PROBLEM SET 2 - ANSWERS

5.3

(a) u_1(□$_1$,I) = 1/4(2) + 1/4(2) +1/4(4) +1/4(3) = 11/4
(b) u_1(□$_1$,O) = 21/8
(c) u_1(□$_1$, □$_2$) = 1/4(2) + 1/4(2) +(1/4)(1/3)(4) +1/4(2/3) +3/4(1/3) +1/4(2/3) = 23/12
(d) 7/3

6.4

(a) First, we find the expected payoff to each strategy: u_1(U, □$_2$) = 2/6(2) + 0 + 1/2(4) = 7/3; u_1(M, □$_2$) = 1/6(3) + 0 + 1/2 = 1; u_1(D, □$_2$) = 1/6 + 1 + 1 = 13/6. As the strategy U yields the highest expected payoff to player 1, given □$_2$, BR_1(□$_2$) = {U}.
(b) BR_2(□$_1$) = {R}.
(c) BR_1(□$_2$) = {U}.
(d) BR_1(□$_2$) = {U,D}.
(e) BR_2(□$_1$) = {L,R}.

6.5

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

(a) BR_1(□$_2$) = {P}.
(b) BR_1(□$_2$) = {R}.
(c) BR_1(□$_2$) = {P}.
(d) BR_1(□$_2$) = S_1, the full strategy space for player 1.

7.4

For “give in” to be rationalizable, it must be that x □ 0. The manager must believe that the probability that the employee plays “settle” is (weakly) greater than 1/2.

7.6

R = {7:00, 6:00, 6:00)}

7.7

Yes. If s_1 is rationalizable, then s_2 is a best response to any player 1 strategy that may rationally be played. Thus, player 2 can rationalize strategy s_2.

Additional problem 1:

(a) All; (U,L) and (D,R) + mix with p = 3/4 and q = 2/9.
(b) (D,R); only (D,R)
(c) All; all
(d) (D,R); only (D,R)
(e) All; (U,L) and (D,R) + mix with p = 3/7 and q = 7/9.
(f) All; (U,L), (U,R), and (D,R) + mixes. There is no mix of U and D possible if there is strictly positive weight on L. However, all mixes involving R are Nash equilibria.
(g) All; (U,L), (U,R), and (D,R) + mixes. There is no mix of U and D possible if there is strictly positive weight on L. However, all mixes involving R are Nash equilibria.
(h) All; No pure-strategy Nash equilibrium, only a mix with p = q = 1/2.
(i) (U,R); only (U,R)
(j) U is dominated by 1/2 M, 1/2 D. After eliminating U, R is dominated by 1/3 L, 2/3 C. We can't reduce this further, so (M,L), (M,C), (D,L), and (D,C) are rationalizable; (M,L) and (D,C) are pure-strategy NEs. There's a mix, with prob(M) = 14/15 and prob(L) = 1/2.
(k) (M,C); (M,C)

Additional problem 2

a) {A,B,C,D}. Every player 1 strategy is a best response to some pure strategy of player 2. E.g., A = BR_1(H), B = BR_1(G), etc.
b) H is dominated by E, so a rational player 2 would never play H. Knowing this, player 1 would never play A because it is dominated. Therefore the consistent set is {B,C,D}.
c) Because player 2 knows that player 1 is rational, player 2 knows that player 1 will never play a strategy outside {A,B,C,D} from part a). Player 1 is not assumed to know that player 2 knows that player 1 is rational, so player 2 cannot be assumed to be able do the next step of iterated elimination; thus, the consistent set remains {B,C,D}.
d) Because player 2 knows that player 1 knows that player 2 is rational, player 2 knows that player 1 will not choose a strategy outside of the set {B,C,D} from part b). In the new game {B,C,D}||{E,F,G}, G is dominated by E. Therefore C now dominates B, so the consistent set is {C,D}.
e) Again, as in part c), knowledge that player 1 is rational has no additional bite, so the consistent set remains {C,D}.

f) Now player 2 knows that player 1 will not choose a strategy outside {C,D} from part d). In the new game {C,D}||{E,F}, F is dominated by E, and therefore player 1 chooses D. So the only rationalizable strategy for player 1 is {D}.

Additional problem 3

(i) If a < 0 and c > 0, then player 1 will for sure choose D.

(ii) If a > 0 and b > 0, and *either* c < 0 or d < 0. (It's not necessary that U and L both be strictly dominant strategies in the original game. Only one need be.)

(iii) 0 □ a,b,c,d

(iv) The mixed-strategy Nash equilibrium involves U played with probability d/(b+d), D played with probability b/(b+d), L played with probability c/(a+c), and R played with probability a/(a+c).

(v) Two answers (courtesy of Matthew Rabin): a,b > 0 and c = d = 0, or [(0 □ a, c > 0) or (0 □ c, a > 0)] and b > 0.