Econ 204A: Organization

• Class Page: [www.econ.ucsb.edu/~bohn/204A/204Aindex.html](http://www.econ.ucsb.edu/~bohn/204A/204Aindex.html)
  - Information is updated throughout the quarter.
  - Check for announcements. Class page announcements are assumed known.

• Open door policy for graduate students. NH3016.
  - Official office hours posted on the class page.

• E-mail: henning.bohn@ucsb.edu. Put “Econ 204A” in the subject line.

• Grading: Weekly problem sets, midterm (in class), final exam.
Introduction to Macroeconomics

• Objectives of macroeconomics:
  To analyze the economy as a whole, to explain economic growth and economic fluctuations, and to assess economic policy.

• Outline of Econ 204A:
  1. Introduction: Intertemporal choice problems.
     - Tools: Constrained optimization. Graphical analysis.
     - If time: Introduction to New Growth.
  3. The Ramsey Model: Optimal Consumption and Savings/Investment over Time.
     - Applications to Fiscal Policy and to Money. Digression to discrete time.
**Intertemporal Choice: Consumption**

- Individual decision problem:
  - Given a series of wage incomes $w_t$. Given real interest rate $r_t=r$ (constant).
  - Individuals choose consumption $c_t$ and asset holdings $a_t$ subject to
    Budget equations: $a_t = (1 + r)a_{t-1} + w_t - c_t$

- Building intuition: Two-period version with graphical analysis. Then generalize.
  - Consumption now ($c_1$) vs. consumption later ($c_2$).
  - Assume given initial wealth $A = (1 + r)a_0$.

- Budget equations imply an **intertemporal budget constraint (IBC):**
  - use $a_1 = (1 + r)a_0 + w_1 - c_1 = A + w_1 - c_1$ and
    $a_2 = (1 + r)a_1 + w_2 - c_2$
  - impose the **terminal condition** $a_2 = 0$:
    $a_1 = -\frac{1}{1+r} [w_2 - c_2] \Rightarrow 0 = A + w_1 - c_1 + \frac{1}{1+r} [w_2 - c_2]$
    $\Rightarrow$ IBC: $c_1 + \frac{1}{1+r} c_2 = w_1 + \frac{1}{1+r} w_2 + A$. 
  - Means: Present value of consumption = Present value of income plus initial wealth.
Two Periods: Graphical Analysis

- Budget line has slope \(-(1+r)\). Increase in \(r\) \(\Rightarrow\) steeper slope.

  Feasible set: Area under the budget line.

- Endowment point is \((A+w_1, w_2)\). Higher \(A, w_1, w_2\) \(\Rightarrow\) budget line shifts “out”.
Two Periods: Math

• Optimization problem: maximize \( U = u(c_1) + \beta u(c_2) \)
  - subject to IBC: \( c_1 + \frac{1}{1+r} c_2 = A + w_1 + \frac{1}{1+r} w_2 \)

• Approach #1: substitute constraint into objective. Problem is:
  \[
  \text{Max} \quad U = u\left(A + w_1 + \frac{1}{1+r} w_2 - \frac{1}{1+r} c_2\right) + \beta u(c_2).
  \]
  FOC for c_2: \(-\frac{1}{1+r} u'(c_1) + \beta u'(c_2) = 0 \quad \Rightarrow \quad \frac{1}{1+r} u'(c_1) = \beta u'(c_2)\)

• Approach #2: use Lagrangian. Define shadow value \( \lambda \). Problem is:
  \[
  \text{Max} \quad L = u(c_1) + \beta u(c_2) + \lambda \cdot \left(A + w_1 + \frac{1}{1+r} w_2 - c_1 - \frac{1}{1+r} c_2\right)
  \]
  \[
  \Rightarrow \text{FOC for } c_1 \text{ and } c_2: u'(c_1) = \lambda \text{ and } \beta u'(c_2) = \lambda \cdot \frac{1}{1+r} \Rightarrow \frac{1}{1+r} u'(c_1) = \beta u'(c_2)\)

• Same conditions. If utility is strictly concave, the solution \((c_1,c_2)\) is unique.
Interpretation 1: Consumption Smoothing

**Consumption Smoothing Intuition:**
- Suppose time preference factor is approximately equal to the discount factor:
  \[ \beta \approx \frac{1}{1+r} \Rightarrow u'(c_1) \approx u'(c_2) \Rightarrow c_1 \approx c_2 \]
- Insight: *Consumption is a “smooth” series.* True even if the income series varies.

**Benchmark:** suppose \( c_1 \approx c_2 \). Then \( c_1 + \frac{1}{1+r} c_2 = c_1 \cdot (1 + \frac{1}{1+r}) = A + w_1 + \frac{1}{1+r} w_2 \)

\[ \Rightarrow c_1 = c_2 = \frac{1}{l+(1+r)^{-1}} \cdot (A + w_1 + \frac{1}{1+r} w_2) \]
- Find: *Consumption is approximately a fraction of total lifetime resources.*
- Note: If wage income was a constant \( y \), then \( \frac{w_1 + \frac{1}{1+r} w_2}{l+(1+r)^{-1}} = \frac{y + \frac{1}{1+r} y}{l+(1+r)^{-1}} = y \).

**Permanent Income** = Annuity equivalent of the income actual stream (Friedman 1957)

Here \( y^P = \frac{w_1 + \frac{1}{1+r} w_2}{l+(1+r)^{-1}} \). Then \( c_1 = y^P + \frac{1}{l+(1+r)^{-1}} \cdot A \).
- Find: *Consumption \approx Permanent Income plus a fraction of initial wealth.*
Permanent Income Model with many periods

- Generalize to arbitrary number of periods \( n \). Terminal condition \( a_n = 0 \).

- Intertemporal budget constraint: \( \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} c_t = A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t \)

- Benchmark: If \( \beta \approx 1/(1+r) \), so consumption is constant, the budget constraint implies

\[
c_1 = (A + \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} w_t)/(\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}})
\]

- Simple approximation for large \( n \) and small \( r \): \( 1/\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} \approx \frac{r}{1+r} \approx r \)

=> Consumption is approximately a fraction \( r \) of lifetime resources.

- Lessons from permanent income theory:
  1. Distinguish changes in current income (holding future income constant) from changes in permanent income (current and future income): Permanent changes have a much greater impact than temporary changes.
  2. Future income matters => Expectations about future income matter.
  3. Consumption growth depends on interest rates relative to the rate of time preference
Interpretation 2: Incentives to Save

- High interest rates provide incentives to consume less and save more:

  \[ 1 + r > 1/\beta \ \text{in} \ \frac{1}{1+r} u'(c_1) = \beta u'(c_2), \text{then} \ u'(c_1) > u'(c_2) \ \Rightarrow \ c_2 > c_1. \]

1. High interest rates “tilt” the consumption path upwards. Consumption grows over time.
2. For any given present value, growing consumption must start at a lower level.

  \[ \Rightarrow \ \text{Initial consumption tends to be less than permanent income intuition would suggest.} \]

  **Caveat:** High \( r \) has income effects. Positive for savers, negative for borrowers.

- Example (Power utility):

  \[ u(c) = \frac{1}{1-\theta} c^{1-\theta} \ \text{with} \ \theta > 0, \theta \neq 1 \]

  - Then: \( u'(c) = c^{-\theta} \). FOC is \( \frac{1}{1+r} c_1^{-\theta} = \beta c_2^{-\theta} \Rightarrow \frac{c_2}{c_1} = [(1 + r)\beta]^{1/\theta} \) increasing in \( r \).

  - In logs: \( \ln\left(\frac{c_2}{c_1}\right) = \frac{1}{\theta} [\ln(1+r) + \ln(\beta)] \]

- Definition:

  \[ \frac{\partial \ln(c_2/c_1)}{\partial \ln(1+r)} = \frac{(1+r)}{(c_2/c_1)} \frac{\partial (c_2/c_1)}{\partial (1+r)} = \text{Elasticity of intertemporal substitution (EIS)} \]

  - Convenient property of power utility: EIS = 1/\theta, constant

  - Note: \( u(c) = \ln(c) \) implies \( u'(c) = c^{-1} \) has constant EIS=1.
Decision Problems with Production

- Output is produced with capital and labor: \( Y_t = F(K_t, L_t) \)
  - Properties: Increasing; concave; constant returns to scale. (More later.)
  - Firm profits = Output – wage cost – cost of capital. (All in real terms.)
  - Real wage = Marginal product of labor: \( w_t = F_L(K, L) \)
  - Interest rate = Marginal product of capital – depreciation rate.

- Static model as warm-up: take K as given. No capital investment.
  - Time constraint: available hours = h. Time for work (l) plus leisure (h-l)

- Individual problem: Households maximize utility from consumption and leisure:
  maximize \( U = u(c, h-l) \) subject to \( c = w \cdot l + \pi \) \[ \pi = \text{other income} \]
  FOC: \( u_c(c, h-l) \cdot w = u_{h-l}(c, h-l) \)
  => Consumption-leisure graph with indifference curves and budget line (slope −w).

- Market equilibrium: assume N identical households, firms are owned by households
  - Firms operating profits (given K): \( \Pi = F(K, L) - w \cdot L. \)
  - Allocation is symmetric: \( l = \frac{1}{N} L, \pi = \frac{1}{N} \Pi \)
  => \( c = w \cdot \frac{L}{N} + \frac{1}{N} \Pi = \frac{1}{N} [w \cdot L + F(K, L) - w \cdot L] = \frac{1}{N} F(K, L) = F(\frac{1}{N} K, l) \)
  Aggregate tradeoff between per-capita consumption and leisure is concave.
  - Firms maximize profits => FOC: \( F_L(K, L) = w. \) Determines equilibrium L and c.
**Social Planning Perspective**

- What if a central authority (government) made all economic decisions?
  - Social planner maximizes household utilities subject to the production constraint.
    \[ U = u(c, h - l) \text{ s.t. } c = \frac{1}{N} F(K, N \cdot l). \]
    FOC: \[ u_c(c, h - l) \cdot \frac{1}{N} F_L \cdot N = u_c(c, h - l) \cdot F_L = u_{h-l}(c, h - l). \]
    => Same conditions as in the market equilibrium – same solution.

- What if there was a single household owning and operating a firm?
  Household would maximize \[ U = u(c, h - l) \text{ s.t. } c = F(K, l). \] Same solution.

- Welfare Theory (digression/intro only)
  - **Pareto Optimality** as benchmark: an allocation is Pareto Optimal if no one can be made better off without making someone else worse off. Here symmetric special case.
  - **First fundamental welfare theorem: Competitive equilibrium is Pareto optimal.**
    Assuming competitive, price-taking behavior, increasing utility.
  - **Second fundamental welfare theorem: under some conditions, Pareto optimal solutions can be implemented as market equilibrium**
    In general requires lump sum transfers, a concave production function, a concave utility function. Here no transfers needed because of symmetry.
• Use of welfare theory in macroeconomics:
  - Pareto optimal allocation provides benchmark – use social planning problem to find them.
  - Interpret the social planner as a **representative agent** who also operates a firm.
  - Social planning/representative agent problem is a useful “device” to find the market equilibrium, provided the second welfare theorem applies.

• Approach:
  1. Solve the social planning problem to obtain equilibrium quantities. Here: \((c, l)\)
  2. Given the optimal quantities, find market clearing “prices” from the FOC:
     Here: optimal \((c, l)\) implies a unique factor price
     \[ w = u_{h-l}(c, h-l) / u_c(c, h-l). \]
     - Applies to most problems studied in this class. (Exception: distortionary taxes.)
     - Logical leap: one agent represents a large number.
Dynamic Model: Production Economy with many discrete time periods

- Resource constraints: \( Y_t = F(K_t, L_t) = I_t + C_t \) and \( K_{t+1} = I_t + (1 - \delta) \cdot K_t \).

- Household utility: \( U = u(c_1, h - l_1) + \beta \cdot u(c_2, h - l_2) + ... \)

- Alternative interpretations:
  1. Market allocation with firms and households; markets for labor, goods, capital.
  2. Social planner maximizes \( U \) subject to production constraints.
  3. Representative household maximizes \( U \) s.t. per-capita resource constraints.

- Observations:
  - Marginal increase in \( l_t \) allows a marginal increase in \( c_t \) by \( F_L \).
    \[ \frac{\partial u}{\partial c_t} \cdot F_L() = \frac{\partial u}{\partial l_t} \text{ and } F_L = w_t \] apply, same as in the static production model.

  - Marginal increase in \( K/N \) reduces \( c_t \) by same amount and increases \( c_{t+1} \) by \( F_K + (1 - \delta) \).
    \[ \frac{\partial u}{\partial c_t} = \beta \cdot \frac{\partial u}{\partial c_{t+1}} \cdot [F_K(K_{t+1}, L_{t+1}) + 1 - \delta] \]. Also: \( F_K + 1 - \delta = 1 + r \Rightarrow F_K = r + \delta \)

Interpretation: Marginal product of capital = interest + depreciation = cost of capital

[Note: Concave production => aggregate tradeoff between \( c_t \) and \( c_{t+1} \) is concave.]

- Challenge: track dynamics of capital and output: \( K_t \rightarrow Y_t \rightarrow I_t \rightarrow K_{t+1} \rightarrow Y_{t+1} \ldots \)

How does such an economy evolve over time? How does economic growth occur?

- To start: examine dynamics with fixed saving rate = Solow growth model.
Context: A Brief History of Macroeconomics

- Pre-Keynesian (“Classical”) Analysis
  - Extension of microeconomics. Quantity theory of money.

- John Maynard Keynes. ISLM interpretation by John Hicks.
  - Missing theory of inflation => The Phillips curve => Neoclassical Synthesis (~1960s)


- Real Business Cycles: Fluctuations in a stochastic growth model.

- Current consensus: DSGE = Dynamic stochastic general equilibrium models
  - With infinite-lived dynasties or with overlapping generations of finite-lived agents.
  - With or without informational frictions that yield ‘Keynesian’ features.

- Central issues: Population; preferences; technology
  For this class: omit uncertainty and imperfect information.
Learning Objectives

• Conceptual:
  1. Macroeconomics is based on microeconomic principles – optimal choices subject to
     constraints, market equilibrium, welfare theory.
  2. Microeconomic intuition: income and substitution effects.
  3. Macroeconomic intuition from simplified models:
     - Intertemporal consumption choices and permanent income.
     - Static production models and consumption-leisure tradeoff.

• Technical skills:
  1. Optimization – with and without constraints.
  2. Solving and interpreting intertemporal and static choice problems.

  Problem sets for practice.