

1. Short answers (5 pts each)

a) Give two conditions that might make it cost-effective for a firm to pay to screen workers.

b) When we solved the problem of finding the optimal commission rate, what did the workers get to **choose** (or maximize over)? What did the firm get to **choose**?

c. A firm manufactures cooling systems for nuclear reactors. Should it pay its workers based on piece rates? Explain why or why not.

2.

a. Let production be given by $Y=5Q_H+3Q_L$, where Q_H and Q_L are quantities of high and low skill workers, respectively. Draw an isoquant for 75 units of output, and label the intercept values on the axes. If wages for high and low types are \$6/hr and \$2/hr respectively, plot two isocost lines and indicate the cost-minimizing optimum choice (Q_H^* , Q_L^*).

b. Now suppose a new technology is introduced that allows workers to communicate and enhance one another's productivity. Now $Y=Q_HQ_L$. What is the optimal (Q_H^* , Q_L^*) to produce 75 units?

c. Which production technology (the one in part a or the one in part b) allows the firm to produce 75 units more cheaply? Explain and back up with calculations.

3.

a. A firm wishes to deter low skill workers and attract high skill workers using a probation wage. The unskilled wage in the market at large is w_u , and the skilled wage is w_s . In terms of w_u and w_s , what first and second period wages, w_1 and w_2 , should the firm offer?

b. Suppose you must pay workers a severance package when you fire them and that this is an additional 50% of their first period wage. In terms of w_u and w_s , what first and second period wages, w_1 and w_2 , should the firm offer?

4.

a. Suppose there are two types of workers, low and high, with different lifetime value of marginal products: $VMP_{low} = 90,000\$$ and $VMP_{hi} = 250,000\$$. **Half of the workers are high types and half are low types.**

It is worth 50,000\$ to a low type and 10,000\$ to a high type not to go through a year of college. Firm A requires **5 years** of college in order to pay workers the high wage, 250,000\$, and pays all other workers the low wage, 90,000\$. Firm B pays all workers the low wage, 90,000\$. Firm C pays all the workers 165,000\$. If these 3 firms are the only employers in the market, which firm or firms will stay in business? Explain and back up your explanation with calculations.

b. Now suppose instead that marginal disutility of effort is rising in years of schooling. Specifically effort cost for low types is $C_L = 10000s^2$ and effort cost for high types is $C_H = 5000s^2$. What is the optimal s^* (i.e., what is the lowest s that will attract high types and deter low types.) Explain.

c. Draw the graph showing a signaling equilibrium, given these new cost of effort functions. .

5.

A firm sells bullets and wishes to establish a wage

$$w = a + bE,$$

where E is worker effort and a and b are to be chosen by the firm. The worker maximizes utility given by

$$U = w - 3E^2 \quad (\text{which is to say the worker's cost of effort is } 3E^2.)$$

Each unit of effort, E , generates $9E$ bullets which can be sold for \$2 per bullet. Assume the workers' utility must be at least 0 for him to be willing to accept the job.

a) Calculate the workers choice of effort E^* and the profit-maximizing values of a and b . Calculate the firm's profit, given this optimal wage schedule.

b) On the same graph, plot the line that represents the firm's revenue, plot the line that represents the firm's wage offers as a function of effort, $a^* + b^*E$, and plot the worker's cost of effort $C(E)$ as a function of effort.

c) Now suppose workers get tired when they work long hours and their productivity drops off. Let each unit of effort, E , generate $48E^{1/2}$ bullets which can be sold for \$2 per bullet. (This captures the idea of decreasing marginal product as workers tire). Let worker's effort cost be as in part a. What is the workers choice of effort E^* and the profit-maximizing values of a and b ?

6. A firm pays a worker less than his productivity (V_t) early in his career and more than his productivity (V_t) later in his career. At retirement time, T , the worker's best alternative (A_T) to working for the firm is equal to his wage (W_T) and is also equal to his productivity (V_T). In other words, suppose $V_T = W_T = A_T$. Suppose also that the firm earns a (time-discounted) total of zero profits over the worker's career with the firm.

a) Draw a graph showing the evolution of V_t , A_t , and W_t over time, from $t=0$ to $t=T$.

b) Assume interest rate $r=0$. Two areas in this graph must be equal for the firm to earn a total of zero profits over the worker's career with the firm. On your graph, shade in and label those 2 areas.

c) Suppose V_t falls by \$10,000 (that is, worker productivity falls by \$10,000 at each point in time) because of a decrease in the price of the firm's product. Suppose also that a second firm enters the area and offers jobs in a different industry. This increases the workers' alternative, A_t , by \$20,000 (at every point in time.) The firm does not offer buyouts, but simply asks older workers to **voluntarily** retire.

Will any of them accept? Explain.

7.

a. You are a worker in a co-op. There are 3 other workers in your co-op. Every unit of effort, E , you put forth raises the co-op's revenues by 8 dollars. Your effort cost is E^2 . If you wish to maximize total co-op benefits minus your effort cost, what should be the effort, E^* , you put forth?

b. Suppose the co-op workers decide they will **split revenues equally between them**. If you maximize your personal benefit minus effort cost, how much effort do you put forth? Calculate and explain.

c. Explain the reason for any difference between the answers in part a and part b.

8. a. Let the possible stock prices (per share) of a firm next period be given by: 96, 98, 100, 102, 104, and suppose each of these 5 possibilities occurs with probability .2. The current price of a share is \$100. If the cost of an option is its expected value, how much does it cost to buy 5000 options with strike price \$101?

b. Suppose now that (unobservable) action V' yields stock price outcomes 80, 98, 100, 102, 141, each with probability .2. Would the firm prefer the manager to take this strategy over doing nothing? Calculate and explain.

c. The manager is risk-averse and does not like the high-variance strategy. (He's worried that if the stock price falls to 80 he'll be fired.) Suppose you offer him 1000 options with strike price \$101. What is the highest his disutility of taking action V' (in fear and effort cost, measured in dollars) could be, such that the 1000 options you offer him would still cause him to take action V' (or make him indifferent)?