  $x_1$  and 2 units of  $x_2$ . If  $p_1 = 4$ , what must  $p_2$  equal?
6. My utility function  $U(x_1, x_2)$  depends on the two commodities  $x_1, x_2$ . When  $p_1 = 3$  and  $p_2 = 7$ , my demand functions are such that I spend all my income on commodity 2 and never buy any of commodity 1, no matter how large or small my income may be. Does it follow that  $x_1$  is an economic "bad" for me? (If so, explain why. If not, give an alternative explanation.)
7. If  $x_1(6,3,540)=60$ , what is  $x_2(6,3,540)$ ?
8. My demand functions for goods 1 and 2 are  $x_1(p_1, p_2, I)$  and  $x_2(p_1, p_2, I)$ . If  
$$x_1(2,5,90) = 20 \quad x_2(2,5,90) = 10 \quad \text{and} \quad x_1(2,4,90) = 15 \quad x_2(2,4,90) = 15$$
can you say anything about how I would rank the commodity bundles (20, 10) and (15, 15)? (Hint: This problem requires no calculus, algebra, or even arithmetic. Just a little thinking.)
9. I need 1 bike frame and 2 tires make a bicycle. My utility function for the two goods is then  $U(X_F, X_T) = \min(X_F, X_T/2)$ . What is my demand function for tires? How

many tires to I buy when  $P_F=20$ ,  $P_T=5$  and income is 120\$/wk?

10. Find general expression for demand functions  $x_1^*(p_1, p_2, I)$ ,  $x_2^*(p_1, p_2, I)$  for Cobb Douglas utility,  $U(x_1, x_2) = Ax_1^\alpha x_2^\beta$
11. Joel claims his demand function for good  $x_1$  is  $I/(p_1 + p_2 + p_3^2)$ . Prove he is lying.
12. Jane likes sugar in her beverages. Pepsi has twice as much sugar as lemonade. So Jane's utility for Pepsi and Lemonade is  $U(X_P, X_L) = 2X_P + X_L$ . She faces prices  $P_P$  for Pepsi and  $P_L$  for lemonade. Draw her **demand curve** for Pepsi. Be sure to label the axes and any important points on the graph. Hint: Begin by glancing again at Problem 3.
13. Edsel consumes 50 bottles of beer per week. His price elasticity of demand for beer is ( $E_{x_e, p_e}$ ) is -0.2. The price of beer doubles. How many bottles per week does he consume now?
14. Madonna is spending \$300 per week on eyeliner. Her price elasticity of demand for eyeliner is ( $E_{x_e, p_e}$ ) is -0.1. Suppose the price of eyeliner rises 10%. How much does she spend on eyeliner after the price change?
15. Explain in words what means to say:  $E_{x_1, p_1} = 0$
16. Suppose a consumer is currently buying a positive amount of good 1. and her elasticities are  $E_{x_1, p_1} = E_{x_2, p_1} = \dots = E_{x_n, p_1} = 0$ .
  - a. Explain in words what this means
  - b. Does this make sense? Why or why not?