Fiscal Policy:
I. Government Spending

• Assumption: Public spending $G$ per efficiency unit of labor.
  - Assume spending is tax-financed: $T=G$, lump-sum.
  - Here abstract from productivity and population growth (could be added)
  - Best interpret as government consumption. [Public capital would require different model.]

• Resource constraint per-capita: $\dot{k} = f(k) - \delta k - c - G$
  - Phase diagram with $c(k)$-line: $c(k) = f(k) - \delta k - G$. Higher $G$ shifts the $c(k)$-line down.

• Preferences: utility over private consumption $U = \int_0^\infty e^{-\rho t} u(c) dt$.
  - Treat public spending as exogenous. (Standard macro view. Endogenous in public finance.)
  => Hamiltonian: $H(c,k,\lambda,t) = e^{-\rho t} u(c) + \lambda \cdot [f(k) - \delta k - c - G]$

  - Note: $G$ enters only into the constraint => Same Euler equations as without government.
    $$\frac{\dot{c}}{c} = \frac{1}{\theta(c)} (f'(k) - \delta - \rho)$$

  - Steady state conditions:
    $\dot{k} = 0$ $\Rightarrow$ $c^* = f(k^*) - \delta k^* - G^*$
    $\dot{c} = 0$ $\Rightarrow$ $f'(k^*) = \delta + \rho$

  - Implied phase diagrams: Fixed $k^*$-line. Shifts in $c(k)$-line whenever $G(t)$ changes.
Application #1: Permanent Increase

**FIGURE 2.8** The effects of a permanent increase in government purchases

- Find: Instant jump to the new steady state. Government spending has no impact on capital stock, no impact on the interest rate, crowds out private consumption one-for-one.
Application #2: Temporary Increase

- Graphs: See Romer Fig.2.9 (next slide)

- Key idea: Follow different “arrows” phase diagrams as time passes.
  - Until $t_0$: Economy at the initial steady state with normal.
  - Interval $[t_0, t_1]$: Movements in the phase diagram follow $c(k)$ function with higher $G$.
  - After $t_1$: Movements in the phase diagram follow $c(k)$ function with normal $G$.

- Solve backwards:
  - Return to the steady state in the long run $\Rightarrow$ Must be on the ‘normal’ saddle path at $t_1$.
  - Determine $c(0)$ such that movements during $[t_0, t_1]$ end on the saddle path at $t_1$.

- Find: Temporary increase government spending has
  - a temporary negative effect on capital accumulation: crowding out;
  - a temporary negative impact on consumption.
  - a temporary positive impact on interest rates.

Optional Exercise: Suppose at time $t_0$, the government announces that spending will increase permanently, starting at time $t_1$. Determine the effects.
Note on Preferences

• Suppose individuals have preferences over private and public consumption:

\[ U = \int_{0}^{\infty} e^{-\rho t} u(c,G) dt \]

- Standard approach in public finance: Social planner optimizes over G
- Hamiltonian: \( H(c,G,k,\lambda,t) = e^{-\rho t} u(c,G) + \lambda \cdot [f(k) - \delta k - c - G] \)

- Apply the Maximum Principle with two choice variables:

\[ \frac{\partial H}{\partial c} = e^{-\rho t} \frac{\partial u}{\partial c} - \lambda = 0 \text{ and } \frac{\partial H}{\partial G} = e^{-\rho t} \frac{\partial u}{\partial G} - \lambda = 0 \]

=> Optimality requires \( \frac{\partial u}{\partial c} / \frac{\partial u}{\partial G} = 1 \) = unit marginal rate of substitution.

• Result: Optimal to divide to total consumption \( \tilde{c} = c + G \) into private and public components such that each provides the same utility on the margin.

• Express the model in terms of total consumption:

- write \( c = c(\tilde{c}), G = G(\tilde{c}), \) and \( \tilde{u}(\tilde{c}) = u[c(\tilde{c}),G(\tilde{c})] \)

=> Welfare problem: maximize \( U = \int_{0}^{\infty} e^{-\rho t} \tilde{u}(\tilde{c}) dt \) s.t. \( \dot{k} = f(k) - \delta k - \tilde{c} \)

• Interpretation: The optimal growth model without government sector can be interpreted as model in which public spending is subsumed into private spending.
Fiscal Policy II: Government debt and deficits

- Assumption: Given path of spending can be financed with taxes $T$ or debt $D$.

  - Government budget equation:
    \[ \dot{D} = G(t) + r(t) \cdot D(t) - T(t) \]

  - Integrate over time:
    \[ D(t) = D(0) \cdot e^{\int_0^t r(\nu) d\nu} + \int_0^t (G(s) - T(s)) \cdot e^{\int_0^s r(\nu) d\nu} ds \]

  - Apply present value factors $p_{0,t} = e^{-\int_0^t r(\nu) d\nu}$,
    \[ p_{0,t} D(t) = D(0) + \int_0^t p_{0,s} G(s) ds - \int_0^t p_{0,s} T(s) ds \]

  - Impose the transversality condition $\lim_{t \to \infty} p_{0,t} D(t) = 0$. Obtain the

- Government’s intertemporal budget constraint:
  \[ D(0) + \int_0^\infty p_{0,s} G(s) ds = \int_0^\infty p_{0,s} T(s) ds \]

  - Interpretation: Initial debt and the present value of spending must be backed by the present value of taxes.

- Claim (Ricardian neutrality): Financing of public spending does not matter.
Proof of Ricardian neutrality

• Step 1: Individual budget equation with taxes:
  \[
  \dot{a} = w(t) + r(t) \cdot a(t) - c(t) - T(t)
  \]
  => Intertemporal budget constraint:
  \[
  a(0) + \int_0^\infty p_{0,s}w(s)ds = \int_0^\infty p_{0,s}c(s)ds + \int_0^\infty p_{0,s}T(s)ds
  \]

• Step 2: Combine with the government budget constraint:
  \[
  a(0) + \int_0^\infty p_{0,s}w(s)ds = \int_0^\infty p_{0,s}c(s)ds + D(0) + \int_0^\infty p_{0,s}G(s)ds
  \]

• Step 3: Individual assets = capital + government bonds
  \[
  k(0) + \int_0^\infty p_{0,s}w(s)ds = \int_0^\infty p_{0,s}c(s)ds + \int_0^\infty p_{0,s}G(s)ds
  \]

• Find: Taxes and government bonds cancel out.

  * PV of consumption = Private resources – PV of public spending

• Interpretation:
  1. **Ricardian neutrality**: Given the spending path, taxes and deficits have no impact on individual decisions.
  2. **Ricardian equivalence**: Tax-financed and deficit-financed government spending have equivalent effects on individual decisions.
  3. **Government bonds should NOT be considered net wealth** (Barro)
• Practical relevance: Are budget deficits really irrelevant?

  1. Good intuition – taxpayers own the government debt.
     Caveats about tax distortions, finite lives, credit market imperfections, uncertainty about incidence of future taxes, etc. (later)

  2. Property of representative agent models: Accept or use different model.

  3. If income effects of taxation cancel out, the substitution effects remain
     => Shift emphasis to tax distortions, incentive effects.

  **Fiscal Policy III: Tax Incentives**

• Example: Suppose taxes are imposed on capital income.

  - Individual savings: \( \dot{a} = w(t) + (1 - \tau(t)) \cdot r(t) \cdot a(t) - c(t) \)
    
    => Optimality condition: \( \frac{\dot{c}}{c} = \frac{1}{\theta(c)} [r(1 - \tau) - \rho] \)

  - Steady state conditions:
    \[
    \begin{align*}
    \dot{k} &= 0 \quad \Rightarrow \quad c^* = f(k^*) - \delta k^* - G^* \quad \text{where} \quad G^* = \tau[f'(k^*) - \delta]k^* \\
    \frac{\dot{c}}{c} &= 0 \quad \Rightarrow \quad (1 - \tau)[f'(k^*) - \delta] = \rho
    \end{align*}
    \]

    - Phase diagrams: \( k^* \)-line shifts to the left; \( c(k) \) as before.
      => Taxes with incentive effects do matter.

• Note on decentralization: When taxes are distortionary, market equilibrium is not Pareto-optimal
  => Cannot use planning solutions to compute the equilibrium.
Learning Objectives for Fiscal Policy

• Applied: Ability to do fiscal policy applications
  - Determine the impact of government spending in the optimal growth model.
    
  *Problem sets for practice.*

• Conceptual:
  - Know the impact of government spending in the optimal growth model.
  - Know when Ricardian neutrality applies.
  - Know that distortionary taxes create complications.