22 Repeated Games and Reputation

1.

(U, L) can be supported as follows. If player 2 defects ((U,M) is played) in the first period, then the players coordinate on (C, R) in the second period. If player 1 defects ((C, L) is played) in the first period, then the players play (D, M) in the second period. Otherwise, the players play (D, R) in the second period.

2.

(a) To support cooperation, δ must be such that $2/(1-\delta) \geq 4+\delta/(1-\delta)$. Solving for δ, we see that cooperation requires $\delta \geq 2/3$.

(b) To support cooperation by player 1, it must be that $\delta \geq 1/2$. To support cooperation by player 2, it must be that $\delta \geq 3/5$. Thus, we need $\delta \geq 3/5$.

(c) Cooperation by player 1 requires $\delta \geq 4/5$. Player 2 has no incentive to deviate in the short run. Thus, it must be that $\delta \geq 4/5$.

3.

(a) The Nash equilibria are (B,X) and (B,Y).

(b) Yes. Player 1 plays A in period 1 and B in period 2. Player 2 plays X in period 1. In period 2, player 2 plays X if player 1 played A in period 1, and plays Y if player 1 played B in period 1.

4.

In period 2, subgame perfection requires play of the only Nash equilibrium of the stage game. As there is only one Nash equilibrium of the stage game, selection of the Nash equilibrium to be played in period 2 cannot influence incentives in period 1. Thus, the only subgame perfect equilibrium is play of the Nash equilibrium of the stage game in both periods. For any finite T, the logic from the two period case applies, and the answer does not change.

5.

Alternating between (C, C) and (C, D) requires that neither player has the incentive to deviate. Clearly, however, player 1 can guarantee himself at least 2 per period, yet he would get less than this starting in period 2 if the players alternated as described. Thus, alternating between (C,C) and (C,D) cannot be supported.

On the other hand, alternating between (C,C) and (C,D) can be supported. Note first that, using the stage Nash punishment, player 2 has no incentive to deviate in odd or even periods. Player 1 has no incentive to deviate in even periods, when (D, D) is supposed to be played. Furthermore, player 1 prefers not to deviate in an even period if

$$7 + \frac{2\delta}{1-\delta} \leq 3 + 2\delta + 3\delta^2 + 2\delta^3 + 3\delta^4 + \dots,$$

which simplifies to

$$7 + \frac{2\delta}{1-\delta} \leq \frac{3 + 2\delta)}{1-\delta^2}.$$

Solving for δ yields $\delta \geq \sqrt{\frac{4}{5}}$.

6.

A long horizon ahead.

7.

(a) The (pure strategy) Nash equilibria are (U, L, B) and (D, R, B).

(b) Any combination of the Nash equilibria of the stage game are subgame perfect equilibria. These yield the payoffs (8, 8, 2), (8, 4, 10), and (8, 6, 6). There are two other subgame perfect equilibria. In the first, the players select (U, R, A) in the first round, and then if no one deviated, they play (D, R, B) in the second period; otherwise, they play (U, L, B) in the second period. This yields payoff (9, 7, 10). In the other equilibrium, the players select (U, R, B) in the first round and, if player 2 does not cheat, (U, L, B) in the second period; if player 2 cheats, they play (D, R, B) in the second period. This yields the payoff (8, 6, 9).

8.

(a) Player 2^t plays a best response to player 1's action in the stage game.

(b) Consider the following example. There is a subgame perfect equilibrium, using stage Nash punishment, in which, in equilibrium, player 1 plays T and player 2^t plays D.

1 \ 2	E	D
T	3, –1	6, 0
A	5, 5	7, 0

(c) Consider, for example, the prisoners' dilemma. If only one player is a long-run player, then the only subgame perfect equilibrium repeated game will involves each player defecting in each period. However, from the text we know that cooperation can be supported when both are long-run players.

9.

(a) As $x < 10$, there is no gain from continuing. Thus, neither player wishes to deviate.

(b) If a player selects S, then the game stops and this player obtains 0. Since the players randomize in each period, their continuation values from the start of a given period are both 0. If the player chooses C in a period, he thus gets an expected payoff of $10\alpha - (1 - \alpha)$. Setting this equal to 0 (which must be the case in order for the players to be indifferent between S and C) yields $\alpha = 1/11$.

(c) In this case, the continuation value from the beginning of each period is αx. When a player selects S, he expects to get αz; when he chooses C, he expects $10\alpha + (1 - \alpha)(-1 + \delta\alpha x)$. The equality that defines α is thus $\alpha z = 10\alpha + (1 - \alpha)(-1 + \delta\alpha x)$.

23 Collusion, Trade Agreements, and Goodwill

1.

(a) Consider all players selecting $p_i = p = 60$, until and unless someone defects. If someone defects, then everyone chooses $p_i = p = 10$ thereafter.

(b) The quantity of each firm when they collude is $q^c = (110 - 60)/n = 50/n$. The profit of each firm under collusion is $(50/n)60 - 10(50/n) = 2500/n$. The profit under the Nash equilibrium of the stage game is 0. If player i defects, she does so by setting $p_i = 60 - \varepsilon$, where ε is arbitrarily small. Thus, the stage game payoff of defecting can be made arbitrarily close to $2,500$.

To support collusion, it must be that $[2500/n][1/(1-\delta)] \geq 2500+0$, which simplifies to $\delta \geq 1 - 1/n$.

(c) Collusion is "easier" with fewer firms.

2.

(a) The best response function of player i is given by $BR_i(x_j) = 30+x_j/2$. Solving for equilibrium, we find that $x_i = 30+\frac{1}{2}[30+\frac{x_i}{2}]$ which implies that $x_1^* = x_2^* = 60$. The payoff to each player is equal to $2,000 - 30(60) = 200$.

(b) Under zero tariffs, the payoff to each country is 2,000. A deviation by player i yields a payoff of $2,000 + 60(30) - 30(30) = 2,900$. Thus, player i's gain from deviating is 900. Sustaining zero tariffs requires that

$$\frac{2000}{1-\delta} \geq 2900 + \frac{200\delta}{1-\delta}.$$

Solving for δ, we get $\delta \geq 1/3$.

(c) The payoff to each player of cooperating by setting tariffs equal to k is $2000 + 60k + k^2 - k^2 - 90k = 2000 - 30k$. The payoff to a player from unilaterally deviating is equal to

$$\begin{aligned} &2,000 + 60\left[30+\tfrac{k}{2}\right] + \left[30+\tfrac{k}{2}\right]k - \left[30+\tfrac{k}{2}\right]^2 - 90k \\ &= 2,000 + \left[30+\tfrac{k}{2}\right]^2 - 90k. \end{aligned}$$

Thus, the gain to player i of unilaterally deviating is

$$\left[30+\frac{k}{2}\right]^2 - 60k.$$

In order to support tariff setting of k, it must be that

$$\left[30 + \frac{k}{2}\right]^2 - 60k + \frac{200\delta}{1-\delta} \leq \frac{[2000 - 30k]\delta}{1-\delta}.$$

Solving yields the condition

$$\frac{[30 + \frac{k}{2}]^2 - 60k}{1800 - 90k + [30 + \frac{k}{2}]^2} \leq \delta.$$

3.

The Nash equilibria are (A, Z) and (B, Y). Obviously, there is an equilibrium in which (A, Z) is played in both periods and player 2^1 sells the right to player 2^2 for 8α. There is also a "goodwill" equilibrium that is like the one constructed in the text, although here it may seem undesirable from player 2^1's point of view. Players coordinate on (A, X) in the first period and (A, Z) in the second period, unless player 2^1 deviated from X in the first period, in which case (B,Y) is played in the second period. Player 2^1 sells the right to player 2^2 for 8α if he did not deviate in the first period, whereas he sells the right for 4α if he deviated. This is an equilibrium (player 2^1 prefers not to deviate) if $\alpha > 3/4$.

4.

(a) Each player 2^t cares only about his own payoff in period t, so he will play D. This implies that player 1 will play D in each period.

(b) Suppose players select (C, C) unless someone defects, in which case (D, D) is played thereafter. For this to be rational for player 1, we need $2/(1-\delta) \geq 3 + \delta/(1-\delta)$ or $\delta \geq 1/2$. For player 2^t, this requires that $2 + \delta p^G \geq 3 + \delta p^B$, where p^G is the price he gets with a good reputation and p^B is the price he gets with a bad reputation. (Trade occurs at the beginning of the next period, so the price is discounted). Cooperation can be supported if $\delta(p^G - p^B) \geq 1$.

Let α be the bargaining weight of each player2^t in his negotiation to sell the right to player 2^{t+1}. We can see that the surplus in the negotiation between players 2^t and 2^{t+1} is $2 + \delta p^G$, because this is what player 2^{t+1} expects to obtain from the start of period $t+1$ if he follows the prescribed strategy of cooperating when the reputation is good. This surplus is divided according to the fixed bargaining weights, implying that player 2^t obtains $p^G = \alpha[2 + \delta p^G]$. Solving for p^G yields $p^G = 2\alpha/(1-\delta\alpha)$. Similar calculations show that $p^B = \alpha/(1-\delta\alpha)$. Substituting this into the condition $\delta(p^G - p^B) \geq 1$ and simplifying yields $\delta\alpha \geq 1/2$. In words, the discount factor and the owner's bargaining weight must be sufficiently large in order for cooperation to be sustained over time.

5.

(a) The Nash equilibria are (x, x), (x, z), (z, x), and (y, y).

(b) They would agree to play (y,y).

(c) In the first round, they play (z, z). If no one defected in the first period, then they are supposed to play (y, y) in the second period. If player 1 defected in the first period, then they coordinate on (z, x) in the second period. If player 2 defected in the first period, then they coordinate on (x, z) in the second period. It is easy to verify that this strategy is a subgame perfect equilibrium.

(d) The answer depends on whether one believes that the players' bargaining powers would be affected by the history of play. If deviation by a player causes his bargaining weight to suddenly drop to, say, 0, then the equilibrium described in part (c) seems consistent with the opportunity to renegotiate before the second period stage game. Another way of interpreting the equilibrium is that the prescribed play for period 2 is the *disagreement point* for renegotiation, in which case there is no surplus of renegotiation. However, perhaps a more reasonable theory of renegotiation would posit that each player's bargaining weight is independent of the history (it is related to institutional features) and that each player could insist on some neutral stage Nash equilibrium, such as (x, x) or (y, y). In this case, as long as bargaining weights are positive, it would not be possible to sustain (x, z) or (z, x) in period 2. As a result, the equilibrium of part (c) would not withstand renegotiation.

6.

(a) If a young player does not expect to get anything when he is old, then he optimizes myopically when young and therefore gives nothing to the older generation.

(b) If player $t-1$ has given $x_{t-1} = 1$ to player $t-2$, then player t gives $x_t = 1$ to player $t-1$. Otherwise, player t gives nothing to player $t-1$ ($x_t = 0$). Clearly, each young player thus has the incentive to give 1 to the old generation.

(c) Each player obtains 1 in the equilibrium from part (a), 2 in the equilibrium from part (b). Thus, a reputation-based intergenerational-transfer equilibrium is best.

7.

(a) Any δ.

(b) $\delta \geq \frac{3}{7}$.

(c) $m = \frac{4}{3(1-\delta)}$.

8.

(a) Cooperation can be sustained for $\delta \geq \frac{2}{3}$.

(b) Cooperation can be sustained for $\delta \geq \frac{k}{k+1}$.

(c) Cooperation can be sustained for $\delta \geq \frac{4(k-2)!}{4(k-2)!+k!}$.