Economics 171: Midterm Exam

Question 1: Basic Concepts (12 points)

Define and briefly explain the following terms (4 points each):

(a) Dominant strategy – *A strategy that always does better than every other possible strategy, no matter what the other players do.*

(b) Best response – *A strategy that, given some belief about the choices of the other player(s), yields the best result that a player can achieve (can be ties)*

(c) Nash equilibrium – *A strategy profile under which no player can do better with a unilateral deviation; a strategy profile consisting of mutual best responses.*

Question 2: Evaluating payoffs (18 points)

Evaluate the following payoffs for this game:

		Player 2	
		L	R
Player 1	A	2, 1	3, 7
	B	-3, 0	4, 4
	C	0, 9	6, -5
	D	8, 11	5, 5

(a) (4 points) $u_1(\sigma_1,R)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$. (In words, this means "What is the expected utility for player 1 if s/he plays a mixed strategy of A 1/4 of the time, B 1/4 of the time, C 1/4 of the time, and D 1/4 of the time and player 2 plays R?")

1/4(3 + 4 + 6 + 5) = 18/4 = 9/2*

(b) (4 points) $u_2(\sigma_1,L)$ for $\sigma_1 = (1/9, 1/3, 1/3, 2/9)$.

*1/9*1 + 1/3*(0) + 1/3*9 + 2/9*11 = 50/9*

(c) (5 points) $u_1(\sigma_1,\sigma_2)$ for $\sigma_1 = (1/4, 1/4, 1/4, 1/4)$, $\sigma_2 = (1/2, 1/2)$.

1/8(2 + (-3) + 0 + 8 + 3 + 4 +6 + 5) = 25/8*

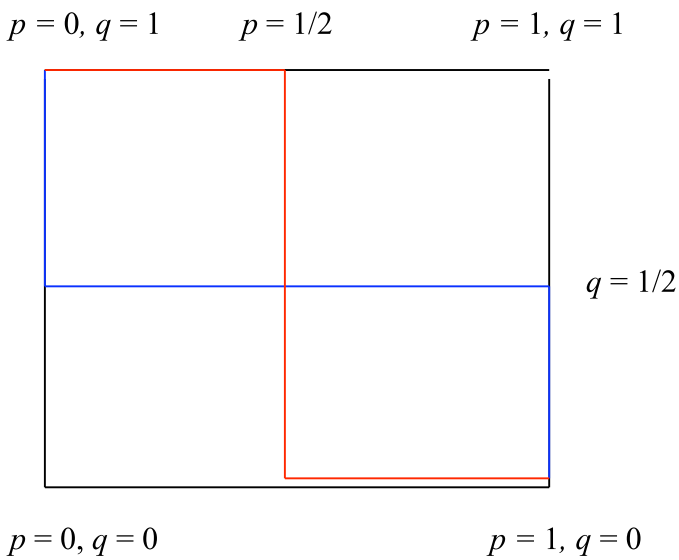
(d) (5 points) $u_2(\sigma_1,\sigma_2)$ for $\sigma_1 = (1/7, 2/7, 4/7, 0)$, $\sigma_2 = (5/9, 4/9)$.

*5/63*1 + 4/63*7 + 10/63*0 + 8/63*4 + 20/63*9 + 16/63*(-5) = 165/63 = 55/21*

Question 3: Best response graph (20 points)

		Player 2	
		L	R
Player 1	U	0, 0	1, 2
	D	1, 2	0, 0

(12 points) Draw the best response functions for Player 1 and Player 2, assigning *p* as the probability that Player 1 chooses U and *q* as the probability that Player 2 chooses D:



(6 points) Where and how many times do these best response functions intersect? What do we call such intersections?

Here the blue line segments represent the best-response function for player 1, while the red line segments represent the best-response function for player 2.

There are three intersections, at (p, q) = (0, 1), (1,0), and (1/2, 1/2). These are the two pure-strategy Nash equilibria and the mixed-strategy Nash equilibrium.

Question 4: Stackelberg competition (20 points)

Stackelberg competition is similar to Cournot competition, except that one of the firms chooses a quantity to produce before the other firm does, the second firm observes the first firm's production choice and then makes a production choice, the first firm knows that the second firm observes the first firm's production choice and then makes a production choice, etc.

Let firm 1 be the firm that produces first (the Stackelberg *leader*) and firm 2 be the firm that produces last (the Stackelberg *follower*). Let q_1 denote firm 1's quantity and q_2 denote firm 2's quantity, where each of q_1 and q_2 is non-negative; thus, the industry's total output is $q_1 + q_2$.

Suppose the price is given by the function p = 2400 - q_1 - q_2, and that there is no cost of production. Note that the profit for each firm is given by that firm's production multiplied by the industry price given by the formula above, and assume that each firm is interested only in maximizing its own profit.

What quantity does each firm produce in equilibrium? (Hint: Think about the firms' best-response functions; in particular, realize that firm 1 can derive firm 2's best-response function and will choose a quantity that maximizes its own profits given this best-response function).

*Firm 1 knows that if it chooses quantity q_1, firm 1 will wish to maximize: Profit = Revenue = Price*Quantity produced = $(2400 - q_1 - q_2)*q_2$*

*Taking the derivative with respect to q_2 yields $2400 - q_1 - 2q_2$; at the maximum, this equals 0, so $q_2 = 1200 - q_1/2$. This is firm 2's best response function. Taking this into account, firm 1 wishes to choose q_1 to maximize: Profit = Revenue = Price*Quantity produced = $(2400 - q_1 - q_2)*q_1 = (2400 - q_1 - (1200- q_1/2)*q_1 = (1200 - q_1/2)*q_1$*

Taking the derivative with respect to q_1 yields $1200 - q_1$; at the maximum, this equals 0, so $q_1 = 1200$. Since $q_2 = 1200 - q_1/2$, we know that $q_2 = 600$.

Question 5: Iterated elimination of dominated strategies, pure-strategy and mixed-strategy Nash equilibrium (30 points)

Consider this game, where the numbers in each cell represent the utilities for Player 1 and Player 2, respectively:

		Player 2			
		E	F	G	H
Player 1	A	2, 20	2, 20	2, 20	2, 110
	B	9, 18	9, 30	14, 70	110, 2
	C	10, 12	12, 60	110, 50	110, 2
	D	16, 18	0, 15	10, 10	10, 2

(a) (10 points) Which pure strategies (A, B, C, D, E, F, G and/or H) survive the iterated deletion of strictly dominated strategies? Explain fully the sequence you used for your iterated elimination, including specifying the probabilities involved in any cases where a mix of two pure strategies is used to eliminate a third pure strategy.

1. *First note that strategy A is strictly dominated by strategy B (or strategy C), so we can eliminate it from consideration.*
2. *Once this is eliminated, note that strategy H is dominated by strategies E, F, and G, so it can be eliminated.*

3. *After eliminating H, strategy B is dominated by strategy C.*
4. *Once strategy B is eliminated, strategy G is dominated by strategy F, so we can eliminate it.*
5. *We can't do any further elimination, so the strategies that survive are C, D, E, and F.*

(b) (10 points) What are the Nash equilibria in this game, both pure and mixed (in the case of a mixed-strategy equilibrium, specify the probabilities that each strategy is played in the mix)?

1. *Given that only strategies C, D, E, and F survive, we can examine the reduced game. We see that there are two pure-strategy Nash equilibria, (C, F) and (D, E).*
2. *There is also a mixed-strategy Nash equilibrium; to solve for it, let p be the probability that Player 1 plays C and let q be the probability that Player 2 plays E. Then, we have that 10q + 12(1-q) = 16q and 12p + 18(1-q) = 60p + 15(1-p). Solving, we have p = 1/17 and q = 2/3.*

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(c) (10 points) For every Nash equilibrium, what is the utility or expected utility for each player? If the players could write a binding contract involving side payments, what do you think would be the outcome (strategy profile) played in this game?

*The pure-strategy Nash equilibria are trivially easy: With (C, F), player 1 gets 12 and Player 2 gets 60; with (D, E), player 1 gets 16 and Player 2 gets 18. For the mixed-strategy Nash with p = 1/17 and q = 2/3, the expected utility for Player 1 is 1/17*2/3*10 + 1/17*1/3*12 + 16/17*2/3*16 + 16/17*1/3*0 = 20/51 + 12/51 + 512/51 + 0 = 544/51 = 32/3, and the expected utility for Player 2 is 1/17*2/3*12 + 1/17*1/3*60 + 16/17*2/3*18 + 16/17*1/3*15 = 24/51 + 60/51 + 576/51 + 240/17 = 900/51 = 300/17.*

If the players could make a binding contract and make side payments, I would expect them to choose the strategy profile (C, G), as this offers the highest joint payoff, 160. There would be a range of side payments that would make this a better choice for both players than any of the Nash equilibria without side payments.