Technical Note on the Keynesian Model
Econ 135 - Henning Bohn

This note presents the Keynesian model in algebraic form and comments on the IS curve. It is complementary to the class -- not a substitute -- and assumes that you have read Mishkin ch.21-23.

In class, I focus on explaining the Keynesian model in words and with diagrams. I don’t walk through the underlying math, mainly because most students find diagrams easier to follow. However, going through the math at least once -- at your own pace -- will help you better understand the diagrams and how to use them correctly; and some students may find the math easier than the diagrams, or useful to double-check answers from graphs.

A second motivation for this note is to comment on the IS curve. The IS curve is covered in Mishkin ch.20, which is not required (though recommended). This note explains why I don’t follow ch.20 and then provides a streamlined presentation of the IS curve.

1. Basic Equations

The Keynesian model has three basic equations:

1. **AS-curve:**
   \[ \pi = \gamma (Y - Y^p) + \pi^e + \rho \]
   Here \( \gamma > 0 \) (gamma) is a slope coefficient that indicates how strongly inflation responds to the gap between output and potential; \( \rho \) symbolizes a price shock: normally \( \rho = 0 \), but it can be positive or negative.

2. **IS-curve:**
   \[ Y = Y^d (r, ...) = Y - d \cdot r. \]
   Mishkin’s algebra for the IS curve is quite cumbersome; to simplify, I write IS as generic downward-sloping function with intercept \( Y \) and negative slope \( d > 0 \). I will explain below how this relates to Mishkin ch.20.

3. **MP-curve:**
   \[ r = \bar{r} + \lambda \cdot \pi \]
   Here \( \lambda > 0 \) (lambda) is a slope coefficient that indicates how strongly the central banks responds to the inflation; \( \bar{r} \) is the intercept, i.e., the interest rate the central bank would set at zero inflation.

Overall, the model has three equations for the three endogenous variables (\( \pi, Y, r \)). Potential output, expected inflation, and the various intercepts and shocks are exogenous factors. For any given combination of exogenous determinants, the model delivers a unique solution for (\( \pi, Y, r \)). Most applied questions are about the impact of various changes. To solve problems, one must determine how the solution changes when one (or more) of the exogenous factors change.
2. Algebraic Solution

To solve the model, it helps to note that each of the three equations includes only two of the three endogenous variables. This suggests a solution by successive elimination of variables is convenient. In terms of pure math, one could do the substitution in any sequence. Mishkin’s strategy, which I follow here, is to eliminate \( r \) first. The main step is to substitute the MP equation into the IS equation to obtain

\[
(*) \quad Y = \bar{Y} - \bar{d} \cdot r = \bar{Y} - \bar{d} \cdot (\bar{r} + \lambda \cdot \pi) = \bar{Y} - \bar{d} \cdot \bar{r} - \bar{d} \cdot \lambda \cdot \pi
\]

The result is a curve that expresses the demand for goods as function of inflation, called AD-curve. Since \( \lambda > 0 \) and \( \bar{d} > 0 \), the AD curve has a negative slope. The intercept \( \bar{Y} - \bar{d} \cdot \bar{r} \) is increasing in \( \bar{Y} \) and decreasing in \( \bar{r} \).

After eliminating \( r \), the AS- and AD-curves are system of two linear equations for \((\pi, Y)\). Because AS has positive slope whereas AD has negative slope, there is always a unique intersection, which provides the solution for \((\pi, Y)\). To obtain a final solution for output, one may substitute AS into AD:

\[
Y = \frac{1}{1 - \bar{d} \cdot \gamma} \left\{ \bar{Y} - \bar{d} \cdot \bar{r} + \bar{d} \cdot \lambda \cdot \gamma \cdot Y^p - \bar{d} \cdot \lambda \cdot \pi^e - \bar{d} \cdot \lambda \cdot \rho \right\} - \bar{d} \cdot \lambda \cdot \gamma \cdot Y
\]

Moving all \( Y \)-terms to the left and dividing yields a solution for output:

\[
Y = \frac{1}{1 - \bar{d} \cdot \gamma} \left\{ \bar{Y} - \bar{d} \cdot \bar{r} + \bar{d} \cdot \lambda \cdot \gamma \cdot Y^p - \bar{d} \cdot \lambda \cdot \pi^e - \bar{d} \cdot \lambda \cdot \rho \right\}.
\]

Thus \( Y \) increases when \( \bar{Y} \) increases, when \( \bar{r} \) decreases, when potential output rises, when expected inflation declines, and when \( \rho \) is negative. Knowing the solution for \( Y \), one can substitute \( Y \) into AS to obtain a solution for \( \pi \) and then substitute \( \pi \) into MP to obtain a solution for \( r \) (both are too long and messy to be insightful, hence the math is omitted).

The point of this derivation is to show you (a) that algebraic solutions exist – inviting you to work with them if you are mathematically inclined; and (b) that algebraic produces “big” formulas; the latter suggests that most students are probably better off using graphs to solve problems that trying to memorize the formulas. However, understanding the AD-curve is important, so everyone should understand the math leading to (*)

3. AD-AS Analysis

The idea of AD-AS analysis is to graph the AS curve (equation 1) and the AD-curve (equation *) into and output-inflation diagram to solve problems graphically. The AS equation describes inflation (firm’s decisions about pricing) as function of the aggregate demand for goods, so it can drawn as is into \((\pi, Y)\)-diagram. If \( \rho = 0 \), the AS curve runs through the point \((Y^p, \pi^e)\); if \( \rho \neq 0 \), the AS curve is shifted up or down accordingly. The AD-curve economically describes how aggregate demand
depends on inflation; to draw it accurately into a diagram with $\pi$ on the vertical axis, the equation needs to be inverted to read:

\[
\pi = \frac{\bar{F} - \bar{d} \bar{r}}{\bar{d} \bar{\lambda}} - \frac{1}{\bar{d} \bar{\lambda}} Y
\]

You may think of (**) as specifying the inflation rate that would be needed to induce the central bank to set interest rates in a way that produces a given aggregate demand $Y$. The main point of (**) is to point out that in a $(\pi, Y)$-diagram, the slope of AD is $-\frac{1}{\bar{d} \bar{\lambda}}$. That is, the AD-curve is steep when $\bar{d}$ and $\bar{\lambda}$ are small, and it is relatively flat when $\bar{d}$ and $\bar{\lambda}$ are large. The intercept is proportional to $\bar{F} - \bar{d} \bar{r}$, which means the curve shifts to the right (or equivalently, up) when $\bar{F}$ increases, and to the left (down) $\bar{r}$ increases.

Now we have the tools to solve problems. We start with an equilibrium position of the economy – some initial solution for $(\pi, Y, r)$. Then a shock hits. The key question is always which of the basic equations are affected. To review (details in Mishkin ch.21-23):

1. AS shifts when potential output changes, when expected inflation changes exogenously, or when firms’ cost of production changes (so $\rho$ is non-zero). If AS shifts up, so in the $(\pi, Y)$-diagram, $Y$ declines and $\pi$ increases. Given higher $\pi$, the MP curve implies higher $r$.

2. MP shifts when monetary policy changes; this can be expressed in terms of interest rates (e.g., “Fed decides to raise the interest rates” means shift up in $\bar{r}$) or in terms of money supply and liquidity effect (e.g., “Fed increases money growth” means the interest rate drops via the liquidity effect, so it’s shift down in $\bar{r}$). If MP shifts up, AD shifts left, so in the $(\pi, Y)$-diagram, $Y$ and $\pi$ decrease. Note that the IS curve is unchanged, so lower $Y$ requires higher $r$.

3. IS shifts when a component of $C+I+G+NX$ changes exogenously, either autonomously ($C, I, NX$) or because of a change in fiscal policy; and increase/decrease implies an increase/decrease in $\bar{F}$. If $\bar{F}$ increases, IS shifts right, AD shifts right, so in the $(\pi, Y)$-diagram, $Y$ and $\pi$ increase. Here the MP curve is unchanged, so higher $\pi$ implies higher $r$.

Here are three items that require care (or produce wrong answers):

1. Monetary vs. fiscal policy: Monetary policy is about Federal Reserve policy with respect to money or interest rate targets, and it affects MP. Fiscal policy is about government spending or taxes, and it affects IS.

2. The direction of supply shocks: Economic changes that force firms to raise prices will shift the AS curve up, or equivalently, to the left. Algebraically, such shifts are represented by positive values for
\( \rho \) (meaning: higher inflation). In the press, such shocks are sometimes described as “negative” supply shocks; this is intuitively appealing because the impact on output is negative, but mathematically misleading. Best think of \( \rho \) as shock to prices.

3. Expected inflation: The relevant expectation in the Keynesian model is a measure summarizing the beliefs about inflation held by firms and workers when they set prices and nominal wages. The model assumes that \( \pi^e \) is given in the short and changes slowly in response to changes in actual inflation. In the context of Mishkin ch.7 (which discusses different types of expectations), these are adaptive expectations. Importantly, the \( \pi^e \) in the AS curve may differ from rational expectations held by economists or investors in financial markets. (It’s almost worth having different symbols, but Mishkin does not, so you will have to remember the distinction when we turn to financial markets.)

4. Notes on the IS curve

Mishkin comments on the IS curve in ch.20. Here is my synopsis and some comments how to interpret the IS relationship \( Y = Y - d \cdot r \).

The starting point is that aggregate demand has four components, consumption \( C \), capital investment \( I \), government spending \( G \), and net exports \( NX \).

For consumption, Mishkin assumes that \( C \) depends on disposable income \( Y - T \) and on exogenous (“autonomous”) component \( \bar{C} \), so \( C = \bar{C} + mpc \cdot (Y - T) \), where mpc stands for marginal propensity to consume out of current disposable income. If you remember intermediate macro, this might strike you are overly simplistic. Notable omissions are that incentives to save (rather than consume) depend on real interest rates, that permanent and temporary changes in income and taxes have different effects, and that expectations about future income and taxes should influence decisions about consumption versus savings. Implicitly, the omitted elements are reflected in \( \bar{C} \); this means \( \bar{C} \) shifts when expectations about future income and taxes change. Also, the effect of a tax cut on consumption may differ from the mpc from current income. This suggests writing

\[
C = \bar{C} + mpc \cdot Y - mpc_T \cdot T - z \cdot r
\]

with separate coefficients for \( Y \) and \( T \) and with an interest rate response \( (z) \). [This is more elaborate than in Mishkin, worth thinking through, but I will simplify below.]

For capital investment, it’s uncontroversial that capital investment depends negatively on the cost of borrowing, which Mishkin writes as linear function \( I = \bar{I} - d \cdot (r + \tilde{f}) \), with slope \( d \) and indicator \( \tilde{f} \) for financial frictions that raise corporate borrowing costs. If you remember intermediate macro, there are again omission that are implicitly subsumed into the autonomous component \( \bar{I} \), notable
expectations about the profitability of investment, which depends on expectations about future sales, marginal tax rates, business regulations, etc. News about any of these factors would trigger shifts in $\bar{T}$.

For net exports, Mishkin assumes that $\textit{NX} = \bar{\textit{NX}} - x \cdot r$ has an autonomous component and depends negatively on the interest rate.

Combining all four components, one obtains

$$Y = C + I + G + \textit{NX}$$

$$= \bar{C} + mpc \cdot Y - mpc_T \cdot T - z \cdot r + \bar{I} - d \cdot (r + \bar{f}) + G + \bar{\textit{NX}} - x \cdot r$$

or equivalently

$$Y = \frac{1}{1-mpc} [\bar{C} - mpc_T \cdot T + \bar{I} + G + \bar{\textit{NX}} - d \bar{f}] - \frac{d + x + z}{1-mpc} \cdot r,$$

which is the IS curve. This is equation (12) in Mishkin ch.20, except I allow for $mpc_T \neq mpc$ and $z \neq 0$. To summarize all the autonomous components, define

$$\bar{Y} = \frac{1}{1-mpc} [\bar{C} - mpc_T \cdot T + \bar{I} + G + \bar{\textit{NX}} - d \bar{f}]$$

and let $\bar{d} = \frac{d + x + z}{1-mpc}$ denote the slope, then the IS curve reduces to $Y = \bar{Y} - \bar{d} \cdot r$. To remember for solving problems: The IS curve is negatively sloped and shifts when $\bar{Y}$ changes.

One reason I do not require Mishkin ch.20 is that the chapter tries to describe some complicated issues with oversimplified formulas; addressing them adequately would require even more math, a bigger investment in math than warranted for this class. One issue is the response of $\textit{NX}$ to interest rate changes ($x$). The argument for $x>0$ involves linkages between interest rates and exchange rates, and between exchange rates and the competitiveness of exports that are complicated and not necessarily effective in the short run. Hence I will interpret most changes in $\textit{NX}$ simply as shifts in $\bar{\textit{NX}}$ and disregard the interest rate channel (approximate $x=0$). A second issue is response of consumption to various changes in disposable income. Mishkin’s assumption of a single marginal propensity to consume (mpc) is unfortunate because responses differ depending on the source of the change. (This motivates my distinction between mpc and mpc_T above.) There is good evidence that the marginal propensity to consume from temporary changes in $Y$ is small (less than $\frac{1}{4}$). Moreover, the marginal propensity to consume from taxes (mpc_T) depends significantly on expectations whether future government will close the resulting fiscal deficits/surpluses created through adjustments in taxes or in spending. If consumers expected that a tax change is reversed later, the marginal propensity to consume may small. (Previous macro classes may have discussed the case of ‘Ricardian neutrality’, which says the effect is zero.). The assumption that higher taxes reduce consumption is implicitly assuming that tax changes are generally not reversed within the planning horizon of current consumers. Finally, $\bar{f}$ has a broader interpretation: Economically, this variable captures everything that makes the required “raw” return on capital investment greater than the benchmark real interest
rate. This includes financial frictions, but also applies to risk premiums and to policy issues such as taxes or concerns about business regulations.

Bottom line: Instead trying to describe details of aggregate demand algebraically, I find it more useful to write IS as simple equation that you can remember: $Y = \bar{Y} - d \cdot r$. You should know that the IS curve has negative slope and you should know which disturbances would shift the curve. Moreover, to memorize what goes into $\bar{Y}$, it’s reasonable to approximate $mpc$~0, so

$$\bar{Y} = \bar{C} - mpc \cdot T + \bar{I} - df + G + NX$$

This provides a concise list of forces that shift IS:

- IS shifts right when the $G$ increases or when the autonomous components of $C$, $I$, or $NX$ increase.
- IS shifts left when taxes increase or when financial frictions increase.

In economic applications, you should assume $\bar{C}$ changes whenever households have an economic reason to change their consumption (at given disposable income); assume $\bar{I} - df$ changes whenever firms have a reason to change their capital investment (at given state rate $r$); and assume $NX$ changes whenever importers or exporters have a reason to change their behavior.

5. A more general MP-curve: The Taylor Rule

Mishkin’s MP-curve is simplified version of a more complete description of monetary policy known as the Taylor Rule:

$$r = \bar{r} + \lambda_1 \cdot \pi + \lambda_2 \cdot (Y - Y^p)$$

The term $\lambda_1 \cdot \pi$ has the same role as in the MP-curve and indicates how strongly the central bank responds to inflation at any given level of output. The Taylor principle again calls for $\lambda_1 > 0$. The term $\lambda_2 \cdot (Y - Y^p)$ models a central bank response to the output gap. This is motivated by the insight that expected and actual inflation are likely to rise when the output gap is positive (or fall if the gap is negative). Responding to the output gap with $\lambda_2 > 0$ serves to preempt inflation before inflationary expectations get established. Economist John Taylor famously estimated that the Federal Reserve has follow such a rule with coefficients $\lambda_1 = 0.5$ and $\lambda_2 = 0.5 / Y^p$.

The Keynesian model works similarly with Taylor rule instead of MP-curve. Combining IS-curve and Taylor rule imply $Y = \bar{Y} - d \cdot (\bar{r} + \lambda_1 \cdot \pi + \lambda_2 \cdot (Y - Y^p))$. Like (*), this is a relationship between output and inflation that can be drawn as AD-curve. A new element is that monetary policy will change when there is new information about potential output; otherwise, AD-AS analysis is unchanged. Put differently, the Taylor rule suggests that in the model with MP-curve, new information about potential output may induce the central bank to change $\bar{r}$. 

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