Mishkin ch.4: Interest Rates

Summary

1. Three key concepts:

   Present Value       Yield to Maturity       Total Return

2. Know how to work with the key concepts:

   Task in Exams: Problem solving

3. Applications:

   Real versus nominal rates       PV of stocks: Discounted dividends
   Inflation-indexed bonds          Tracking total returns

[Notes on Mishkin Ch.4 - P.1]
**Fundamentals**

- **Present Value:**

\[
PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N}
\]

\(i = \text{discount rate} = \text{interest rate used to discount future payments}\)

- **Yield to Maturity** (or simply: **Yield**) = Particular discount rate \(i\) that solves

\[
P_B = PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N}
\]

One-period example (Treasury Bill promises $10,000 at maturity)

\[
P_B = \frac{10000}{(1+i)} \Rightarrow (1+i) = \frac{10000}{P_B} \Rightarrow i = \frac{10000 - P_B}{P_B}
\]
Application: Coupon Bonds

- Defined by: Maturity date => Years to maturity = N
  Coupon = C
  Face Value = F => Coupon rate = C/F

- Market data: Price = P_B or P_{B_t} => Current Yield = C/P_B
  Yield to maturity = i or i_t

  (Time subscript t used when timing matters.)

- Common problems: Compute yield from price. Compute price from yield.

Example

- Bond quote from WSJ:

<table>
<thead>
<tr>
<th>Date</th>
<th>Security</th>
<th>Maturity</th>
<th>Coupon Rate</th>
<th>Price</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/17/2013</td>
<td>Treasury note</td>
<td>7/31/2016</td>
<td>1.50%</td>
<td>102.641</td>
<td>0.622%</td>
</tr>
</tbody>
</table>

  Question: How is the yield calculated?
**Example**

<table>
<thead>
<tr>
<th>Date:</th>
<th>7/17/2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security</td>
<td>Maturity</td>
</tr>
<tr>
<td>Treasury note</td>
<td>7/31/2016</td>
</tr>
</tbody>
</table>

- **Task #1:** Set up the present value equation
  - About 3 years to maturity => $N = 3$
  - Price is per $F=100$ face value => $C = (\text{Coupon Rate}) \times 100 = 1.50$
  - Payment at dates $t+1,\ldots,t+N-1$: Coupon = $C$
  - Payment at maturity date $t+N$: Face value + final coupon = $F+C$

\[
PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{1.50}{1+i} + \frac{1.50}{(1+i)^2} + \frac{101.50}{(1+i)^3}
\]

- **Task #2:** Solve the present value equation
  - Cubic equation – solve numerically by exploiting that $PV$ is declining in $i$.

  *Knowing how this works is essential to understanding yield quotes.*
<table>
<thead>
<tr>
<th>Worksheet</th>
<th>Discount</th>
<th>PV</th>
<th>Graph each pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>try coupon:</td>
<td>1.50%</td>
<td>100.00</td>
<td>-&gt; near par</td>
</tr>
<tr>
<td>try low:</td>
<td>0.00%</td>
<td>104.50</td>
<td>-&gt; high PV</td>
</tr>
<tr>
<td>try higher:</td>
<td>1.00%</td>
<td>101.47</td>
<td>-&gt; yield too high</td>
</tr>
<tr>
<td>try again:</td>
<td>0.50%</td>
<td>102.97</td>
<td>-&gt; yield too low</td>
</tr>
<tr>
<td>and again:</td>
<td>0.62%</td>
<td>102.61</td>
<td>-&gt; close enough</td>
</tr>
</tbody>
</table>

WSJ quote: 0.62% 102.64
Notes on Bond Quotes

- Most of the time, economists use published or online quotes to obtain yields.
- Traders sometimes work with yields without mentioning prices. Then $P_B = PV$ is implied. Everyone knows.

- Bond quotes have two parts:
  1. Identify the security: issuer, maturity date, coupon = fixed data.
  2. Market information: price, yield, time of quote = changes over time.

- Implied items that also change: current yield, time to maturity.

- Assumptions to check:
  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "$100 : 8" = $100 \frac{8}{32} = 100.25$)

  *Good sources should have legends or footnotes to confirm interpretation*

- Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard “accrued interest” – use prices are as quoted “clean”
The Total Return
(a.k.a: Return, Rate of Return)

- Definition:

\[ \text{RET} = \frac{\text{Payment} + P_{t+1} - P_t}{P_t} \]

- Measured over a specific time period t to t+1:
  \( P_t \) = Price at the start; known.
  \( P_{t+1} \) = Price at the end; often unknown at time t.
  Payment = current yield or other payout during the time period t to t+1; assumed known.

- Can be computed for ANY financial asset – not only bonds

- Components:

  \[ \text{Current Yield} = \frac{\text{Payment}}{P_t} \]

  \[ \text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} = \frac{\text{Change in Price}}{\text{Initial Price}} \]

[Notes on Mishkin Ch.4 - P.7]
Application in Mishkin:

Total return on a coupon bond for one year:

\[ RET = \frac{C + P_{Bt+1} - P_{Bt}}{P_{Bt}} \]

Mishkin’s formula is a special case: Payment = C. Period = one year

Remember the general principle:

\[ RET = \frac{Payment}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

e.g. if period = X days, then: Payment = (Annualized current yield) * X/365.
if asset = real estate, then: Payment = Rental income minus expenses.

• Caution: Yields are usually annualized. But current yield in RET refers to the period over which RET is calculated – may require conversion.
Illustrations in Mishkin

1. Price – Yield Relation

Example: 10-year coupon bond

<table>
<thead>
<tr>
<th>Price of Bond ($)</th>
<th>Yield to Maturity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>7.13</td>
</tr>
<tr>
<td>1,100</td>
<td>8.48</td>
</tr>
<tr>
<td>1,000</td>
<td>10.00</td>
</tr>
<tr>
<td>900</td>
<td>11.75</td>
</tr>
<tr>
<td>800</td>
<td>13.81</td>
</tr>
</tbody>
</table>
2. Key Linkages: Yield Change - Price Change - Return

Lesson: Price responses increase with maturity

<table>
<thead>
<tr>
<th>(1) Years to Maturity When Bond Is Purchased</th>
<th>(2) Initial Current Yield (%)</th>
<th>(3) Initial Price ($)</th>
<th>(4) Price Next Year* ($)</th>
<th>(5) Rate of Capital Gain (%)</th>
<th>(6) Rate of Return [col (2) + col (5)] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>10</td>
<td>1,000</td>
<td>503</td>
<td>−49.7</td>
<td>−39.7</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>1,000</td>
<td>516</td>
<td>−48.4</td>
<td>−38.4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1,000</td>
<td>597</td>
<td>−40.3</td>
<td>−30.3</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>1,000</td>
<td>741</td>
<td>−25.9</td>
<td>−15.9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1,000</td>
<td>917</td>
<td>−8.3</td>
<td>+1.7</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1,000</td>
<td>1,000</td>
<td>0.0</td>
<td>+10.0</td>
</tr>
</tbody>
</table>

*Calculated with a financial calculator, using Equation 3.

Common measure of price-sensitivity: %\(\Delta P = -\) Duration * \(\Delta i/(1+i)\)

Pure discount bonds have duration = maturity; for coupon bonds, duration < maturity.

Optional reading: Online Appendix to Ch.4
Real Returns and Real Yields

- **Real interest rate** = Nominal interest rate minus expected inflation
  
  $r = i - \pi^e$

  - Traditional measurement: Find nominal interest rates & estimate expected inflation
    
    Problem: expectations are not directly observable.

  - New approach: use yields on inflation-protected securities (in U.S. since 1997)

- **Treasury Inflation-Protected Securities (TIPS):**
  - Face value is fixed in real terms: $F_r = \$100$
  - Nominal face value varies with CPI: $F = F_r \times (\text{CPI})$
  - Nominal coupon varies with CPI: $C = (\text{Coupon rate}) \times F$
  - If Price = Face value, then: nominal return $\approx (\text{Coupon rate}) + (\text{Change in CPI})$

  $\Rightarrow$ **Real yield to maturity = real return $\approx$ Coupon rate**

  $\Rightarrow$ Interpret coupon rate as real interest rate.

- If Price $\neq$ Face value, real return differs from coupon rate & not easy to compute.

  $\Rightarrow$ Rely on published sources for real yields to maturity (e.g. WSJ)

- Remember: **Quoted yields on TIPS are direct measures of REAL interest rates.**

[Notes on Mishkin Ch.4 - P.11]
Other Applications

• Present values of corporate stocks:

  - Payment = Dividend. Example of infinitely lived asset

  \[ PV_t = \frac{Dividend_{t+1}}{1+i} + \frac{Dividend_{t+2}}{(1+i)^2} + \ldots \]

  - Return to stocks in Mishkin ch.7.

• British consols:

  - Coupon bonds without repayment date.

  \[ P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i} \]

  - Nice illustration of negative price-to-yield relationship.